

COGNITIVE PROCESSES ASSOCIATED WITH THE PROFESSIONAL DEVELOPMENT OF THE MATHEMATICS TEACHER

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This paper proposes a model of professional development based on Sfard's stages of interiorisation, condensation, and reification, which highlights the teacher's cognitive processes. The model is applied to the case study of a primary teacher participating in a collaborative project for professional development. This adaptation of Sfard's stages proves to be of special value when interpreting the process of the teacher's professional development from a cognitive perspective.

Keywords: Cognitive model; Collaborative context; In-service primary teacher; Professional development

Procesos Cognitivos Asociados al Desarrollo Profesional del Profesor de Matemáticas

Este artículo propone un modelo de desarrollo profesional que pone de relieve los procesos cognitivos del profesor, en relación con las fases de interiorización, condensación y cosificación del trabajo de Sfard. El modelo se aplica al caso de una maestra que participa en un proyecto colaborativo de desarrollo profesional. La adaptación de las fases de Sfard se muestra valiosa para interpretar el proceso de desarrollo profesional de la maestra desde una perspectiva cognitiva.

Términos clave: Desarrollo profesional; Entorno colaborativo; Formación continua; Modelo cognitivo

In studies of professional development and teacher training, teachers have been considered from various perspectives. Our interest lies in the processes involved in generating teachers' knowledge, and builds on Brown and Borko's (1992) description of professional development in which the teacher is seen as an apprentice. These researchers take "a view of the teacher as an adult learner whose de-

Muñoz-Catalán, M. C., Climent, N., Carrillo, J., & Contreras, L. C. (2010). Cognitive processes associated with the professional development of the mathematics teacher. *PNA*, 4(3), 87-97.

velopment results from changes in cognitive structures; these cognitive structures... are the thinking patterns by which a person relates to the environment.” (p. 227)

From this perspective, we understand that the teacher learns in contact with their peers through a consensual process involving their personal conceptual schema, beliefs and motivations, in which language and communication play a fundamental role. In this respect we coincide with the considerations of ontology, epistemology and learning theory characteristic of social constructivism (Ernest, 1996). We share the view that the individual and the group are interconnected and knowledge is built as part of the social process (Carrillo, Climent, Gorgorió, Rojas, & Prat, 2008). In this paper, however, discussion is focused on a model which takes account of the cognitive processes implemented by the teacher in the course of professional development, highlighting the influence of the group on those processes, for which purpose we refer to Sfard (1991).

TOWARDS A MODEL OF PROFESSIONAL DEVELOPMENT

Sfard states that acquisition of new mathematical notions usually begins with an operative conception of the notion, and that the transition from computational operations to abstract objects is a difficult process requiring three stages: (a) interiorisation, (b) condensation and (c) reification.

Sfard uses *conception* to refer to the internal representations and associations evoked by a concept, reserving this latter term (synonym of *notion*) for a mathematical idea expressed in its conventional form.

Although it would be a mistake to identify the nature of mathematical understanding with that of professional understanding, or the process of abstracting a mathematical notion from operative to structural conception with the process of acquiring professional knowledge, we believe that the process of professional development does share some degree of parallelism with the process of abstraction which is involved in moving from low level work —interiorisation— to higher level work —reification— (from a cognitive point of view). Our aim, however, is to adapt Sfard’s stages to the professional development of teachers, and then to particularise them to the case of a collaborative context (Feldman, 1993).

It is not possible to maintain the differentiation between the conception of the notion (in this case the specific professional issues) as process and as object. Extending the definitions above to the case of professional development, the key to moving from one stage to another would lie in the maturity (understood as the increasing complexity of reflection upon the phenomenon of education, or as the assimilation of elements which deepen professional knowledge) with which the issue in question was approached. Hence, as we will see in the example in the next section, the interiorisation stage is characterised by familiarity with the is-

sue, and is most probably initiated with the analysis of similar situations. It is characterised by the teacher mulling over something that does not seem quite right or that they feel could be improved. This may be accompanied by specific solutions (to these particular circumstances, that is, without making generalisations about the underlying issue or perceiving its wider dimensions), which themselves may or may not be put into practice.

