

Radiative corrections to the S , T , U parameters and their impact on the W boson mass in the 331 model

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We investigate radiative corrections to electroweak precision observables, specifically the Peskin-Takeuchi parameters S , T , and U , in the $SU(3)_C \times SU(3)_L \times U(1)_X$ model (also known as the 331 model) with $\beta = -\sqrt{3}$. Using the SARAH and SPheno packages, we compute the mass spectrum and low-energy observables. We show that these parameters place strong constraints on the model, requiring most of the scalar masses to lie in the TeV range or below, and impose indirect bounds on the newly predicted gauge bosons. Furthermore, we demonstrate that the model can accommodate the W boson mass anomaly reported by the CDF Collaboration, should future measurements confirm its persistence.

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I. INTRODUCTION

A precise experimental determination of the mass of the W boson can serve as a robust test for the Standard Model (SM). Any observed deviations from the predictions of the SM would provide crucial insights into new physics beyond the SM (BSM). Recently, the Collider Detector at Fermilab (CDF) Collaboration announced a new measurement for the mass of the W boson, yielding $M_W^{\text{CDF}} = 80.4335 \pm 0.0094$ GeV [1]. This measurement differs significantly from the SM prediction of $M_W^{\text{SM}} = 80.353 \pm 0.006$ GeV [2], with a significance of 7σ . The recent measurements by the CMS [3] and ATLAS [4] Collaborations are, however, surprisingly close to the SM prediction.

In addition to the discrepancy in the prediction of the W boson mass reported by the CDF, there are various theoretical and experimental motivations to explore BSM. Numerous models have been proposed to address these concerns. One such model is the 331 extension of the SM, which is based on the concept of higher weak isospin

symmetry, denoted by $SU(3)_L$, directly extending from $SU(2)_L$ [5–9]. Notably, this novel gauge framework offers potential solutions to key questions concerning fermion family number [5], the quantization of electric charge [10,11], discrepancies in the third quark family [12], and strong CP conservation [13,14]. Moreover, it has the potential to address issues such as dark matter [15–17], flavor physics [18–20], neutrino mass generation [21–23], cosmic inflation, and the baryon asymmetry [24–26].

Various 331 extensions have been proposed in the literature. Among them, the minimal 331 model [27] and the 331 model with right-hand neutrinos represent two typical examples [5–7]. Additionally, several other intriguing models have emerged, including the economical 331 model [28], the reduced 331 model [27], the simple 331 model [29], the flipped 331 model [30], the 331 model with exotic charged leptons [31], and the 331 model with neutral (heavy) fermions [32]. While certain realizations of the 331 model are known to develop a Landau pole at relatively low energies [33], Refs. [34,35] discuss possible ways to avoid this problem.

The particles proposed by the 331 model have not yet been detected in experimental searches. Given the absence of direct experimental evidence, it becomes crucial to scrutinize the indirect effects of these particles on SM observables. For instance, the existence of new particles as predicted by the 331 models may contribute to the Peskin-Takeuchi parameters S , T , U [36–38], consequently altering the theoretical prediction of the W boson mass and potentially resolving the W boson mass discrepancy. Furthermore, these effects can be used to indirectly

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constrain the parameter space of the model, aiding in the evaluation of its feasibility. Some previous work has been done to study the S , T , U parameters in the 331 model [39–42], with the most recent work [43] focusing on the CDF W boson mass discrepancy. However, these previous attempts were mostly generic and did not quantify the impact of the S , T , U parameters in terms of model input parameters.

The goal of this research is to scrutinize the indirect effects of particles predicted within the framework of the 331 model on electroweak precision observables, particularly focusing on the oblique S , T , U parameters. Initially, we will investigate the impact of new particles, specifically the additional Higgs bosons predicted by the 331 model, on the S , T , U parameters. Subsequently, we will analyze the resulting modifications in the W boson mass and identify the appropriate parameter space that can explain the CDF W boson mass discrepancy while adhering to the S , T , U constraints.

To achieve this goal, we used the *Mathematica* package called SARAH [44–48] to create the SPheno [49] source code for the 331 model. This allowed us to calculate mass spectra and low-energy observables like the S , T , U parameters. We performed the parameter scan using the SARAH Scan and Plot (SSP) computer package [50].

The remainder of the paper is structured as follows: In Sec. II, we offer a concise overview of the 331 model, outlining its key components. Section III addresses the discrepancy in the W boson mass and a discussion on oblique parameters S , T , U . We explore how the variation in these parameters influences the shift in the W boson mass. A numerical analysis is presented in Sec. IV. Finally, in Sec. V, we summarize our results and draw relevant conclusions.

II. MODEL SETUP

The 331 model proposes an extension to the SM with appealing features. It utilizes the gauge group $SU(3)_C \times SU(3)_L \times U(1)_X$, where $SU(3)_L \times U(1)_X$ breaks down to $SU(2)_L \times U(1)_Y$, and subsequently to $U(1)_Q$, featuring an extended Higgs sector. Anomaly cancellation requirements, along with QCD's asymptotic freedom, dictate the number of generations to match the number of colors, explaining the existence of three generations in the SM. Left-handed fermions transform as triplets under $SU(3)_L$, with the number of triplets equaling the number of antitriplets for an anomaly-free theory.

The 331 model suggests that two quark generations transform as triplets, while one transforms as an antitriplet, potentially giving an explanation for large top quark mass. Additional heavy quarks are introduced, and in the minimal version, no new leptons are added, with the third component of the lepton antitriplet chosen as the conjugate of the charged lepton. New heavy neutrinos could also be considered. Moreover, extending $SU(2)_L$ to $SU(3)_L$

implies the existence of five new gauge bosons, including a neutral boson Z' .

The electric charge operator in 331 models is given by

$$Q = T_3 + \beta T_8 + X,$$

where T_3 and T_8 are the diagonal generators of $SU(3)_L$, X is the $U(1)_X$ charge, and β is a model-dependent parameter. The choice of β determines the electric charge assignments and the particle content of the model. Common choices include $\beta = \pm\sqrt{3}$ and $\beta = \pm 1/\sqrt{3}$. These different β values correspond to unique sets of new particles with varying electric charges. Consequently, the particle composition of the 331 model varies according to the specific β chosen. In this work, we consider the 331 model with $\beta = -\sqrt{3}$ based on Ref. [51], where newly predicted charged particles are given in Table I along with their corresponding charges.

We provide a brief overview of the model in the coming sections.

A. Scalar sector

The symmetry-breaking pattern of the 331 model is

$$SU(3)_L \times U(1)_X \rightarrow SU(2)_L \times U(1)_Y \rightarrow U(1)_Q,$$

which is accomplished by the introduction of three scalar triplets ρ , η , and χ ,

$$\rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{-Q_V} \end{pmatrix}, \quad \eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta^{-Q_Y} \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^{Q_Y} \\ \chi^{Q_V} \\ \chi^0 \end{pmatrix}. \quad (1)$$

Here Q_V and Q_Y are the charges on V and Y , respectively. The vacuum expectation values (VEVs) of three scalars are chosen as

$$\rho = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \\ 0 \end{pmatrix}, \quad \eta = \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 \\ 0 \\ 0 \end{pmatrix}, \quad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ v_3 \end{pmatrix}. \quad (2)$$

In the first step of symmetry breaking, the field χ is introduced at a considerably high scale, typically at the TeV scale. This breaking transitions $SU(3)_L \times U(1)_X$ into $SU(2)_L \times U(1)_Y$. Subsequently, fields ρ and η are employed in the second step of symmetry breaking, which

TABLE I. New charged particles predicted by 331 model with $\beta = -\sqrt{3}$ along with their respective charges.

Particles	D, S	T	E	V	Y	H_V	H_Y	H_W
Q	5/4	-4/3	-2	-2	-1	-2	-1	1

further breaks $SU(2)_L \times U(1)_Y$ down to $U(1)_Q$ at the weak scale, approximately $v_1 \sim v_2 \sim M_W$. This arrangement ensures v_3 significantly exceeds $v_{1,2}$.

