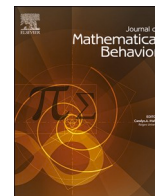


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# Variables in planning and carrying out a problem-posing task in early childhood education

Enrique Carmona-Medeiro<sup>a</sup>, Juan Pedro Martín-Díaz<sup>b,\*</sup>, Nuria Climent<sup>b</sup>

<sup>a</sup> Department of Didactics, Faculty of Education, Area of Didactic of Mathematic, University of Cádiz, Spain

<sup>b</sup> Department of Integrated Didactics, COIDESO, University of Huelva, Huelva, Spain

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## ABSTRACT

This research focused on understanding the variables inherent in the design and implementation of a mathematical problem-posing task. We developed a single case study of a problem-posing lesson by an Early Childhood Education teacher in a classroom with 4- to 5-year-old children who were unfamiliar with such activities. The results of this study show the potential of considering five variables serving as critical points that pose dilemmas linked to the design and implementation of problem-posing tasks. We found that the task changed from its original design during implementation, implying that the choices the teacher made about the variables were not static and were strongly linked to the purpose of the problem-posing task as well as to the contextual characteristics of the early childhood classroom. This study provides a potentially useful framework for analyzing the design and implementation of problem-posing tasks as a dynamic process.

## 1. Introduction

Despite being foregrounded as a fundamental tool for mathematics education in classic works (Freudenthal, 1973), problem posing has only relatively recently attracted the attention of researchers (Cai & Hwang, 2020; Silver, 1994). This attention has been seen across educational stages, including primary (English, 1997), secondary (Koichu, 2020), and graduate (Rosli et al., 2015) levels, but to date little has been done in the sphere of early childhood education (Palmér & van Bommel, 2020).

Early childhood pupils tend to have a more open outlook, are attentive to detail, and are often more sensitive to context than older children (Kwon & Capraro, 2021). Hence, by incorporating problem posing at this stage, teachers can help to establish an early association of mathematics with real situations (Quintero, 2004). At the same time, problems posed by primary pupils can be more complex than teachers expect (as in Xu et al., 2019), suggesting that the approach could be adopted at the early childhood stage.

Because incorporating problem posing is challenging for teachers (Barlow & Cates, 2006), additional research is necessary to better understand the obstacles teachers face when designing and managing problem-posing tasks (Harel et al., 2006). The effective implementation of a problem-posing task requires the effective involvement of the pupils, and the development of both problem-posing abilities and positive attitudes towards problem posing (Cai et al., 2022). In this paper, we consider a mathematical task to be any kind of structured environment (Watson et al., 2013) which the students are invited to explore with the purpose of focusing their attention on a specific idea or mathematical process. Further, we consider that tasks, in both their pre-active and active

\* Corresponding author.

E-mail address: [juan.martin@ddcc.uhu.es](mailto:juan.martin@ddcc.uhu.es) (J.P. Martín-Díaz).

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phases (Jackson, 1968), constitute a dynamic process that develops and changes during the lesson.

To explore the life cycle of a task from the teacher’s perspective, this study considered variables along the timeline of the design and implementation of a problem-posing task, throughout which tensions—or, as we shall call them, *dilemmas*—are produced. A dilemma can be defined as a critical point at which the teacher is obliged to choose from a range of options (Sullivan et al., 2015).

In this study, we show how an experienced early childhood teacher planned and executed a problem-posing task with pupils who had no previous experience of this kind of activity. We give particular attention to what can be learned from the dilemmas that emerged at both the planning and implementation stages of the task.

Thus, the subject of this paper is the management of a problem-posing mathematics task with a particular focus on the way it develops from its design to its implementation. In doing so, we aim to answer the following research question: What variables should be taken into consideration in the design and implementation of a problem-posing task in early childhood education? With this study, we aim to contribute to the construction of theoretical frameworks for analyzing problem-posing teaching, the scarcity of which has been highlighted (Zhang & Cai, 2021). We also provide examples of teaching through problem posing in early childhood education, which can serve to counteract the lack of familiarity among teachers regarding how to put this approach into practice (Lee et al., 2018). We hope the study is of relevance to teacher education in terms of the implementation of problem posing in early childhood education.

## 2. Theoretical framework

### 2.1. Problem posing

There are numerous definitions of problem posing (Baumanns & Rott, 2022). We consider the posing of mathematical problems by pupils themselves either based on a given problem situation or generated by modifying existing problems (Cai & Hwang, 2020; Silver, 1994). Our focus henceforth will be on problems based on a given problem situation. In these cases, posing the problem occurs before attempts to solve it are made, and the aim is in the posing more than in the solution (Silver, 1994).

Cai and Hwang (2023) and Cai et al. (2022) divided problem-posing tasks into two parts: the *problem situation* and the *prompt*. The problem situation provides the context and the data that pupils can employ to create a problem, whereas the prompt provides instructions for guiding the pupils’ posed problems, making them aware of what the task expects of them (for example, “pose as many problems as you can”; Fig. 1). According to Cai et al. (2022), the problem situation can be purely mathematical or real. The data provided can be qualitative or quantitative and can include mathematical expressions and diagrams (Cai & Hwang, 2020). In addition, a problem-posing task can be structured or unstructured (Baumanns & Rott, 2020, 2022) according to the level of openness made available to the problem poser.

Following Cai (2022), we use the terms “task” and “problem” in a broad sense. This allows any mathematical question and any mathematical task based on the problem situation to be deemed problem posing, giving rise to the posing of problems along any point of the routine-nonroutine continuum (Baumanns y Rott, 2020). Therefore, we will not differentiate between an exercise (or routine task, where the learner knows what to do to solve it) and a genuine problem (for which the learner has not immediate answer).

### 2.2. Dilemmas in the planning and execution of a problem-posing task

Sullivan et al. (2015), drawing on Barbosa & de Oliveira (2013) underlined five dilemmas facing the teacher at the planning stage of tasks. Applying these dilemmas to a problem-posing task can provide insights into the configuration of key aspects of problem posing from design to implementation. In what follows, we describe the five dilemmas differentiated by Sullivan et al. (2015) and how we reinterpret them in relation to problem-posing tasks.