The condensation stage involves freeing oneself from the particular and seeing the issue as something more general, which means introducing a new variable into mapping the terrain of professional practice. It can be seen not only when a teacher reflects on their practice prospectively (that is, when planning) or retrospectively, but also while activities are being put into practice and decisions are being taken about them.

The reification stage would add to the above an understanding of the issue in its complexity (relative to the level at which the teacher is operating, one cannot speak of an “absolute” understanding), along with its relations and derivations.

We can thus imagine the teacher progressing along a kind of professional development helix, where the role of the teacher trainer becomes that of providing learning contexts which will further the progress. The contents of the helix have been thoroughly studied and range from teachers’ everyday activities to professional knowledge (Hill, Ball, & Schilling, 2008; Shulman, 1986). This professional knowledge can partly be seen deployed in situations directly related to the classroom (e.g., planning). The helix also involves direction, defined by the three stages of interiorisation, condensation and reification. Finally, the helix has a specific shape, given to it by reflection. Reflection is both content and generatrix of the helix —content in that we have included it within professional knowledge, taking reflexive practice as reference—; generatrix in that it is the means by which the helix is created. Reflection likewise allows progress through the three stages mentioned above, and in its turn leaves a trace in the contents of professional knowledge.

JULIA’S DEVELOPMENT VIEWED THROUGH THE LENS OF THE CONTINUOUS HELIX MODEL

In the study and promotion of professional development, collaborative contexts (involving teachers and researchers) have shown themselves to be especially appropriate (Llinares & Krainer, 2006). Climent and Carrillo (2003) and Climent (2005) analyse the professional development of primary teachers participating in a collaborative research project (PIC: Proyecto de Investigación Colaborativa), in terms of both reflection on practice (for which the PIC has proved a fruitful context) and teacher training.

The PIC started in 1999 and is made up of two experienced primary teachers, a novice teacher (Julia), a novice and two experienced researchers. Its work pays

special attention to the participants' reflections on their conceptions of their pedagogical content knowledge concerning school mathematics, among other issues, in an atmosphere of co-operation in which discussion and reflection play a vital role.

Julia showed great interest in forming part of the project, as she considered it an opportunity to continue her training with other teaching professionals. We collected information during two years, in which Julia acted for the first time as group tutor (to 6-year-old pupils). We used a wide variety of data collection techniques in two contexts: Julia's classes and the PIC; chief amongst these were her classroom diaries, interviews, classroom recordings and recordings of the PIC sessions.

In this paper we focus on a single aspect of her practice: planning. It was highly significant that initially Julia considered each day's plan as a rigid document which had to be followed to the letter, independently of local factors (the tiredness of the students, particular difficulties, etc). Due in large part to her individual reflection and the group reflection in the PIC, Julia began to consider the planning stage in a more flexible light and was able to foresee such difficulties at this stage and to incorporate ideas to deal with them. We briefly describe this process below, highlighting the key role that reflection played as a force for moulding her professional development.

Interiorisation

Julia approached her teaching in conformity with the culture at her school, where teaching was traditional, with minimal use made of manual exercises and a high degree of reliance on the textbook. From the start, Julia used the textbook as the principal source of her teaching material, showing reluctance to leave a section half done, even when her pupils showed clear signs of fatigue. When she started the second teaching unit, she began to keep a class diary, a practice that she continued into the following unit.

Analysing the diary entries for the instances where her pedagogical sensibility was overridden by her desire to complete a particular section, we noted that her personal reflections showed little potential for change. They did not seek to prioritise and select key sections from the textbook, or to modify this in any way, but remained at the level of sequencing the activities in terms of difficulty or conceptual demands so as to improve the pupils' chances of getting through them. In short, the diary entries only allowed her to become aware of her difficulties and to consider ideas for improvements which never materialised. However, they do indicate that such issues were beginning to stir in Julia, awakening in her certain dissatisfaction.