The scalar potential in the case of scalar triplets mentioned earlier can be expressed as follows:

$$\begin{aligned}
 V_H = & \mu_1^2(\rho^\dagger\rho) + \mu_2^2(\eta^\dagger\eta) + \mu_3^2(\chi^\dagger\chi) + \lambda_1(\rho^\dagger\rho)^2 \\
 & + \lambda_2(\eta^\dagger\eta)^2 + \lambda_3(\chi^\dagger\chi)^2 + \lambda_{12}(\rho^\dagger\rho)(\eta^\dagger\eta) \\
 & + \lambda_{13}(\rho^\dagger\rho)(\chi^\dagger\chi) + \lambda_{23}(\eta^\dagger\eta)(\chi^\dagger\chi) \\
 & + \lambda_{12}(\rho^\dagger\eta)(\eta^\dagger\rho) + \lambda_{13}(\rho^\dagger\chi)(\chi^\dagger\rho) \\
 & + \lambda_{23}(\eta^\dagger\chi)(\chi^\dagger\eta) + \sqrt{2}f(\varepsilon_{ijk}\rho^i\eta^j\chi^k + \text{H.c.}). \quad (3)
 \end{aligned}$$

Here, f has mass dimension, and it is typically assumed to be of the same order as v_3 in order to avoid introducing an additional energy scale into the model. The parameters λ_i , λ_{ij} , and λ'_{ij} , where i and j range from 1 to 3, are dimensionless couplings.

After the symmetry breaking, we have one CP -odd scalar A and three CP -even scalars denoted by h, H_2 , and H_3 . For CP -even scalars, the mass matrix is provided as

$$M_{H^2}^2 = \begin{pmatrix} 2\lambda_1 v_1 + \frac{f v_2 v_3}{v_1} & -f v_3 + v_1 v_2 \lambda_{12} & -f v_2 + v_1 v_3 \lambda_{13} \\ -f v_3 + v_1 v_2 \lambda_{12} & 2\lambda_2 v_2^2 + \frac{f v_1 v_3}{v_2} & -f v_1 + v_2 v_3 \lambda_{23} \\ -f v_2 + v_1 v_3 \lambda_{13} & -f v_1 + v_2 v_3 \lambda_{23} & 2\lambda_3 v_3^2 + \frac{f v_1 v_2}{v_3} \end{pmatrix}. \quad (4)$$

In the limit $v_3 \gg v_{1,2}$, the three eigenvalues of the above mass matrix are expressed as

$$M_h^2 = \lambda_1 v_1^2 + \lambda_2 v_2^2, \quad (5)$$

$$M_{H_2}^2 = \frac{(v_1^2 + v_2^2)}{v_1 v_2} f v_3, \quad (6)$$

$$M_{H_3}^2 = 2\lambda_3 v_3^2. \quad (7)$$

The Higgs boson h is identifiable as the particle detected at the Large Hadron Collider (LHC) [52,53] at CERN, with a mass of 125 GeV. Supposedly, H_2 and H_3 are heavier. h and H_2 emerge from ρ and η , while H_3 arises from χ , as a consequence of the two-step symmetry breaking. Additionally, within the Higgs sector of the 331 model, there are three charged scalars denoted as $H_W^\pm, H_V^{\pm\pm}$, and H_Y^\pm . The tree-level masses of the Higgs bosons within the 331 model can be summarized as

$$M_{H_W} \approx M_{H_2} \approx M_A \approx v_3 \sqrt{\frac{k(v_1^2 + v_2^2)}{v_1 v_2}}, \quad (8)$$

$$M_{H_3} = \sqrt{2\lambda_3} v_3, \quad (9)$$

$$M_{H_Y} = v_3 \sqrt{k \frac{v_1}{v_2} + \frac{\lambda'_{23}}{2}}, \quad (10)$$

$$M_{H_V} = v_3 \sqrt{k \frac{v_2}{v_1} + \frac{\lambda'_{13}}{2}}, \quad (11)$$

where k is defined by $k \equiv f/v_3$.

B. Gauge sector

With the introduction of $SU(3)_L$ symmetry into the gauge framework of the 331 model, four charged gauge bosons ($Y^\pm, V^{\pm\pm}$) and one neutral gauge boson (Z') are required in addition to W and Z bosons. Following the first step of symmetry breaking, these newly introduced gauge bosons obtain their masses, approximately at v_3 . Assuming $v_3 \gg v_{1,2}$, the mass spectra of the vector bosons can be expressed as

$$M_W = \frac{1}{2} g \sqrt{v_1 + v_2}, \quad (12)$$

$$M_Y = M_V = \frac{1}{2} g v_3, \quad (13)$$

$$M_Z = \frac{1}{s_w} M_W, \quad (14)$$

$$M_{Z'} = \frac{2c_w}{\sqrt{3[1 - (1 + \beta^2)s_w^2]}} M_Y, \quad (15)$$

where $c_w = \cos \theta_w$, $s_w = \sin \theta_w$, and θ_w is the weak mixing angle.

C. Fermion sector

Extending the $SU(2)_L$ gauge group to $SU(3)_L$ leads to left-handed fermions transforming as triplets or antitriplets under $SU(3)_L$. An anomaly-free theory requires an equal number of triplets and antitriplets. One approach designates the three lepton generations as antitriplets, leading to two quark generations as triplets and one as an antitriplet. Consequently, the first two generations of left-handed quark fields are represented as triplets within the $SU(3)_L$ group, while the third generation is represented as an antitriplet. This yields the following expressions for the quark fields:

$$q_{1L} = \begin{pmatrix} u \\ d \\ D \end{pmatrix}_L, \quad q_{2L} = \begin{pmatrix} c \\ s \\ S \end{pmatrix}_L, \quad q_{3L} = \begin{pmatrix} b \\ -t \\ T \end{pmatrix}_L. \quad (16)$$

In Eq. (16), only left-handed quarks are depicted, while their corresponding right-handed counterparts exist as singlets. Furthermore, a different configuration is adopted for the third generation of quarks compared to the SM,

driven by the requirement of anomaly cancellation. Additionally, a negative sign is introduced in the q_{3L} triplet to align the t couplings with those of the SM. The lepton fields are represented as

$$l_{1L} = \begin{pmatrix} e \\ -\nu_e \\ E_e \end{pmatrix}_L, \quad l_{2L} = \begin{pmatrix} \mu \\ -\nu_\mu \\ E_\mu \end{pmatrix}_L, \quad l_{3L} = \begin{pmatrix} \tau \\ -\nu_\tau \\ E_\tau \end{pmatrix}_L. \quad (17)$$

The masses of the new fermions originate from the following Yukawa Lagrangian:

$$-\mathcal{L}_{\text{Yuk}} = y_{ik}^j \bar{q}_{iL} \chi J_{kR} + y_{33}^j \bar{q}_{3L} \chi^* J_{3R} + y_{mn}^E \bar{\ell}_{mL} \chi^* E_{nR} + \text{H.c.} \quad (18)$$

Here, i (k) runs from 1 to 2, while j (m, n) runs from 1 to 3. The right-handed heavy quarks J_{kR} correspond to the quarks D and S , while J_{3R} refers to T . Similarly, E_{nR} represents the right-handed heavy leptons E_e, E_μ, E_τ . The scalar field χ contributes mass only to the third component of the triplet fermions ($E_e, E_\mu, E_\tau, D, S, T$), meaning there is no direct coupling of χ to SM particles. Once the first stage of symmetry breaking occurs, the new fermions acquire mass through the VEV of χ . For simplicity, assuming a diagonal mass matrix for the new fermions, their masses are given by

$$M_D = \frac{y_{11}^j}{\sqrt{2}} v_3, \quad M_S = \frac{y_{22}^j}{\sqrt{2}} v_3, \quad M_T = \frac{y_{33}^j}{\sqrt{2}} v_3, \quad (19)$$

$$M_{E_e} = \frac{y_{11}^j}{\sqrt{2}} v_3, \quad M_{E_\mu} = \frac{y_{22}^j}{\sqrt{2}} v_3, \quad M_{E_\tau} = \frac{y_{33}^j}{\sqrt{2}} v_3. \quad (20)$$

III. THE ELECTROWEAK PRECISION OBSERVABLES

In the subsequent sections, we offer a concise introduction to the S, T, U parameters, followed by an examination of the current experimental status regarding S, T, U and the mass of the W boson.