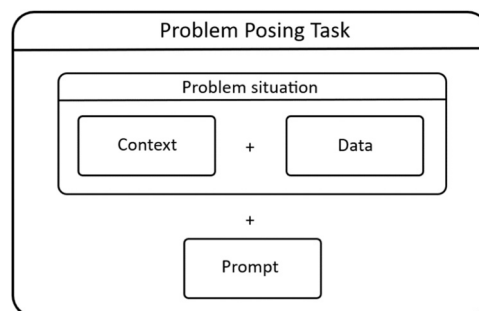


Fig. 1. Schematic representation of problem-posing tasks based on Cai and Hwang (2023).

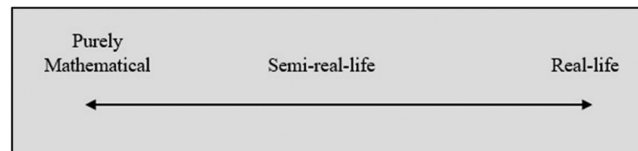


Fig. 2. The context dilemma in the design and implementation of a problem-posing task.

### 2.3. Dilemma related to the context variable

For [Barbosa and de Oliveira \(2013\)](#), one of the dilemmas a teacher faces in designing a task is whether to situate it in a real or mathematical context. The context of the problem situation in a problem-posing task can be based on real-world referents or, at the other extreme, purely mathematical or abstract referents ([Cai & Hwang, 2023](#)). An intermediate alternative is to locate the problem situation in a semi-real-life context ([Ponte & Quaresma, 2012](#)), which, unlike the real context, allows the incorporation of imprecisions that are not fully consistent with reality ([Fig. 2](#)).

A problem-posing task in a purely mathematical context might request connections to real-life contexts, as in the case of asking pupils to create story problems to go with mathematical expressions ([Cai & Hwang, 2023](#)). Nevertheless, the context and data provided in such tasks remain purely mathematical. With respect to the context dilemma, we would say that teachers tend towards the mathematical end of the continuum and set their pupils to pose problems considering given mathematical contexts.

The real-life context in which the problem situation is located can be provided in terms of (a) words or text; (b) pictures, figures, or other graphical material; or (c) a table. Conversely, a purely mathematical context can be provided by (a) a mathematical expression; (b) pictures, figures, or other graphics; (c) a table; or (d) a pattern ([Cai and Hwang, 2023](#)).

### 2.4. Dilemma related to the language variable

Sociocultural perspectives of human learning are founded on the idea that humans construct and comprehend knowledge in a social and cultural context through interactions promoting communication, discussion, and negotiation ([Vygotsky et al., 1978](#)). Language is placed at the heart of learning ([Lee, 2010](#)), and communication, thought, and learning are interrelated processes shaped by culture ([Mercer, 2004](#), p. 138). The dilemma for the teacher in planning a task arises from the need to consider, on the one hand, that an essential part of learning is mathematical precision and, on the other hand, that there is a need for language to be accessible for the pupils if it is to promote learning ([Sullivan et al., 2015](#)).

In terms of a task, the language includes the language promoted (i.e. the one the teacher ask for his or her students) as well as the language used by the teacher in setting and managing the task. The teacher can use and promote precision and formality in terms of mathematical language when they use and promote a linguistic register that prizes mathematical rigor above student comprehension. Conversely, using and promoting a conversational or colloquial register can be recognized in the teacher's use of terms that are accessible to their pupils. We can conceptualize these registers as comprising a continuum with invariable modes at the extremes, one involving a high and the other a low degree of mathematical precision ([Fig. 3](#)):

### 2.5. Dilemma related to the structure variable

As with language above, the structure of a task—the degree of liberty it confers—can be placed on a continuum from closed to open ended ([Barbosa & de Oliveira, 2013](#)). In this respect, [Stoyanova and Ellerton \(1996\)](#) identified three types of structures in problem-posing tasks: *free* when pupils pose problems based on an open mathematical situation ([Silber & Cai, 2016](#)) with no restrictions placed on how they go about this activity ([Baumanns & Rott, 2020](#)); *semi-structured* when the task provides more detailed information than free tasks and requires the posers to create problems using knowledge, skills, concepts, and mathematical relationships that have been previously learned ([Baumanns & Rott, 2022](#)); and *structured* when the task requires the new problems to be based on a previously solved problem. Considering the difficulty in distinguishing between free and semi-structured problem-posing tasks, we will adopt [Baumanns & Rott's \(2020\)](#) adaptation in which free and semi-structured are located at the end of a continuum running from open-ended (unstructured) to more structured tasks as the restrictions increase ([Fig. 4](#)).

Here, the dilemma in determining the structure of the task lies in the trade-off between taking account of the pupils' developmental stage and the learning goals. Settling on a free problem-posing task could be fruitful in terms of stimulating pupils' mathematical

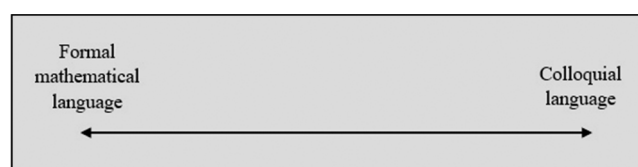


Fig. 3. The language dilemma in the design and implementation of a problem-posing task.

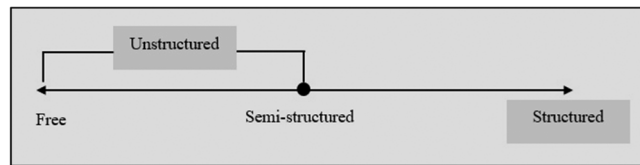


Fig. 4. The structure dilemma in the design and implementation of a problem-posing task.

creativity (Silver, 2013), providing opportunities to make strategic decisions on routes and destinations (Sullivan et al., 2015). At the same time, however, such tasks necessitate a greater cognitive load than do semi-structured and structured tasks (Ngah et al., 2016). By contrast, opting to increase the structure can provide scaffolding to support pupils through each stage of the task (Barbosa & de Oliveira, 2013) and can allow the teacher to focus pupils’ attention on the mathematical elements.

The structure variable is heavily dependent on the problem situation and prompt provided by the teacher (Cai & Hwang, 2023). This is because imposing or modifying a problem situation (the context or data) can shift the situation towards the structured end of the continuum (without it necessarily becoming fully structured); the prompt can likewise introduce a greater degree of structuring.

2.6. Dilemma related to the content focus variable

Barbosa and de Oliveira (2013) noted another dilemma in the choice of content focused on by the task, which is connected to the cognitive demand it involves. The dilemma for the teacher is judging the appropriate cognitive load, which can range from memorization at one extreme to doing mathematics at the other. Doing mathematics requires complex and non-algorithmic thinking, exploration and understanding of the nature of mathematical concepts, processes or relationships (Stein & Smith, 1998).