It was the collective reflection in one of the group sessions analysing a video of her teaching which contributed a new outlook. At the beginning of the lesson, Julia asked the pupils, on the spur of the moment, to accurately define a rectangle. Such were the demands of this task that she spent nearly an hour trying to

get the pupils to deduce from a series of examples the defining features of a rectangle —such as parallel or perpendicular sides— even though they were clearly feeling extremely tired. In her subsequent reflection, in both her diary and the PIC, Julia acknowledged the pupils' intuitive understanding of the concept, but she didn't consider leaving the activity unfinished, because she wanted them "to be a little bit more precise... because I think that they generalise too much". The response of one of the experienced teachers in the PIC to this rationale was especially interesting:

Inés: You don't think it was because you wanted to get somewhere and you saw that they weren't getting there?...

Julia: There was a moment when I saw it was too much for them.

Inés: Why do they have to get to that point?

Julia: Because it doesn't seem right to me not to finish things...

Inés: ... No, what happens, Julia, is that sometimes we get involved in something and we have to... know how to go back over something, in the sense of saying, "OK, for whatever reason, this isn't working out, and it's OK if this doesn't work out," and you say, "OK, we'll have another go at this tomorrow." ... because... the one who goes away with the sense that things haven't been finished is you, but not the children.

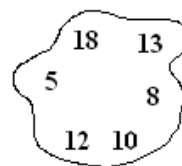
Reflection with other professionals helped Julia become more fully aware of what her decisions were aiming to achieve, and of the importance of noting the pupils' reactions and responding to them. Her improved understanding of the situation provided Julia with new elements of judgment which would later be useful for facing situations from another perspective. In the case described above, in which the pupils' difficulties condition the course of the planning, we can say that Julia was undergoing an interiorisation phase, in that it was now that she was becoming aware that such situations could arise in the classroom and that there were various ways of responding to them.

Condensation

At the end of the first year, Julia conducted four problem-solving activities on number decomposition, which had been designed in the PIC. Below we describe the implementation of the fourth of these, called "The Same, Bigger, or Smaller", and for which Julia had no teaching notes to follow (Figure 1).

Select the numbers which add up to:

- 27
- 36
- 43



A) Is the addition of 23 and 15 greater than 20? Than 30? Than 40? 50?
(Repeat with less than)

B) Is the addition of 23 and 32 greater than 20? Than 30? Than 40? 50?
(The same with less)

C) Is the addition of 19 and 32 greater than 20? Than 30? Than 40? 50?
(The same with less)

Figure 1. “The Same, Bigger, or Smaller”

At the preparation stage, Julia predicted various difficulties that could arise, stemming principally from her students’ limited familiarity with making estimations: “... there will be some who, without meaning to, or thinking about it, will try to add up all the combinations of numbers and this makes it a really laborious task for them”. Once in class, Julia looked very doubtful, foreseeing the difficulties. She started the explanation without making overt mention of specific numbers and writing symbols instead. Then, she decided to introduce some numerical examples. By means of leading questions, she focused the pupils’ attention on the numbers which were grouped together, with the aim that these became meaningful to them and they realised which were the smallest and the largest. She also drew their attention to cases in which it was not possible to express them as the sum of three others, such as 15 (which can be obtained only with two numbers: 10 and 5). Next, taking three numbers to be added together from the group (12, 10 and 13), she gave 35 as an example, and explained the strategy of estimation which they could follow in order to find out the numbers required to be added together and so solve the problem.

At first, the work was done individually. It was noticeable that many pupils found it hard to concentrate. In the feedback phase, she was interested not only in the results, but in the reasoning followed. She tended to select the more able pupils. In the second phase of the activity, the whole class worked together, with the pupils having to do mental calculations so as to estimate the result of the addition of two numbers. She frequently had to ask pupils to pay attention because they seemed tired (the previous activity was 40 minutes long) and she tried to keep them participating, but in the end it became very difficult to continue. Finally, Julia worked through all three estimations, but went through the last two more quickly.