A. The S, T, U parameters

In various BSM models, the dominant BSM effects appear in the corrections to the vacuum polarization of gauge bosons, commonly referred to as oblique corrections [54,55]. If the BSM scale significantly exceeds that of the weak interaction, these corrections can be accurately represented by three distinct parameters: S, T, U [36–38]. These parameters are defined as

$$\hat{\alpha}(M_Z)T \equiv \frac{\Pi_{WW}^{\text{new}}(0)}{M_W^2} - \frac{\Pi_{ZZ}^{\text{new}}(0)}{M_Z^2}, \quad (21)$$

$$\frac{\hat{\alpha}(M_Z)}{4s_w^2 c_w^2} S \equiv \frac{\Pi_{ZZ}^{\text{new}}(0)}{M_W^2} - \frac{\Pi_{ZZ}^{\text{new}}(0)}{M_Z^2} - \frac{c_w^2 - s_w^2}{s_w c_w} \frac{\Pi_{Z\gamma}^{\text{new}}(M_Z^2)}{M_Z^2} - \frac{\Pi_{\gamma\gamma}^{\text{new}}(M_Z^2)}{M_Z^2}, \quad (22)$$

$$\frac{\hat{\alpha}(M_Z)}{4s_w^2} (S + U) \equiv \frac{\Pi_{WW}^{\text{new}}(M_W^2) - \Pi_{WW}^{\text{new}}(0)}{M_W^2} - \frac{c_w}{s_w} \frac{\Pi_{Z\gamma}^{\text{new}}(M_Z^2)}{M_Z^2} - \frac{\Pi_{\gamma\gamma}^{\text{new}}(M_Z^2)}{M_Z^2}. \quad (23)$$

Here, $\hat{\alpha}(M_Z)$ denotes the renormalized electromagnetic fine-structure constant evaluated at the Z boson mass scale, whereas $s_w \equiv \sin \theta_w$ and $c_w \equiv \cos \theta_w$ represent the sine and cosine of the weak mixing angle, respectively. The vacuum polarization functions $\Pi_{VV'}^{\text{new}}(q^2)$ encode the contributions from BSM to the gauge boson two-point functions, with the subscripts VV' referring to specific gauge boson combinations: Π_{WW}^{new} for the W bosons, Π_{ZZ}^{new} for the Z bosons, $\Pi_{Z\gamma}^{\text{new}}$ for the Z -photon mixing, and $\Pi_{\gamma\gamma}^{\text{new}}$ for the photon self-energy. The latest values for these parameters [2] are given by

$$S = -0.04 \pm 0.10, \quad (24)$$

$$T = 0.01 \pm 0.12, \quad (25)$$

$$U = -0.01 \pm 0.11. \quad (26)$$

By definition, these values only arise from BSM and align remarkably well with the zero value predicted by the SM. Within the 331 model framework, the introduction of new quarks, which are singlets under $SU(2)_L$, does not influence the oblique corrections to the S, T, U parameters. These corrections are solely sensitive to the breaking of $SU(2)_L$. Similarly, the contribution of the Z' boson is minimal, except through its involvement in $Z - Z'$ mixing. Additionally, the impact of the gauge bosons Y^\pm and $V^{\pm\pm}$ on the S, T, U parameters is expected to be negligible [43]. This is attributed to their contribution being contingent upon the mass difference between Y^\pm and $V^{\pm\pm}$, which are nearly degenerate. However, dominant contributions are expected from the scalar sector of the 331 model where neutral as well as charged Higgs bosons can play significant role. The representative Feynman diagrams for the scalar bosons are shown in Fig. 1.

B. W boson mass discrepancy

The CDF Collaboration has announced a new measurement for the W boson mass, suggesting the potential existence of BSM physics. Their measurement provides a value [1]

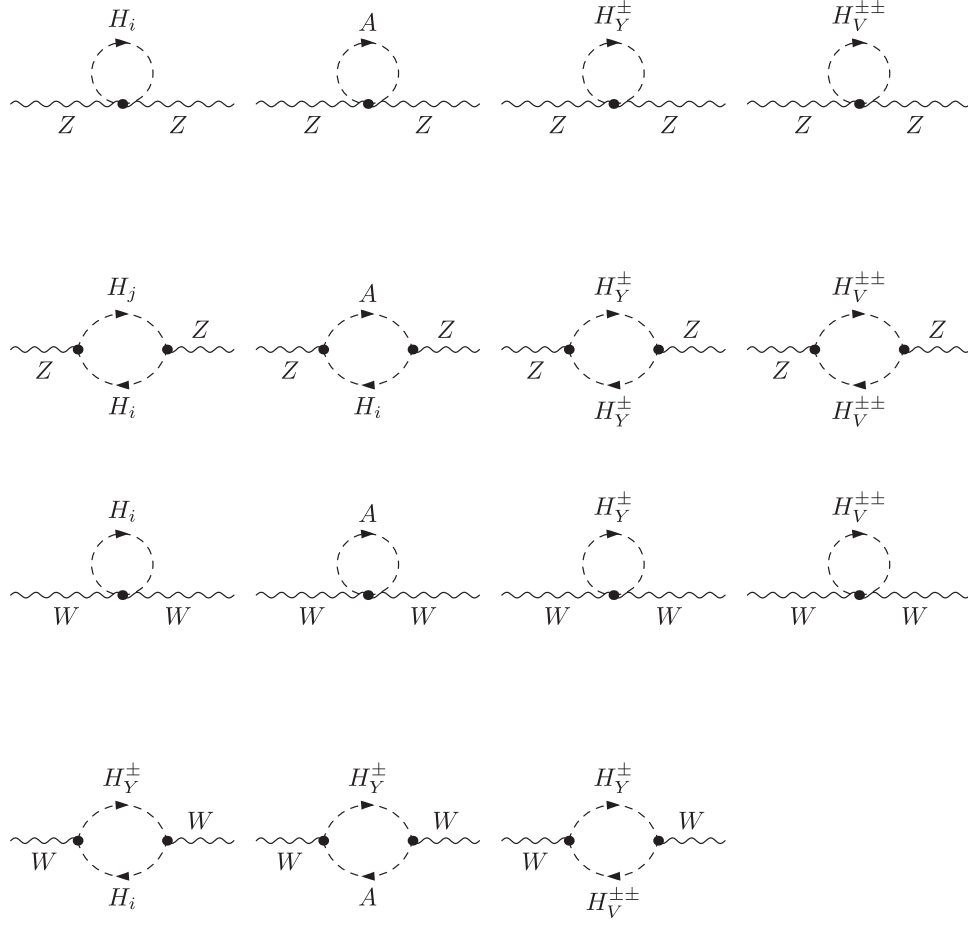


FIG. 1. Representative Feynman diagrams for the one-loop self-energy of gauge bosons featuring Higgs bosons in the loop, where H_i (with $i = 1, 2, 3$) represent the neutral Higgs bosons.

$$M_W^{\text{CDF}} = 80.4335 \pm 0.0094 \text{ GeV},$$

$$M_W^{\text{avg}} = 80.3692 \pm 0.0133 \text{ GeV}.$$

showing a notable deviation of around 7σ from the SM prediction [2], where

$$M_W^{\text{SM}} = 80.353 \pm 0.006 \text{ GeV}.$$

Upon combining previous results from experiments such as ATLAS and LHCb, the world average is determined as [56]

$$M_W^{\text{avg}} = 80.4133 \pm 0.0080 \text{ GeV}. \quad (27)$$

However, as the current measurements by the CMS [3] and ATLAS [4] Collaborations are very close to the SM predictions, it has been reported [57] that a combination of all W boson mass measurements yields a probability of compatibility of only 0.5% and is therefore disfavored. A significantly improved compatibility of 91% is obtained when the CDF-II measurement is excluded from the combination, resulting in the new world average [2]