Problem-posing tasks have high cognitive demands, requiring creativity and a proactive attitude towards mathematics (Chico et al., 2022; Silver, 1994). In terms of the choice of content on which the problem-posing task focuses, problem posing can be oriented as a means, where the objective is to mobilize mathematical content linked to any area of mathematical knowledge, for example, to use a problem posing task to learn a determinate mathematical content such as when students are asked to posed a problem to show the application of  $3/4 \div 1/2$ , or as an goal in itself, where the objective focuses on the posing process and on the notion and nature of a mathematical problem, for example, to learn the different parts that a problem statement have (English, 1997). Working on problem posing as both a means and goal implies that the objective has a twin focus, that of mobilizing knowledge on the problem posing process and other mathematical knowledge. Hence, when a teacher is planning a problem-posing task, their dilemma can be understood to consist of deciding which goal to prioritize (Fig. 5).

2.7. Dilemma related to the levels of interaction variable

The levels of interaction concerning a task refer to the interactions between the teacher and pupils as the latter seek to find the solution (Sullivan et al., 2015). In problem-posing tasks, interaction is largely determined by how the pupils are organized (Cai, 2022)

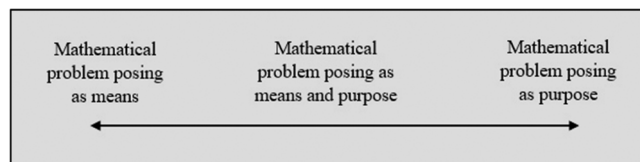


Fig. 5. The content focus dilemma in the design and implementation of a problem-posing task.

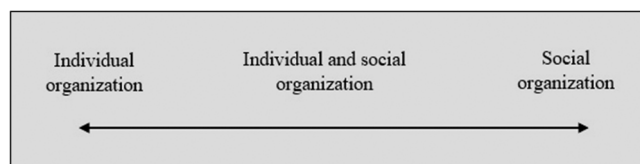


Fig. 6. The dilemma of the levels of interaction in the design and implementation of a problem-posing task.

and by the teacher's role. The pupils can be organized in different ways to tackle the task (individually, in pairs, in small and large groups), and this can change as they work their way through the task. The teacher's role in managing the lesson implies different actions (when to intervene, how to intervene, how much to guide the flow of discussion, etc.) that modulate the levels of interaction (Leiß & Wiegand, 2005). Establishing roles can be done, in some cases, via the prompts, which can have a significant influence on the levels of interaction. Hence, for example, the prompts "pose a problem for a classmate to solve" and "pose a difficult problem for the whole class to think about" configure two roles, those of emitter and receiver, whereas another prompt might give all the pupils the same role. The teacher's dilemma in planning (including implementing) a problem-posing task lies in determining which kind of interactions to prioritize (Fig. 6).

### 3. Methodology

To explore the teaching of problem posing with early childhood pupils, a case study was carried out based on a single case design (Yin, 2009). The unit of analysis for this study was the planning and carrying out of a problem-posing lesson. The case study approach was suitable for this research because the locus of interest was a learning environment in which the researchers aimed to achieve a detailed account of the teacher's task management. The specific design was an intrinsic case study (Stake, 1995). The case was of interest in itself due to the uniqueness of using a problem-posing approach at the early childhood stage of education. Allied to this was the expertise of the teacher in question, who had training in the use of problem-posing methodology and was an active participant in a collaborative research group involving study of and reflection on mathematical problem-posing literature.

#### 3.1. Context and participants

The case selected is that of an early childhood teacher, Beatriz for the purposes of this paper, because of the insights she provided into teaching the mathematical content in question (Yin, 2009). At the time of the study, Beatriz, a 39-year-old female, had 15 years' experience of teaching in the early childhood phase (ages 3–5). Her initial training had been in primary education, although all her teaching experience had been in early childhood education. The lesson observed (described in the following section) was Beatriz's designated class at the state school where she worked. The class consisted of 20 pupils between 4 and 5 years of age. There were no children with specific needs among her pupils, although two required special attention on account of their behavior and learning difficulties. Beatriz had been their class teacher since they started school. The school was a state-run early childhood and primary school located in a small town of around 13,000 inhabitants in southwest Spain.

#### 3.2. Data collection and analysis

With respect to lesson planning (prior to implementation), we drew on Beatriz's original handwritten lesson plan along with audio transcripts from three sessions of a Collaborative Research Project (known as the PIC based on its initials in Spanish). These sessions, in which Beatriz and two of the authors participated (in addition to another researcher and six teachers), involved discussion of the task design (Sessions 29/11, 17/01, and 31/01<sup>1</sup>). Specifically, Session 29/11 involved discussion of problem posing with early education pupils and the possible ways this could be achieved. In the following session (17/01), Beatriz went through her lesson plan and the group shared their thoughts, whereas in the last session (31/01), the group rehearsed the plan, with Beatriz carrying out her intended lesson and the other participants taking the role of the pupils. There then followed a discussion of the task design based on the run-

**Table 1**

Summary of variables used for analysing the design and management of the problem posing tasks.

Variables	Types
Context	Purely mathematical (provided by (a) a mathematical expression; (b) pictures, figures, or other graphics; (c) a table; or (d) a pattern). Semi-real-life (provided by (a) words or text; (b) pictures, figures, or other graphical material; or (c) a table) Real-life
Language	Formal mathematical language Colloquial language
Structure	Structured Unstructured
Content focus	Mathematical Problem posing as means Mathematical Problem posing as means and purpose Mathematical Problem posing as purpose
Level of interaction	Individual organization Individual and Social organization Social organization

through.

<sup>1</sup> The sessions took place on the 29th of November and 17th and 31st of January.



Fig. 7. Poster shown to pupils at the start of the problem-posing task (Martín-Díaz et al., 2020).

Beatriz also informed the PIC members that prior to the lesson on which this study is focused, she had carried out some preliminary work with her class on problem posing ([TP 29/11 8–10<sup>2</sup>]). A videorecording of the lesson (as described below) was also made by one of the members of the PIC (in the capacity of nonparticipating observer) and subsequently transcribed. All data pertaining to the lesson implementation was drawn from this source.