Although she managed to do the three estimations, her impression was that of having left the activity half way: “I see that the problem is that they normally solve almost everything successfully, and this time it began to be a bit frustrating... and we had to finish earlier than expected.” (PIC session)

This episode is representative of the condensation stage. Her decision not to continue in the way that had been foreseen would seem to indicate that Julia was more inclined to take into account the pupils’ learning difficulties and to adapt her original plan to meet them. Also significant is that this decision was made while the class was in progress (favoured to a certain extent by her previous reflection), whereas at the interiorisation level Julia only arrived at such a realisation after the event.

She was beginning to view her planning with a certain degree of flexibility, and to be able to explain her decisions openly in the PIC. It seems the joint reflection had a significant influence, providing her with the necessary pedagogic support to take decisions like these.

Reification

One year later, the possibility of Julia repeating the decomposition activities was discussed in the PIC. Taking into account her experience and her pupils’ current knowledge, she decided that they would be appropriate, albeit with various modifications. With regard to the activity “The Same, Bigger or Smaller”, she omitted it and selected another from an activity bank created in the PIC. Julia took this decision not only because of the feelings of failure experienced the previous year, but also as a result of questioning the rationale of the activity itself, considering that the difficulty lay in the degree of abstraction demanded, by a mental task involving numbers without reference to a specific context. Nevertheless, when she was planning the lesson, she considered the possibility of including a modified version:

I could make the numbers easier... But it was going to look like the one we did yesterday... about estimation... Maybe I’d have been wrong and I’d have got a pleasant surprise, but as last year was far from being productive, I thought, “Where’s the point of wasting an hour by repeating the experience?” (PIC session)

She choose an alternative activity, which consisted in completing four dominoes laid out in a square so that the total represented along each of the four sides always came to be the same —10— (Figure 2).

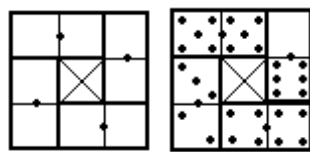


Figure 2. Example of the activity “Domino Squares”

In fact, this activity is very similar to the first part of “The Same, Bigger or Smaller” in that there is still a fixed amount (10 in the example) which is to be obtained by adding the other three numbers which are also given (from 0 to 6). The concepts involved in “The Same, Bigger, or Smaller” were focused on separately in two of the activities ultimately included in her plan. It can be noted that what had been modified was the mode of presenting these concepts, but not the concepts themselves.

We identify her reasoning with the development of the reification stage because Julia now showed herself able to consider the pupils’ learning difficulties at the actual time of planning. She makes use of her knowledge of the features of the activities and their corresponding cognitive demands to look for an alternative means of working on the mathematical contents. She had a degree of professional knowledge, with respect to activities and learners, which was also available to her during her lesson planning (and not merely an awareness of this variable). Previously, she had been making use of an activity designed by the group and acted more as a guide for what had been decided elsewhere (with a stake in the debate, certainly, but with an understandably diminished sense of her own authority given that it had been her first year as teacher and member of the group). Her greater professional understanding, then, made possible and was closely linked to the increased flexibility of her lesson planning with respect to the pupils’ difficulties.

She no longer became aware of such difficulties only if they arose, either afterwards upon reflection (as in the interiorisation stage), or during the class itself (as in the condensation stage). This behavior led her to make suggestions for improvements to subsequent lessons (although not always put into practice) in the former case, and to make slight adjustments as she went, in the second. However, at the reification stage she no longer took a reactive attitude to difficulties, but rather anticipated them in the planning process, so giving a fuller pedagogical treatment to the content. In other words, Julia successfully converted consideration of pupils’ difficulties into an element to be treated independently of the circumstances in which they arose.

Final Reflections

Each stage is defined by an “advance” in Julia’s cognitive and/or teaching dimension. The interiorisation stage supposes the deployment of the idea that the pupils’ difficulties require consideration once the class plan is in operation. It is accompanied in this case by suggestions for treatment which for the moment are

not put into action. The condensation stage treats pupils' difficulties cognitively as one more variable in the analysis of what happens in the classroom. This is reflected in the decisions made during the course of the lesson. In this case the progress is less cognitive, or unconnected to action, as in the previous stage. It is knowledge in action (Schön, 1983). In the reification stage, the potential pupils' problems are taken into account from the planning onwards; giving less importance to externally established pedagogic treatment. So it is also a case of cognition in action (at the planning phase).