Nevertheless, in this work, we choose to retain the CDF-II result in our analysis. Excluding it would leave little to no room for deviations from the SM, thereby potentially overconstraining BSM scenarios such as the 331 model under consideration. By including the CDF-II result, we explore the potential implications of a higher W boson mass, should this measurement be confirmed, on the parameter space of the model. Therefore, we define W boson mass shift as

$$\begin{aligned} \Delta M_W^2 &= (M_W^{\text{CDF}})^2 - (M_W^{\text{SM}})^2 \\ &= (80.433^2 - 80.357^2) \text{ GeV}^2. \end{aligned} \quad (28)$$

The mass shift of the W boson may be influenced by the $S, T,$ and U parameters as [36–38]

$$\Delta M_W^2 = \frac{c_w^2 m_Z^2}{c_w^2 - s_w^2} \left(-\frac{S}{2} + c_w^2 T - \frac{c_w^2 - s_w^2}{4s_w^2} U \right). \quad (29)$$

IV. NUMERICAL RESULTS

A. Computational setup

Here is a simplified description of our computational workflow: Using the *Mathematica* package called SARAH [44–48], we initially generated the SPheno [49] source code for the 331 model. The source code comprises the analytical expressions essential for computing mass spectra and low-energy observables such as the S , T , U parameters. To conduct parameter scans, we employed an additional *Mathematica* package called SARAH Scan and Plot [50].

B. Input parameters

The 331 model is characterized by several parameters, including the VEV of the Higgs triplet χ denoted as v_3 , a parameter f , and dimensionless couplings within the Higgs sector represented by λ_i , λ_{ij} , and λ'_{ij} , where i and j range from 1 to 3, respectively. Additionally, the model introduces masses for new fermions and defines $\tan\beta$ as the ratio of the VEVs of the two electroweak-scale scalar triplets, with $\tan\beta \equiv v_2/v_1$, where v_2 and v_1 correspond to the VEVs of the neutral components of η and ρ , respectively, as shown in Eq. (2). For the purposes of our numerical analysis, we set the masses of the newly introduced fermions to 800 GeV, although this is not relevant to our current discussion. As for the remaining parameters, we conducted random scans using the following set:

$$\begin{aligned} 0 &\leq v_3 &\leq 5000 \text{ GeV}, \\ 0 &\leq f &\leq 10 \text{ GeV}, \\ 0 &\leq \lambda_i, \lambda_{ij}, \lambda'_{ij} &\leq 0.5, \\ 0 &\leq \tan\beta &\leq 60. \end{aligned}$$

We restrict v_3 to values up to 5000 GeV to avoid the Landau pole [43]. The parameter f is expressed as $f \equiv kv_3$, where, typically, the constant k is assumed to be around unity. However, the masses of the Higgs bosons exhibit sensitivity to the magnitude of f . This sensitivity leads to large values for the S , T , U parameters, consequently imposing constraints on the value of f , as will be explained in the following section. Additionally, the mass of the light Higgs boson M_h is highly sensitive to the input parameters, particularly the couplings λ_1 and λ_2 . Consequently, we only consider points that result in M_h falling within the range of 123–127 GeV to allow for theoretical uncertainties from unknown higher-order corrections and model-dependent effects.

C. Higgs boson masses

In this section, we present our results concerning the masses of Higgs bosons plotted on the f - v_3 plane in Figs. 2–7. Since M_h is independent of f and v_3 , we do not present the predictions for M_h . Figure 2 displays the value of M_{H_2} in the f - v_3 plane. The left plot showcases our findings without imposing S , T , U constraints, while the right plot displays results where only points satisfying S , T , U experimental constraints within the 3σ range are included. The color bar indicates the magnitude of M_{H_2} . As expected, the value of M_{H_2} is influenced by both parameters, f and v_3 . It can be observed that the S , T , U constraints preclude values of v_3 below 1500 GeV and above 2300 GeV. In the left plot, the dominance of green and yellow points in the region where $v_3 > 2000$ and $f > 3$ GeV indicates $M_{H_2} > 1000$ GeV. However, the right plot reveals that this region is excluded by the S , T , U parameters. As anticipated based on Eq. (8), M_{H_w} and M_A exhibit nearly degenerate behavior. Consequently, a

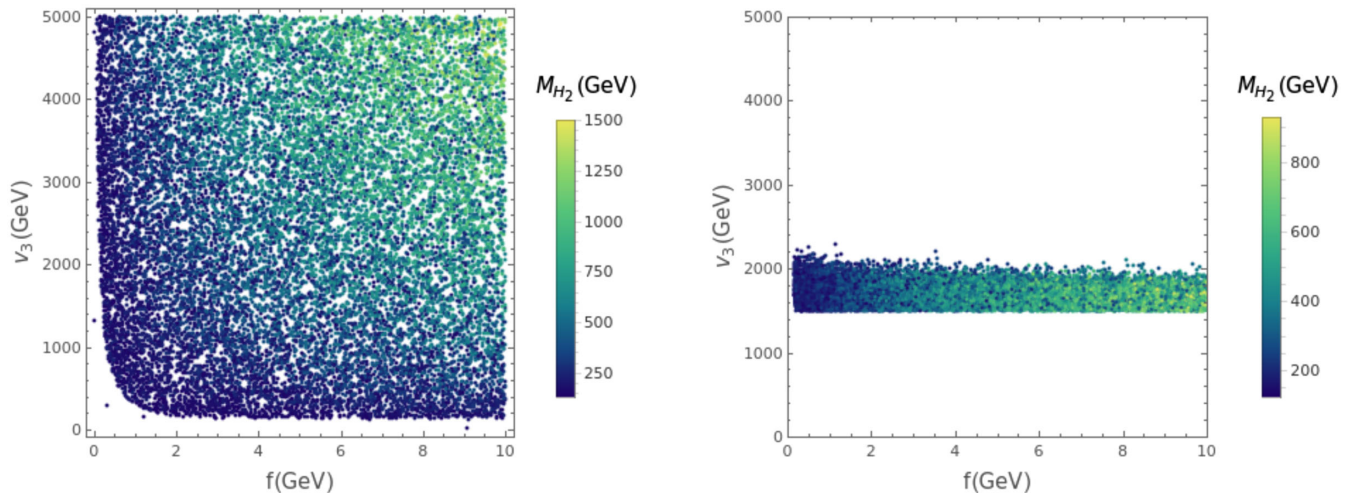


FIG. 2. The 331 predictions for M_{H_2} with (right) and without (left) S , T , U constraints in the f - v_3 plane. The color bar represents the values of M_{H_2} .