In preparation for the analysis, the authors undertook a careful reading of the dilemmas proposed by Sullivan et al. (2015) (based on Barbosa & de Oliveira, 2013) and discussed how these could be applied to problem-posing tasks. The discussion led to the incorporation of the dilemmas in Sullivan et al. (2015) guided by the theoretical principles of problem posing, as outlined in the theoretical framework and summarised in Table 1. As noted in the theoretical framework, instantiations of the variables should be seen as located along a continuum, rather than as discrete categories, with a view to capturing the development of the task over time.

Once all the information had been collated and the criteria for analyzing the dilemmas had been agreed on, the three authors carefully went over the transcriptions of the PIC meetings and watched the lesson recording with the corresponding transcription. This task served a dual purpose: It familiarized the researchers with the data that had been collected for the research and provided an opportunity for them to identify and interpret the dilemmas, based on Sullivan et al. (2015), that Beatriz encountered in planning and delivering the problem-posing lesson.

Each of the authors then carried out an individual content analysis (Krippendorff, 2004) of the transcriptions, identifying aspects of the complete problem-posing cycle, from planning to execution, linked to the five dilemmas described by Sullivan et al. (2015) and falling within the scope of the theoretical framework of this study. The purpose of this analysis was to identify units of information in which evidence could be identified for Beatriz's position with respect to one of the dilemmas.

The three individual analyses were then jointly discussed by the three authors until a single analysis was agreed on, which became the analysis report. We first analyzed and agreed on the documentation relating to the lesson plan and then repeated the process for the transcription of the videorecording of the actual lesson. Once all information had been analyzed, a new document was written, incorporating results from both the planning and execution stages with respect to each dilemma and noting any possible shifts in these with time. During the joint analysis, it was noted that Beatriz adapted her original problem-posing task during the lesson. In total, five points of variance were identified, which are considered in the data analysis reported in the results section below.

### 3.3. Description of the lesson

Beatriz settled on an open-ended problem-posing task based on a large poster (Fig. 7).

The task carried out during the problem-posing session lasted around an hour. The lesson followed a whole-class format throughout (with all the pupils sitting in a U shape so they could see the teacher and each other). The lesson evolved as follows (times are approximate):

First part (0–3 min): Beatriz reminds students of the assembly rules and tells them the story of the Fields Medal, which served to introduce and contextualize the mathematical activity they were about to do.

Second part (3–11 min): Beatriz uncovers the poster, letting the pupils see what was to serve as the first prompt for the problem-posing task (“Let’s look for any problems that might be in the poster”), and invites the pupils to name the items they recognize in the poster (“First we have to look very closely at the poster and all the different things here”). The pupils spontaneously name the items that attract their attention. The children begin to identify details in the poster, occasionally prompted by Beatriz. Exchanges such as the

<sup>2</sup> TP29/11 8–10 indicates that this is the transcription of the 29/11 PIC meeting between lines 8 and 10. (Idem for the 17/01 and 31/01 meetings).

following occur:

31 <sup>a</sup>	P	<i>I can see a firefighter... and a ladder</i>
32	T	<i>Where is the firefighter?</i> [Taking a plastic rod]
33	P	<i>There</i> [Pointing at the poster from their place]
34	T	<i>Not there, where is he?</i>
35	P	[The pupil attempts to develop their reply but is interrupted]
36	T	<i>On the left or on the right? Up or down?</i> [Using the rod to indicate the relative positions on the poster]

<sup>a</sup> The transcription records are labelled as follows: the number of each turn in sequential order; the speaker (T for teacher and P for pupil); the transcription of each speaker's utterance; accompanying gestures in square brackets.

42	T	<i>Let's see, Adrián, your turn.</i>
43	P	<i>I can see a traffic sign</i>
44	T	<i>Adrián sees a traffic sign. Where is it?</i>
45	P	<i>Up</i>
46	T	<i>Up. Where? Here?</i> [Pointing to the roof of a house with the rod]
47	P	<i>No, next to the firefighter.</i>
48	T	<i>Next to the firefighter, oh yes, that's right</i> [in a surprised tone]. <i>And what is the traffic sign like? What's it like?</i>

Third part (12–50 min): Beatriz commences the problem-posing task itself:

149	T	<i>Now we're going to look for problems... let's think of some stories we can make up ... and questions we can ask... [...] Let's make up some problems that come from here</i> [pointing to the poster] <i>from what we can see here... OK?</i>
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The pupils who come up with problems raise their hands and say them out loud. They are unable to formulate their problems in one go and require various attempts, supported by numerous interventions on the part of the teacher and their classmates. Each problem is posed collectively. The following excerpts illustrate the problem-posing processes.

155	T	<i>Let's see, who can think of a problem? Come on Luis... Let's all listen to Luis.</i>
156	P	<i>A lady is walking along with a bag</i> [pointing to the relevant part of the poster] <i>... she starts running and she gets hit by a car ...</i>
157	T	<i>But this is a story... you have to tell the story and then ask a question which we have to think about to get the answer ... come on... this woman is going along with a dog</i> [points to the poster with the rod] <i>... and now what happens? The question is which one is it?</i>

The previous problem develops such that the woman wants to find her way home. The pupils pose the question, "Which street does she need to take?" and between them all identify the route.

169	T	<i>You see? It's here</i> [pointing to the starting point on the poster] <i>... and then you have to go round the corner and take this road here, and here</i> [pointing out on the poster the route given by a pupil to the yellow house] <i>... OK?. And if...? Let's see, Lucía, and if...?</i>
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At a later point, Beatriz tries again to elicit problems:

309	T	<i>... And if I say a word ... and you all have to invent a problem for me with the word ... inside</i>
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312	T	<i>Invent a problem, Thiago, using the word 'inside'...</i>
313	P	[Seven seconds go by without an answer]
314	T	<i>Can you think of one?</i>
315	P	<i>It's really difficult</i> [Another pupil calls out]
316	T	<i>Can you think of one?. Or another problem that you can think of... come on... look, notice, for example, that there's a fruit and veg shop here...</i>
317	T	<i>... look closely, for example, here there are some things with a shape... there's a square shape, a circle...</i> [Pointing on the poster to the windows in the house and some traffic signs].
318	P	<i>Rectangle</i> [Another pupils answers]
319	T	<i>.which is in the shape of a rectangle... come on, let's make up a problem with the shapes...</i>

Multiple turns follow in which the pupils pose problems based on situations which for them represent problems, such as the following:

396	P	<i>I've got a problem now.</i>
397	T	<i>You've got a problem now.</i>
398	P	<i>A car goes to the barber's... and a police car is right behind him</i>

Beatriz then tries another strategy to get the pupils to pose problems.