We are aware that various factors influence this process of abstraction, but we highlight the context of the PIC because it has shown itself to be a meeting point which promotes and enriches reflection on classroom practice, and encourages the consideration of other variables in lesson planning (Ticha & Hospesova, 2006).

From the analysis of teachers' learning systems (Krainer, 2004), we can note that, through joint reflection with others (reflection-networking) and acting and reflecting on her own practice (action-autonomy-reflection), Julia gained additional competence and self-confidence in autonomous planning and interaction (autonomy and action) and in her ability to reflect on mathematical teaching practice and to reflect and communicate with colleagues and take advantage of their ideas (reflection and networking). There is no doubt that Julia's participation in the PIC, providing resources and principles on which she could base her decisions, was a source of valuable support, but credit must also be given to her individual reflection.

We can say that in her process of professional development, Julia completed a preliminary cycle of interiorisation-condensation-reification, with respect to one element of her teaching, that of planning. Once "reified", we consider that this new conceptualisation of planning itself undergoes a process of development, in which she would begin to take account of more and more considerations in her lesson planning and to be able to foresee more alternatives for dealing with particular elements.

REFERENCES

- Brown, C. C., & Borko, H. (1992). Becoming a mathematics teacher. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 209-239). New York: McMillan.
- Carrillo, J., Climent, N., Gorgorió, N., Rojas, F., & Prat, M. (2008). Análisis de secuencias de aprendizaje matemático desde la perspectiva de la gestión de la participación. *Enseñanza de las Ciencias*, 26(1), 67-76.
- Climent, N. (2005). *El desarrollo profesional del maestro de Primaria respecto de la enseñanza de la matemática. Un estudio de caso*. Michigan, MI: Proquest Michigan University.

- Climent, N., & Carrillo, J. (2003). Developing and researching professional knowledge with primary teachers. In J. Novotná (Ed.), *Proceedings of the Second Conference of the European Society for Research in Mathematics Education (CERME2)* (pp. 269-280). Praga, Czech Republic: Charles University.
- Ernest, P. (1996). Varieties of constructivism: a framework for comparison. In L. Steffe & P. Nesher (Eds.), *Theories of mathematical learning* (pp. 335-350). Hillsdale, NJ: Lawrence Erlbaum Associates.
- Feldman, A. (1993). Promoting equitable collaboration between university researchers and school teachers. *Qualitative Studies in Education*, 6(4), 341-357.
- Hill, H., Ball, D. L., & Schilling, S. (2008). Unpacking “pedagogical content knowledge”: Conceptualizing and measuring teachers’ topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372-400.
- Krainer, K. (2004). Theory and practice: facilitating teachers’ investigation into their own teaching: Reflection on Barbara’s teaching experiment. In M. A. Mariotti (Ed.), *Proceedings of the Third Conference of the European Society for Research in Mathematics Education (CERME3)*. Bellaria, Italy: Edizioni plus.
- Llinares, S., & Krainer, K. (2006). Mathematics (student) teachers and teacher educators as learners. In A. Gutiérrez & P. Boero (Eds.), *Handbook of research on the Psychology of Mathematics Education. Past, present and future* (pp. 429-459). Rotterdam, Holland: Sense Publishers.
- Schön, D. (1983). *The reflective practitioner: How professionals think in action*. New York, NY: Basics Books.
- Sfard, A. (1991). On the dual nature of mathematical conceptions: reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, 22(1), 1-36.
- Shulman, L. S. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.
- Ticha, M., & Hospesova, A. (2006). Qualified pedagogical reflection as a way to improve mathematics education. *Journal of Mathematics Teacher Education*, 9(2), 129-156.

This document was originally published as Muñoz-Catalán, M. C., Carrillo, J., & Climent, N. (2009). Cognitive processes associated with the professional development of the mathematics teachers. In M. Tzekaki, M. Kaldrimidou, & H. Sankonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for Psychology of Mathematics Education* (pp. 177-184). Thessaloniki, Greece: PME.

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