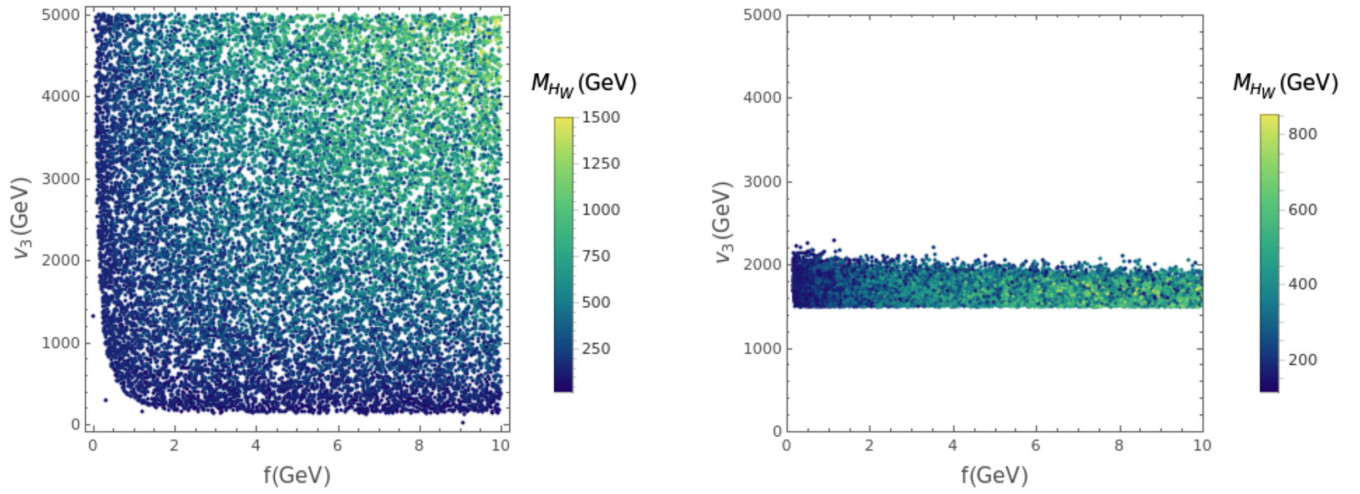


FIG. 3. The 331 predictions for M_{H_W} with (right) and without (left) S, T, U constraints in the f - v_3 plane. The color bar represents the values of M_{H_W} .

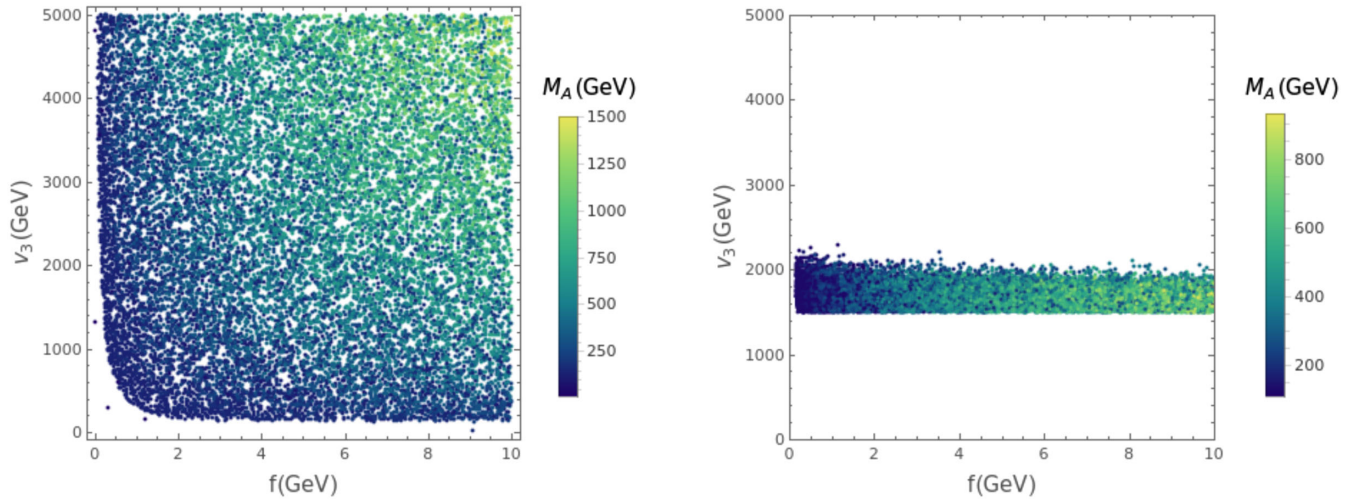


FIG. 4. The 331 predictions for M_A with (right) and without (left) S, T, U constraints in the f - v_3 plane. The color bar represents the values of M_A .

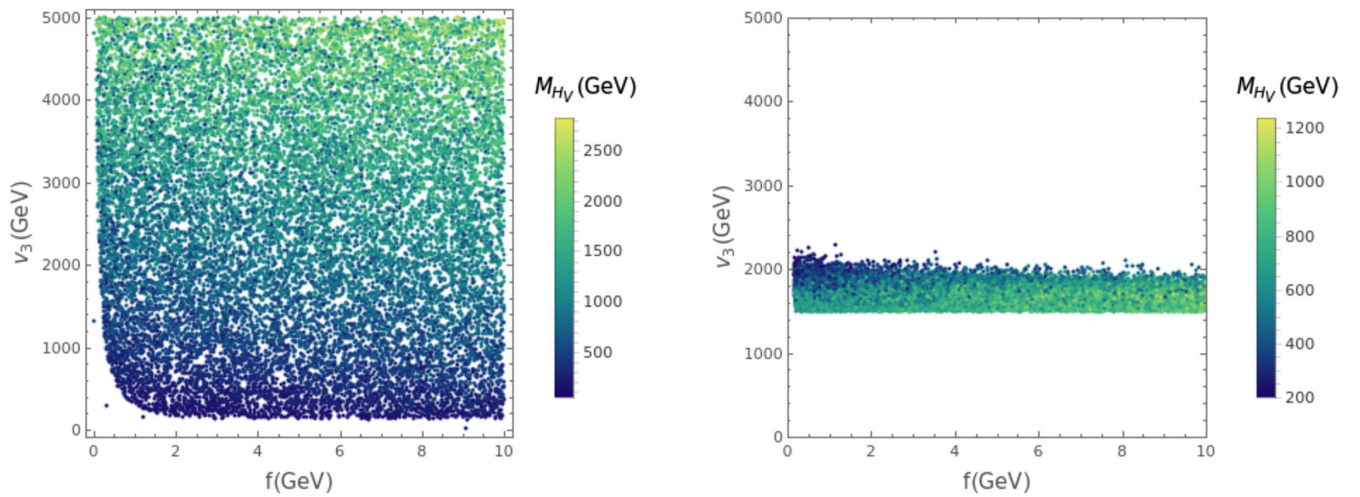


FIG. 5. The 331 predictions for M_{H_V} with (right) and without (left) S, T, U constraints in the f - v_3 plane. The color bar represents the values of M_{H_V} .

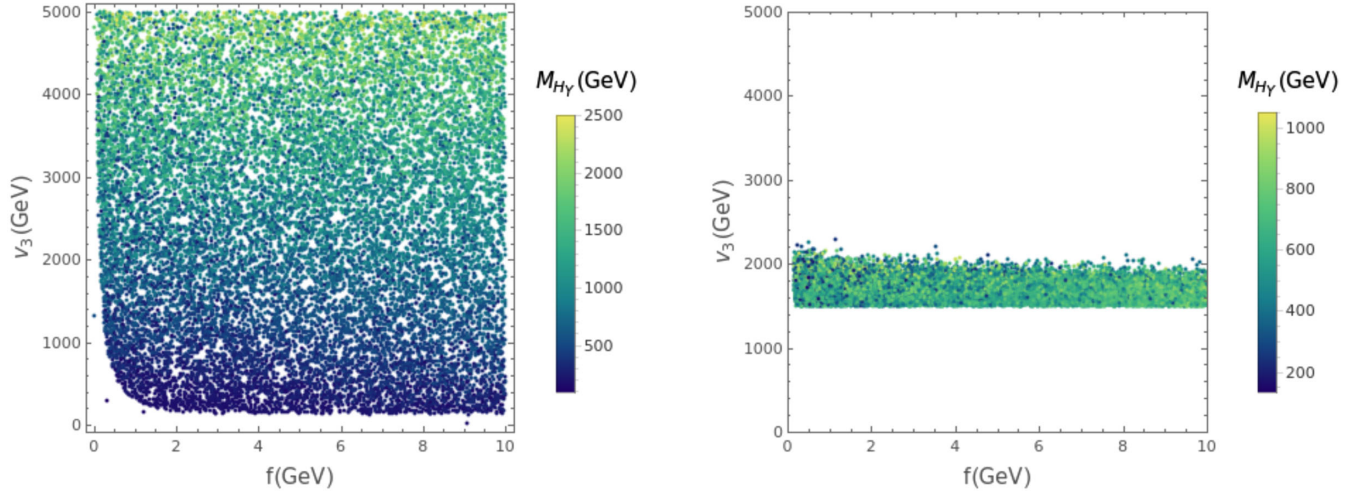


FIG. 6. The 331 predictions for M_{H_Y} with (right) and without (left) S, T, U constraints in the f - v_3 plane. The color bar represents the values of M_{H_Y} .