413	T	<i>... I'm going to begin telling a story and you all ask me questions, OK? Otherwise this is going to be tragic, everyone is going to end up being run over by cars, falling out of trees ...</i>
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424	T	<i>Come on, I'm going to start a new problem, come and look... look, this mother wants to buy her son...</i>
425	P	<i>Some fruit</i>
426	T	<i>And what happens, María?</i>
427	P	<i>He wants an Apple, a grape and a pear [Pointing to the greengrocer's on the poster]</i>
428	T	<i>They want an apple, a grape and a pear ... so the question is...</i>
429	P	<i>How many pears, grapes and apples will there be?</i>

At this point in the lesson, Beatriz uses a drawing to take the posing of a problem further.

487	T	<i>So come on, say it again... let's draw it because I haven't understood very well [Taking a small blackboard and placing it next to the poster as she speaks]</i>
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491	P	<i>There's only an apple, a pear, an orange and a grape</i>
492	T	<i>An apple, a pear, an orange [drawing on the board]</i>
493	T	<i>One bunch of grapes or one grape?</i>
494	P	<i>Just one</i>
495	T	<i>One grape...</i>
496	T	<i>And now what? What is the question?</i>
497	P	<i>How many fruits are there?</i>
498	T	<i>How many fruits are there? Very well, how many are there?</i>

516	T	<i>One, two, three and four... it makes four... [Going over the fruit drawn on the board and the operation underneath] You all remember that it's written like this? [pointing to the operation written on the board] An apple, plus a pear, plus an orange, plus a grape makes four [going over the fruit drawn on the board and the operation underneath] (Fig. 8)</i>
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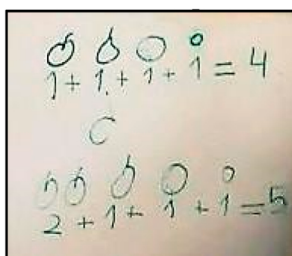


Fig. 8. The teacher's representation of one of the posed problems.

517	P	<i>Two</i>
518	T	<i>Two, no?... and if, for example, there were two apples [Draws another apple on the board] and now a pear, an orange and a grape... now what have we got, how many are there here [Going over the fruit drawn on the board] (Fig. 8)</i>
519	P	<i>Five</i>

With some scaffolding on the part of Beatriz, the contributions on the part of the pupils gradually develop into a coherent story together with an appropriate question. Once most pupils have had a turn at posing a problem, the task is brought to a close.

#### 4. Results

The data was analyzed by pinpointing variations in and developments to Beatriz's original design (which was designated Task 0) in terms of any substantive modification to the problem situation, the prompt, or both. In Task 0, the problem situation was supplied by the poster and the prompt by the instruction to "look for any problems that could be in the poster."

In the first modification to the original task, which we will designate Task 1, Beatriz invites the pupils to pose their own problems

based on a problem she has just set them. In the first instance, this involves identifying and explaining a short journey (“And if...? Let’s see, Lucía. And if...?” Turn 169). In the second instance, towards the end of the lesson, the problem consists of counting items of fruit (“And if, for example, there were two apples and now a pear, an orange and a grape...?” Turn 518). In both instances, the invitation to pose a problem follows directly from a problem that has just been solved, with the teacher eliciting an extension or new condition for the problem in keeping with the what-if-not strategy noted by [Brown and Walter \(1993\)](#). In the latter case, the teacher herself models the formulation of the new conditions.

In the second modification (Task 2), the teacher invites the pupils to pose problems based on a word evoking a mathematical object (“to invent a problem with the word ‘inside’”; Turn 312). In this instance, the problem situation is the whole poster and the locus of the modification is the prompt, which is reformulated to introduce a new condition.

A third modification occurs (Task 3) when Beatriz invites the pupils to pose problems based on a specific item in the poster, in this case the fruit shop (Turn 316). In this instance, the problem situation is still provided by the poster, but only a designated section is relevant to the task whereas the prompt remains to pose a problem.

In the fourth modification of the task (Task 4), the teacher invites the pupils to pose problems based on a mathematical concept (“like a rectangle”; Turns 317, 318, and 319). The problem situation is again provided by the poster, from which context and problem data can be derived. However, the prompt is modified with the addition of a new condition to the problem in the form of the phrase “like a rectangle.”

Finally, in the fifth modification (Task 5), the teacher invites the pupils to pose problems based on a story which she herself begins (Turns 413 and 424). In this instance, the problem situation is supplied by a combination of the unfinished story and the poster (as this provides the source of elements with which the pupils can complete the story). The prompt is to “finish the story to make a story problem.”

Although the original task was adapted and modified to such an extent that it generated five variations, these do not represent the totality of the lesson. Between the points at which she introduced these modifications, Beatriz attempted to direct her pupils’ efforts towards posing problems. Hence, in addition to the problem-posing tasks, it is important to consider below various aspects of the variables that emerged during the lesson. In like fashion, we also consider points of decision-making regarding the lesson plan and task management.

#### 4.1. Real-life or purely mathematical context

The problem situation from which the task started was a poster depicting a stylized real-life scene of a local neighborhood with people going about their daily business (traffic flow, buying and selling, and other professional activities; [Fig. 7](#)). The decision to use this problem situation had been made at the planning stage (“The pupils all go to their places at the start of the lesson and a poster is set up”; LP3<sup>3</sup>). This particular poster was chosen, rather than others that were considered, because it embodied various mathematical elements connected with the current teaching unit (such as larger-smaller, the same-different, inside-outside, first-second-last, problem-solving, addition, and flat shapes [TP17/01 39–43]). Thus, from the start, in terms of this dilemma, Beatriz located the problem-posing task in a semi-real-life context, with both the context and the data provided via a poster (problem posing based on an picture, semi-real-life context). It is interesting to note that in making the choice of picture to use, she considered its potential for drawing out mathematics.

In the lesson itself, Beatriz’s opening gambit for the task is to invite the pupils to pose problems based on the poster (problem posing based on an picture, semi-real-life context; see Turn 149 in the lesson excerpts). Her utterance clearly requests the pupils to pose a story-problem given that she refers to a story and questions (“let’s think of some stories we can make up ... and questions we can ask...”). The reference to the story continues throughout the lesson, situating her turn as the first step in the problem-posing process.

At first, the pupils’ responses diverge from the intended mathematical orientation, as they direct their attention towards identifying accidents (e.g., a car running over a pedestrian<sup>4</sup>—Turn 398). In response to this, Beatriz redirects their attention to new tasks that will

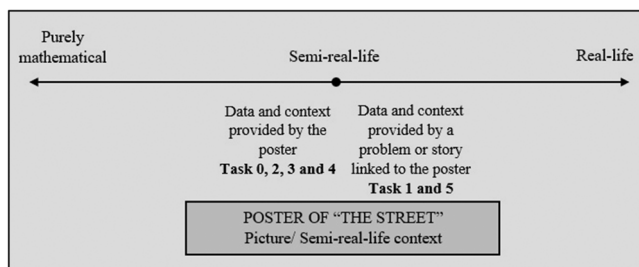


Fig. 9. The context dilemma in the design and implementation of the lesson.

<sup>3</sup> LP*i* refers to line *i* of the lesson plan, and LP*i*-*j* to an excerpt from the lesson plan consisting of lines *i* to *j*.

<sup>4</sup> At this point, the pupils activate their notion of “problem,” which they understand as a difficulty with negative connotations.

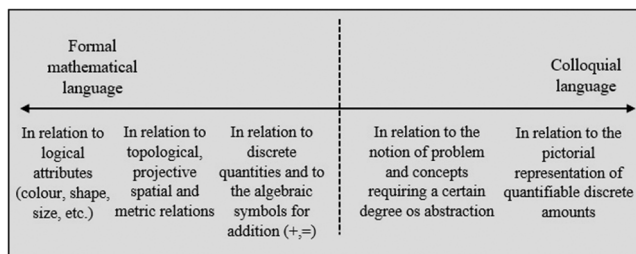


Fig. 10. The language dilemma in the design and implementation of the lesson.

enable the pupils to pose problems.

Nevertheless, in terms of the dichotomy between purely mathematical and real-life contexts, the context is invariable throughout the lesson. This is because the problem situations of all the tasks are provided by the poster. For example, in Task 1 the problem derives from one they have just solved and hence retains the poster as the source of the problem context, whereas in Task 3 the problem requires the pupils to focus on the fruit shop depicted in the poster.

The other variation of the task that we associate with the variable context is found in Task 5, when Beatriz asks the students to continue the story she has started (Turns 413 and 424). In this variation, she models the posing of a story problem, changing the mode of presentation of the problem situation (text instead of picture), although the type of context remains semi-real-life. Fig. 9 summarizes the development of this dilemma over the course of the lesson.

#### 4.2. Formal mathematical or colloquial language

Considering further the task modifications, it can be noted that Beatriz used a colloquial register most of the time. In Task 0, this can be seen clearly when she employs the notion of a problem (Turn 149). She introduces this notion to her pupils using an informal register carefully modulated to be comprehensible to them and approaching the concept through a story and “a question which we have to think about to get the answer” (Turn 157). Likewise, in Task 1, building on the fact that the pupils have posed a problem, she invites them to develop it with a familiar expression (“and if...?”). Again, in Task 3, Beatriz draws on situations familiar to the pupils when she suggests they pose a problem based on the fruit shop depicted in the poster.

By contrast, we can identify dilemmas governing the kind of language used in two adaptations of the task: Task 2 and Task 4. These modulations occur when Beatriz uses slightly more mathematical language to help the pupils focus their problem posing on basic mathematical concepts (using the words “inside” and “rectangle,” which are associated with concepts appropriate to early childhood education).

Although, in general, Beatriz’s classroom voice can be located in the colloquial register, mathematical language can be identified at the points at which Beatriz considers the use of more rigorous language to be compatible with her pupils’ knowledge and stage of development. Hence, in both the language used by the teacher and that required by the task, a demand can be observed for the pupils to identify and communicate the logical attributes of the elements in the poster and their position.

Thus, in the first phase of the lesson when she is encouraging the pupils to identify and recognize items in the poster (objects, people, action), Beatriz repeatedly uses a procedure in which she asks each child who correctly identifies an item for one or more of its logical attributes (e.g., its position and shape in Turn 48). Also, Beatriz uses and encourages mathematical language when talking about relative positions and going from one point to another in the poster, employing topological relations (inside-outside) (Turn 36), projective spatial relations (left-right, above-below) (Turn 46), and metric relations (near-far; Turn 48).

Beatriz also uses mathematical language when she designates discrete quantities using different representative registers (spoken, pictorial, conventional symbols) and additive operations. A significant episode occurs while solving one of the problems posed about quantifying the amount of fruit. Although the pupils use counting as their basic strategy, Beatriz attempts to encourage the use of addition as a strategy to solve the problem, using formal and precise language in both the spoken and written modes. This enables her to introduce the concept of addition and its attendant symbols (+, =), articulating this alongside the strategies used by the pupils (Turn 516). Fig. 10 summarizes the development of this dilemma over the course of the lesson.

Thus, there is evidence of a flexible use of language employed and encouraged by the teacher. The precision and rigor of mathematical language is employed alongside a colloquial and mathematically imprecise language in situations requiring the teacher to attempt to make certain concepts comprehensible to the pupils even though these are still inaccessible.

#### 4.3. Unstructured or structured problem-posing task

The main criterion for determining the structure to give the problem-posing task during the design phase arose in one of the PIC sessions. Here, Beatriz seemed especially concerned not to give her pupils too much information on which to base their problems, fearing that this might overload them and stifle their creativity [TP31/01 637–646].

Given that, in Task 0, Beatriz provides her pupils with the context and specific data for the problems they are to pose via the poster while also providing a free prompt, this original task can be considered an unstructured task. The five adaptations that the task

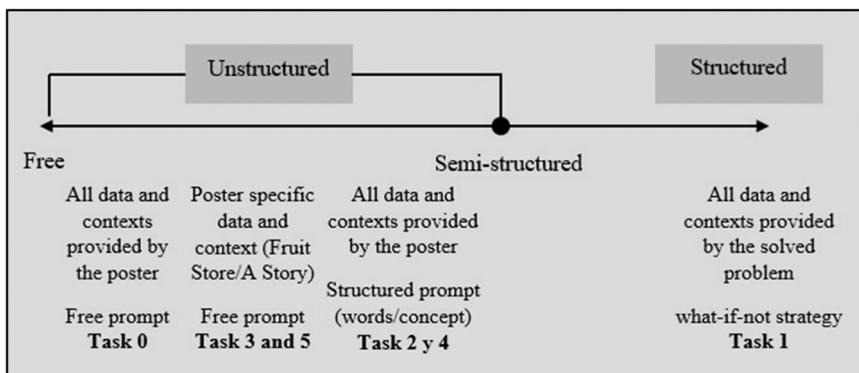


Fig. 11. The structure dilemma in the design and implementation of the lesson.

underwent indicate how the degree of structuring was gradually increased, though it predominantly remained an unstructured problem-posing task.