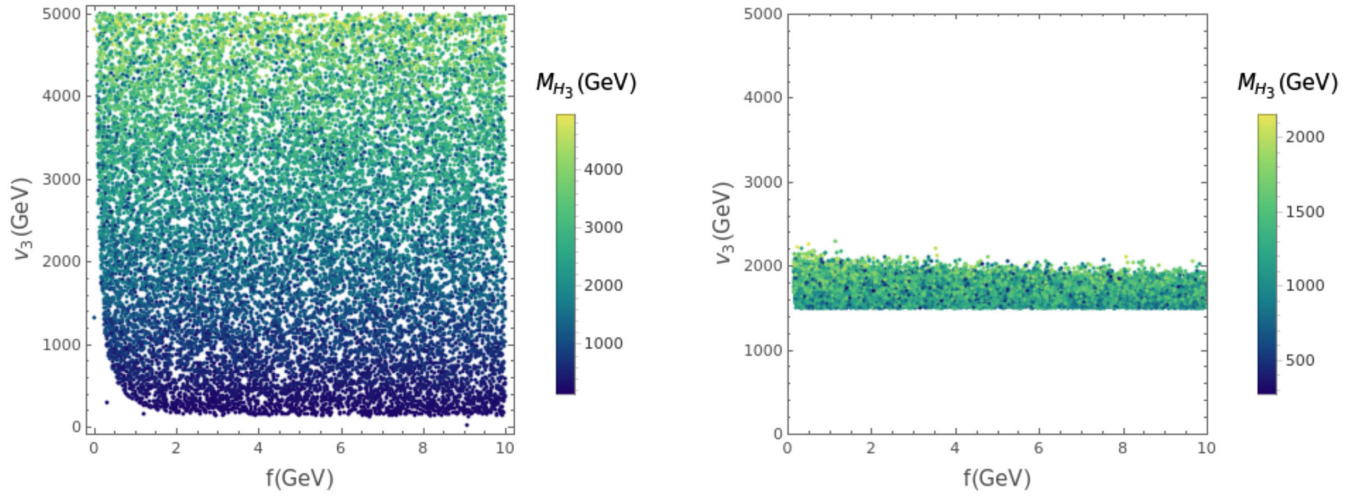


FIG. 7. The 331 predictions for M_{H_3} with (right) and without (left) S, T, U constraints in the f - v_3 plane. The color bar represents the values of M_{H_3} .

comparable trend to that of M_{H_2} is observed for M_{H_W} and M_A , as can be seen in Figs. 3 and 4, respectively.

The correlation of M_{H_V} to f and v_3 exhibits slight variations compared to M_{H_2} and M_{H_W} . The left plot of Fig. 5 illustrates that M_{H_V} shows only a mild dependency on f , whereas its relationship with v_3 is noticeable. Conversely, the right plot in Fig. 5 demonstrates that lower values of M_{H_V} are favored by the S, T, U parameters. Figure 7 displays the value of M_{H_3} in f - v_3 plane. It indicates that M_{H_3} does not exhibit a discernible dependence on f , although its reliance on v_3 is apparent. Similar conclusions can be drawn for M_{H_Y} , as evidenced by Fig. 6.

D. Gauge boson masses

In this section, we present our results for gauge boson masses, specifically the $M_{Z'}$, M_{Y^\pm} , and $M_{V^{\pm\pm}}$. Although we do not anticipate significant contributions from the Y^\pm and $V^{\pm\pm}$ gauge bosons to the S, T, U parameters due to their nearly degenerate masses, it is still useful to determine their possible mass ranges considering the influence of the S, T, U parameters. In Fig. 8, we present 331 predictions for $M_{Z'}$ on the f - v_3 plane, illustrated in both the right and left plots with and without S, T, U constraints, respectively. The color bar indicates the values of $M_{Z'}$. It is evident that the S, T, U constraints restrict the value of $M_{Z'}$ to nearly 7 TeV, contrasting with its potential range up to 13 TeV in their absence.

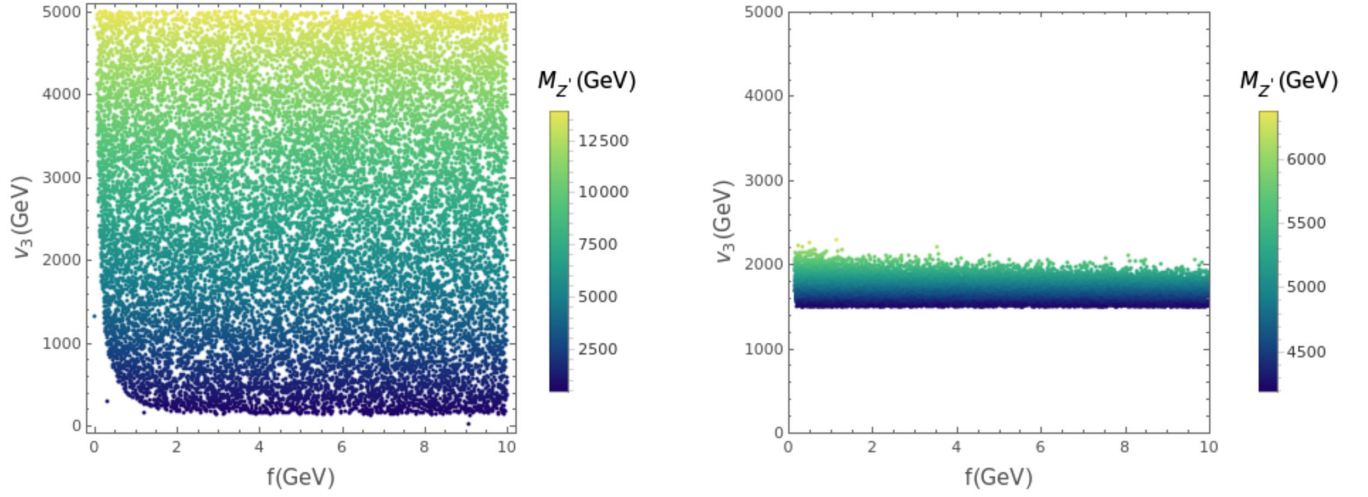


FIG. 8. The 331 predictions for $M_{Z'}$ with (right) and without (left) S, T, U constraints in the f - v_3 plane. The color bar represents the values of $M_{Z'}$.

The analysis in Refs. [58,59] places lower bounds on $M_{Z'}$ that can be translated into constraints on the 331 symmetry-breaking scale v_3 via the relation given in Eq. (15), valid in the decoupling limit $v_3 \gg v_{1,2}$. The experimental mass limit $M_{Z'} \gtrsim 4$ TeV quoted there corresponds to the value at which the theoretical prediction for $\sigma(pp \rightarrow Z') \times \text{BR}(Z' \rightarrow \ell\ell)$ meets the experimental upper bound. Among other factors, this prediction depends sensitively on the Z' couplings to quarks and leptons. These couplings control both the production rate, which at LHC energies is dominated by light-quark parton luminosities and scales approximately as $\sigma(pp \rightarrow Z') \propto (g_{Z'}^q)^2$, and the leptonic branching ratio,

$$\text{BR}(Z' \rightarrow \ell\ell) = \frac{\Gamma(Z' \rightarrow \ell\ell)}{\Gamma_{\text{tot}}}, \quad \Gamma(Z' \rightarrow f\bar{f}) \propto (g_{Z'}^f)^2 M_{Z'}.$$

Here, $g_{Z'}^f$ denotes the effective coupling of the Z' to a generic fermion f . Any nonuniform change between quark and lepton couplings modifies both the production rate and the leptonic branching fraction, while the opening or closing of additional decay channels alters Γ_{tot} and hence $\text{BR}(Z' \rightarrow \ell\ell)$. Since the fermion charge assignments—and therefore the coupling factors $g_{Z'}^f$ —differ among 331 variants, in particular, between $\beta = \pm 1/\sqrt{3}$ and $\beta = -\sqrt{3}$, a faithful translation of the published $\beta = \pm 1/\sqrt{3}$ limit into our $\beta = -\sqrt{3}$ setup requires recomputing $\sigma \times \text{BR}$ with the appropriate charge assignments, while adopting the same acceptance and K-factor assumptions as in the experimental analyses. In practice, the $\beta = \pm 1/\sqrt{3}$ bound is expected to be somewhat stronger. In the absence of a dedicated recast, and allowing for plausible $\mathcal{O}(1)$ variations in the couplings and width effects, we conservatively assign a “model-dependent uncertainty” of 1–2 TeV, under which the limit remains compatible with our results.