The task with the greatest degree of structure which can be located on the fringes of a structured task, is Task 1 (Turn 169), in which Beatriz asks the pupils to pose a problem based on one they have just completed. Some of the adaptations the variable structure underwent were the result of introducing new prompts. For example, in Task 2, Beatriz modifies the task prompt, asking the pupils to “invent a problem with the word ‘inside’” (Turn 309). The prompt does not in itself convert the task into a structured situation, but it does constrain the kind of problems that can be posed because they must necessarily incorporate the term. Hence, the degree of structuring of the task increased compared to Task 0. The same occurred when the pupils were invited to pose a problem using the word “rectangle” in Task 4.

Elsewhere, the task structuring increased due to a narrowing of the problem situation. This was the case in Task 3 (Turn 316), in which, although the poster continues to supply the frame of reference, it is the fruit shop which is considered the context and potential source of data. The original free prompt (pose a problem) remains unchanged.

Finally, in Task 5 (Turns 413 and 424), an increased degree of task structuring is achieved by modifying both the problem situation and the prompt: The context was narrowed to a shopping activity, thus constraining the complete poster to only the retail locations (the fruit shop, the clothing store, and the pharmacy). Furthermore, compared to posing a problem, the task is more tightly restricted by the addition of the instruction to start from the beginning of the given story. In this respect, this second prompt (pose a problem by completing the story) is more structured than the first (pose a problem). Fig. 11 summarizes the development of this dilemma.

- OBJECTIVES
- To learn what a problem is (parts: data, question)
  - To pose problems
  - To know how to pose problems from the same situation and recognise this variety
  - (To a lesser extent) To apply the content worked on to a problem
  - Given a problem, to pose problems based on the given one
  - To better understand a situation by posing problems about it

Fig. 12. The teacher’s objectives as listed in her lesson plan.

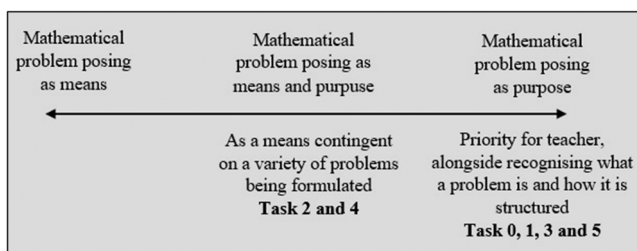


Fig. 13. The content focus dilemma in the design and implementation of the lesson.

#### 4.4. Mathematical problem posing as a means or as a purpose

This dilemma reflects Beatriz’s position on whether problem posing should be a purpose in itself or a means through which the pupils can explore other mathematical content. This position can be seen clearly in her lesson plan and in the run-through with the PIC group prior to executing the plan in class. Her lesson plan makes it explicit that she considers problem posing a purpose in itself. Indeed, she gives the plan the title of “Strategies for facilitating problem posing using a complex picture” [LP1].

In both the run-through with the PIC group and the actual lesson itself, it can be seen that Beatriz’s emphasis is for the pupils to identify what a problem is and how to pose one, stressing the need to “ask a question” (Turns 149, 157, and 413). After the run-through has been completed, Beatriz makes it clear that her priority is for her pupils to pose a variety of mathematical problems rather than focusing attention on other mathematical content: “What I want is not to teach anything, or even try to ... what I want is a variety of problems” [TP31/01 1143–1144].

The set of objectives listed at the start of the lesson plan ([LP42–48] Fig. 12), reiterates the emphasis on problem posing as a purpose in itself. Although the approach is also recognized as a means of tackling content beyond problem posing and problem solving, it is clear that this is focused on “to a lesser extent.”

Beatriz herself notes a variety of content items that could be tackled through problem posing based on the poster (quantity, which is larger, inside and outside, ordinals, problem solving, addition, flat shapes; [TP46–49]), but underlines that her chief interest in noting these areas is above all to ensure that a variety of problems are posed (“not all arithmetical, like ‘How many?’” in her own words; [TP31/01 674–688]).

Across the five adaptations, it can be seen that the content focus is problem posing as the purpose, especially in the case of Tasks 0, 1, 3, and 5, in that Beatriz is highly focused on her pupils managing to produce problems irrespective of the content they might have. Nevertheless, in both Task 2 and Task 4, it is possible that the aim of the task, to a much lesser extent than the posing in itself, was to activate certain mathematical content the pupils had worked on previously. This change could have been due to the teacher’s intention mentioned in the previous paragraph in which she stated that she did not want all the problems to be of an arithmetic type. For this reason, we have located the focus of the problem-posing lesson to be on purpose although in some cases it shifts towards problem posing as both a means and a purpose. Fig. 13 summarizes the development of this dilemma.

#### 4.5. Individual or social organization

In terms of class organization, the lesson was conducted at the whole-class level (Cai et al., 2022), with the pupils seated in a U-shaped “assembly” formation and the poster positioned in front of them to ensure everybody had a good view so as to pose a problem or comment on the contributions of their classmates.

Two roles established by the pupils were identified: emitters and receivers. The framework established by the task was for each pupil in turn to pose a problem for the rest of the class to solve (Turn 157).

With a view to encouraging the pupils to solve the task together, Beatriz listened actively to their contributions and carefully monitored what they said. She responded to almost every pupil’s intervention so as to involve the others in the discussion, encouraging them to amplify their responses (e.g. Turns 32, 34, 36) and eliciting from them the elements constituting the problem (Turns 157, 428). In this regard, teacher-pupil interaction was directed towards the whole class, such as when Beatriz proposed a different version of the task being suggested (e.g. Turns 149, 155, 309) or elaborated on a pupil intervention for the benefit of the group as a whole.