Additionally, the values of M_{Y^\pm} and $M_{V^{\pm\pm}}$ are similarly bounded to approximately 750 GeV by the S, T, U constraints, as evidenced by Figs. 9 and 10. We note that a recent collider recast analysis [60] of ATLAS multilepton data sets a lower limit of about $M_{V^{\pm\pm}} \gtrsim 1.3$ TeV (90% confidence level) under the assumption that the doubly charged vector boson decays almost exclusively into same-sign SM lepton pairs. At face value, this lower bound appears in tension with the upper bound $M_{V^{\pm\pm}} \lesssim 750$ GeV inferred from our analysis. However, several important caveats should be emphasized. First, the collider bound depends sensitively on the branching ratio into SM leptons: if new decay channels of $V^{\pm\pm}$ into exotic fermions or scalar states are kinematically open, the effective leptonic branching fraction is reduced and the ATLAS limit correspondingly weakens. Second, the S, T, U constraints are themselves highly sensitive to the scalar-sector quartic couplings. While the requirement $M_h \simeq 125$ GeV tightly constrains combinations of λ_1 and λ_2 , other quartic couplings (notably λ_3 and the primed λ'_{i3}) remain free within our scan and directly control the heavy scalar masses and their isospin splittings. Modest variations of these couplings can both reduce the T parameter by making heavy scalar multiplets more degenerate and open additional cascade decay channels for $V^{\pm\pm}$. These effects can substantially relax the apparent tension between the S, T, U -derived upper limit and collider-derived lower bound. A dedicated combined study of oblique parameters, scalar potential parameters, and collider signatures is therefore required before any definitive claims.

E. ΔM_W^2

In this section, we present our findings concerning ΔM_W^2 plotted in the f - v_3 plane. Figure 11 shows the values of ΔM_W^2 both with (right plot) and without (left plot)

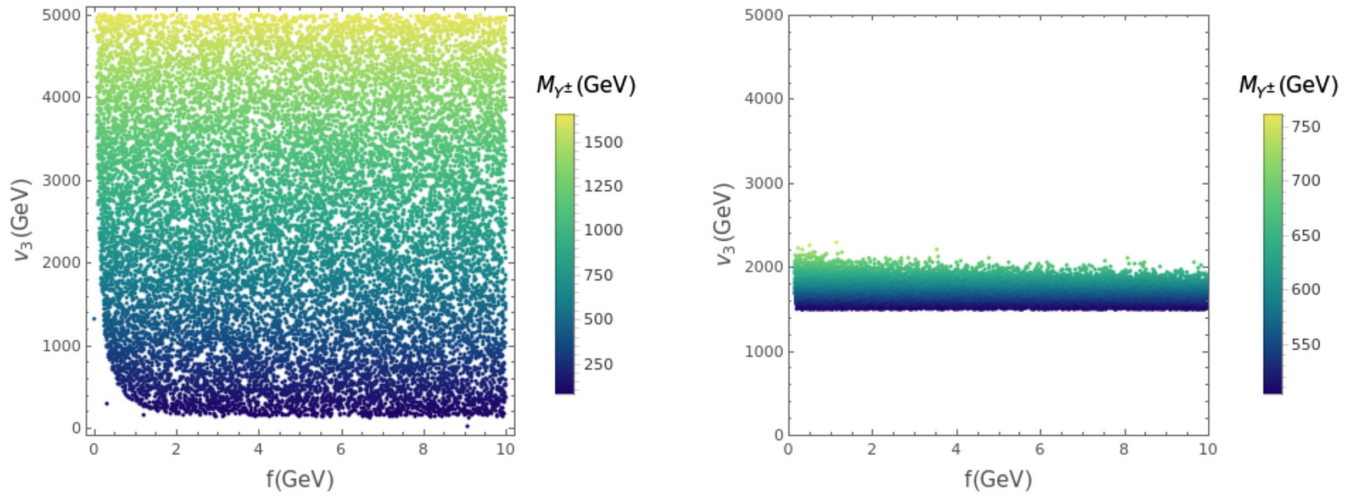


FIG. 9. The 331 predictions for M_{Y^\pm} with (right) and without (left) S, T, U constraints in the f - v_3 plane. The color bar represents the values of M_{Y^\pm} .

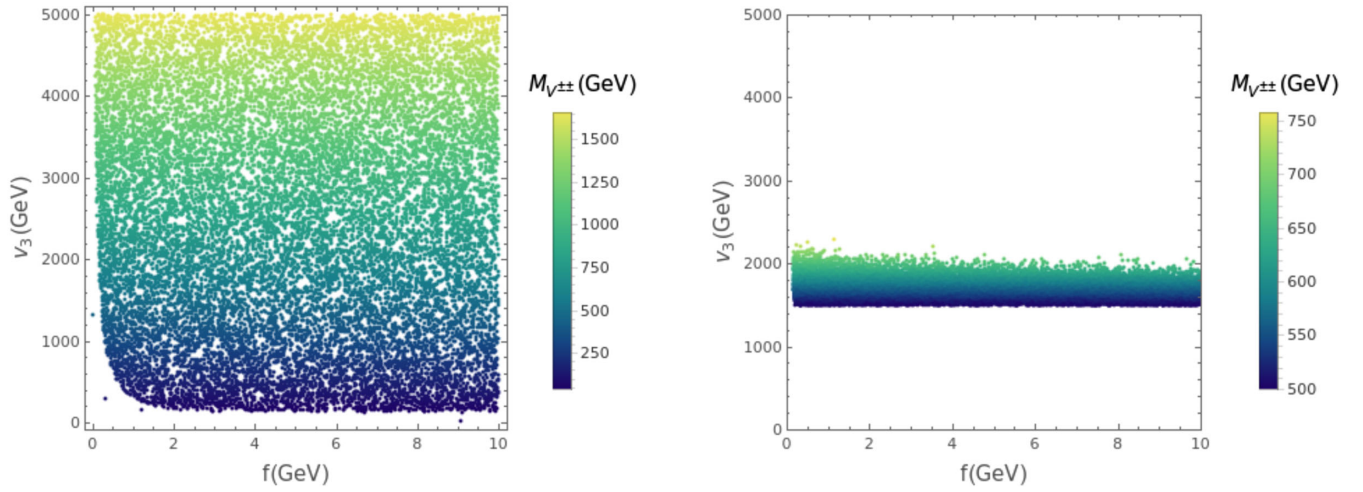


FIG. 10. The 331 predictions for $M_{V^{\pm\pm}}$ with (right) and without (left) S, T, U constraints in the f - v_3 plane. The color bar represents the values of $M_{V^{\pm\pm}}$.