### 5. Development of the problem-posing task

Table 2 summarizes the developments that the original task underwent with respect to the variables used in the study. It includes

**Table 2**  
Development of the lesson by task and variable.

Variables	Task 0	Task 1	Task 2	Task 3	Task 4	Task 5
Context	Semi Real (picture)	Semi Real (problem)	Semi Real (picture)	Semi Real (picture)	Semi Real (picture)	Semi Real text/picture
Structure	Un-structured	Structured	Un-structured	Un-structured	Un-structured	Un-structured
Language	Colloquial	Colloquial	Colloquial / Mathematical	Colloquial	Colloquial / Mathematical	Colloquial
Level of interaction	Social	Social	Social	Social	Social	Social
Content Focus	Purpose	Purpose	Means / Purpose	Purpose	Means / Purpose	Purpose

the five variables (context, language, structure, content focus, and levels of interaction) and the tasks described above. Each variable is shaded according to the variations it underwent with each task. The darker shades are associated with one end of the continuum and the lighter shades with the other.

The table reads from left to right, starting with the original task presented in the form of a poster (Task 0) to the final adaptation (Task 5). As can be seen, the task did not always maintain the same constitution regarding the variables but rather underwent modifications as a result of the dilemmas inherent in each of the variables. These dilemmas were not linear but rather zigzagged according to the teacher's intentions.

## 6. Conclusions

This study contributes to our understanding of how a problem-posing task is planned and carried out. In this sense, it aligns with the need to enrich our understanding of the design and implementation of problem-posing tasks (Yao et al., 2021), given that the greater part of research into problem-posing tasks focuses on the results of the task—that is, on the characteristics of the resultant problems—and to a much lesser degree on the actual design of the problem-posing task itself (Lee, 2021).

This research has demonstrated that the adaptation of Barbosa and de Oliveira's (2013) dilemmas to the field of problem posing can be linked to variables (context, language, structure, levels of interaction, and content focus) which could be useful with respect to analyzing the design and management of mathematical problem-posing tasks.

The analysis has shown that the variables are not static but rather can fluctuate and undergo changes depending on the teacher's choices or intentions at different stages of the lesson. The most significant modifications were in response to the structure variable. The difficulty the students encountered in posing mathematically oriented problems within the parameters of the original task (semi-real-life context, unstructured, free prompt) resulted in new tasks being improvised with a concomitant modification of the structure. The teacher's starting hypothesis that an unstructured problem-posing task would elicit a greater diversity of problems was reconsidered in favor of more structured tasks. This greater structuring was achieved both through more structured prompts (as referred to in Cai & Hwang, 2023), more constrained problem situations, or both.

One variable which provided interesting results was that of context, which underwent various adaptations in terms of its representation (whether an illustration, a previous problem, or text). The element that remained constant throughout these reformulations was the poster, to which the variations made slight adaptations in order to provide support for the pupils in their problem-posing.

Although the problem situation of the original task underwent some adaptations, it remained a semi-real-life context throughout, chiefly because the poster was used as the reference during the whole lesson.

In general, with the exception of the variable levels of interaction, although the location of the dilemmas on their respective continuums never reached extremes, there were some shifts. These shifts, and consequently the decisions they represented with respect to the dilemmas, did seem related. Thus, some ways of increasing the degree of structure provided for the task, in task 0, were achieved by using more mathematically oriented language in the prompt (e.g. in tasks 2 and 4, which included words evoking mathematical objects such as "inside" and "rectangle"). These new prompts brought an additional focus on learning certain mathematical concepts to that of learning to pose problems. On the other hand, other means of providing the task with greater structure, such as narrowing its context or formulating it as a text rather than an illustration, maintained the focus on problem posing and invited more colloquial language.

Further, this study shows the importance of considering a problem-posing task from a dynamic perspective (or an emergent design; Bikner-Ahsbabs & Janßen, 2013) in which the problem situation and prompt anticipated in the design stage are adapted as the task is put into action. This is another possible contribution of this study in contrast with others in which this kind of contingency was not contemplated (Silber & Cai, 2021; Singer & Voica, 2013).

Given the scarcity of research on this topic in early childhood education (Cai & Hwang, 2020; Martín-Díaz et al., 2020) this case study is especially valuable and could contribute to our understanding of how the characteristics of this educational stage constrain the manner in which the dilemmas are solved. In addition, this paper provides an analytical description of a problem-posing lesson at the classroom level (Cai et al., 2022) and thus offers a valuable case study for teaching problem posing (Liljedahl & Cai, 2021).

One of the most critical factors shaping Beatriz's position vis-a-vis dilemmas are the contextual characteristics of the early childhood classroom.

On the one hand, although problem solving is a mainstay of Beatriz's teaching, this lesson represented the pupils' first encounter with mathematical problem posing and therefore represented an interruption to their familiar classroom routines (Herbst, 2012). Her decision to orient problem posing as a purpose in itself underlies the need to encourage the understanding of the notion and nature of mathematical problems (English, 1997). The pupils' first attempts evoked an incipient manifestation of the notion of problem that was closer to real life than to mathematics. The initial choice to attempt the activity with the pupils via an unstructured problem-posing task (Baumanns & Rott, 2020) with a free prompt (Cai & Hwang, 2023) might have had a greater impact in stimulating a personal conception among the pupils of what constitutes a problem. Our experience in this study suggests that the notion of a problem constituted by a story with a question could be particularly appropriate for early childhood education. Nevertheless, the effectiveness of the task with respect to the notion of the problem involved requires further study.

On the other hand, the pupils' developmental stage—making it impossible to provide written information and imperative to use spoken language and visual cues to support the posing process—constricted the teacher's choice of task design regarding the context in

which to locate the problem situation and the means of presenting the contextual information. In addition, the levels of interaction established among the pupils to encourage the problem posing were determined by the teacher's need to mediate the dialogue, discussion, and negotiation involved (Palmér & Bommel, 2020), leading her to introduce problem posing as a whole-class activity (Kwon & Capraro, 2021).

Given that the study focused on early childhood education, future research should serve not only to correlate and uncover the workings of these variables in the planning and carrying out of problem-posing tasks at different levels (Guo et al., 2021) but also to find out whether any additional variables arise at other educational stages.

### Declaration of Competing Interest

The authors of the manuscript, Enrique Carmona Medeiro, Juan Pedro Martín-Díaz and Nuria Climent, declare that: They have not received any financial or non-financial support provided by a third party for the reported work. They have no interest or financial relationship, within the last 3 years, related to the subject or the manuscript.

### Data availability

Data will be made available on request.

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