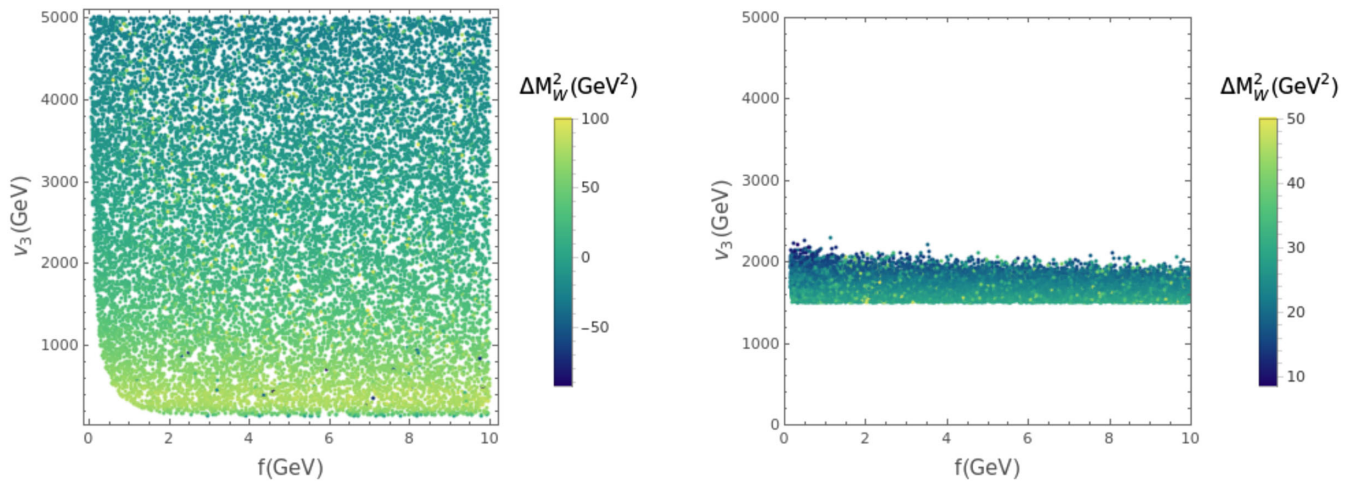


FIG. 11. The 331 predictions for ΔM_W^2 with (right) and without (left) S, T, U constraints in the f - v_3 plane. The color bar represents the values of ΔM_W^2 .

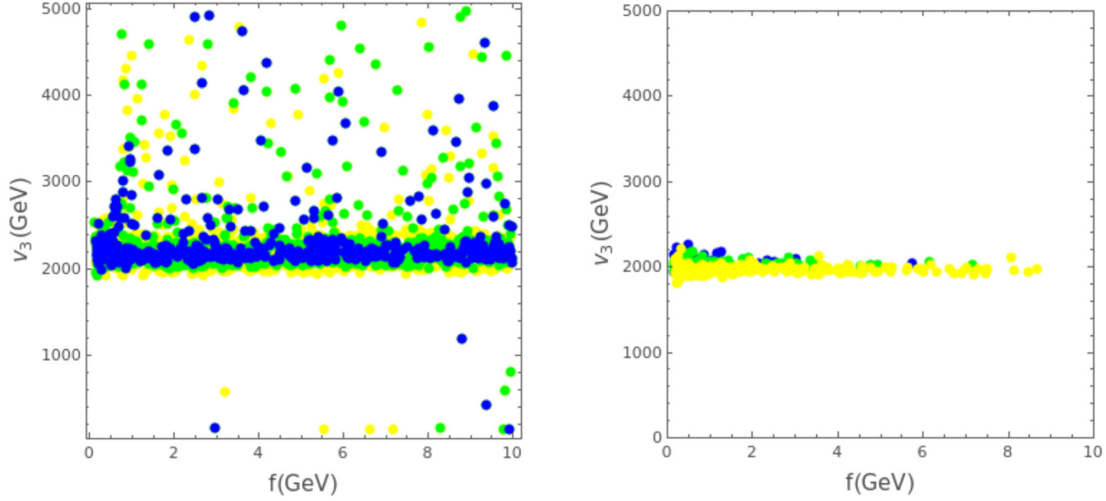


FIG. 12. The 331 predictions for ΔM_W^2 with (right) and without (left) S, T, U constraints in the f - v_3 plane. The blue, green, and yellow points denote values of ΔM_W^2 within the 1σ , 2σ , and 3σ ranges, respectively.

adherence to S, T, U constraints. Notably, even when considering S, T, U constraints, the magnitude of ΔM_W^2 can be very large, reaching up to 50 GeV^2 . Figure 12 further explores ΔM_W^2 in the f - v_3 plane, focusing specifically on points falling within the 3σ range of ΔM_W^2 . The left plot disregards S, T, U constraints, whereas the right plot exclusively includes points that satisfy S, T, U constraints within the 3σ range. In these plots, blue (green) points denote values within 1σ (2σ) of ΔM_W^2 , while yellow points signify values within the 3σ range.

In the right plot, an interesting trend emerges, where the majority of points cluster in the region characterized by small values of f . This phenomenon can be attributed to the correlation between the masses of the Higgs bosons, particularly M_{H_2} , M_{H_W} , and M_A , and the magnitude of f . As the magnitude of f increases, the masses of these Higgs bosons also increase, consequently leading to significant contributions to the self-energy diagrams of the gauge bosons involved in the computation of S, T, U parameters. Notably, most of the blue points are situated in the region where $f < 3 \text{ GeV}$. No points satisfying both S, T, U constraints and ΔM_W^2 constraints are observed for $f > 9 \text{ GeV}$. Furthermore, there are no points outside the range $1800 < v_3 < 2300 \text{ GeV}$ that meet both the S, T, U and ΔM_W^2 constraints within the 3σ range simultaneously.

V. CONCLUSIONS

The extension of the Standard Model's gauge structure from $SU(3)_C \times SU(2)_L \times U(1)_Y$ to $SU(3)_C \times SU(3)_L \times U(1)_X$, known as the 331 model, offers a rich phenomenology due to its extended gauge, Higgs, and fermion sectors. Various versions of this model exist that are characterized by a parameter called β , which determines the types of new particles present. Although no new

particles have been detected in experimental searches so far, they could manifest their presence through indirect effects on SM observables, allowing for indirect constraints on the model's parameter space.

Electroweak precision observables, particularly the Peskin-Takeuchi parameters S, T, U , have been measured with high precision and serve as excellent probes for new physics effects. These parameters have the potential to alter the W boson mass predictions, offering a possible resolution to the discrepancy between the SM prediction and the recently reported experimental measurement by the CDF experiment.

In this work, we have investigated the indirect effects of particles predicted by the 331 model, particularly the additional Higgs bosons, on the Peskin-Takeuchi parameters S, T, U . Using the SPheno setup generated via the *Mathematica* package SARAH, we conducted a parameter scan using the SARAH Scan and Plot package as an interface to SPheno. Our analysis revealed that the S, T, U parameters are highly sensitive to the masses of the Higgs bosons, constraining their values to the TeV range or below. In contrast, the masses of the gauge bosons do not significantly affect the S, T, U parameters. However, the S, T, U parameters constrain the vacuum expectation value of the third Higgs triplet v_3 to the range $1500 < v_3 < 2300 \text{ GeV}$. This restriction, in turn, imposes upper bounds on the gauge boson masses: $M_Z < 7000$, $M_{Y^\pm} < 700$, and $M_{V^{\pm\pm}} < 700 \text{ GeV}$.

We also examined the potential contributions of the S, T, U parameters to the W boson mass. We found that the discrepancy in the W boson mass can be successfully explained if the value of v_3 lies in the range $1800 < v_3 < 2300 \text{ GeV}$. The parameter f , which appears in the Higgs potential, is typically assumed to be of the same order as v_3 . However, the S, T, U parameters, as well as ΔM_W^2 , favor smaller values for this parameter, restricting

its magnitude to $f < 9$ GeV. While the current study was conducted for $\beta = -\sqrt{3}$, we posit that the results outlined herein are transferable to other 331 versions that share a similar Higgs sector with the one under consideration. We hope that this work will aid in the experimental searches for 331 particles.

This article does not contain any studies with human participants or animals performed by any of the authors.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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