Control of Series Active Power Filter by State Feedback

S. P. Litrán, P. Salmerón, R. S. Herrera, J. R. Vázquez

Electrical Engineering Department
Escuela Politécnica Superior, Huelva University
Ctra. de Palos de la Frontera s/n 21819, Palos de la Frontera
Teléfono: 959 217585, Fax: 959 217304
e-mail: salvador@uhu.es; patricio@uhu.es; reyes.sanchez@die.uhu.es; vazquez@uhu.es

Abstract. The active power filters in series connection or series APF, are static compensation systems based on an electronic converter PWM (Pulse Width Modulation). The systematic use of series APFs in the power systems allows the elimination of the harmonics caused by specific loads named voltage source harmonic loads. To obtain this compensation objective, several strategies have been proposed up now. Nevertheless, each strategy development has been based on the equivalent electrical circuit and on the results obtained in simulation and/or experimental tests. Generally, the control structure formal analysis is not carried out. In this work, the whole system state model has been obtained. Besides, the system behavior has been analyzed from the state equations for each compensation strategy. As a consequence, the analysis has allowed the establishment of design rules respect to the resultant control strategy. Finally, a practical case is presented, whose results illustrate the proposed method.

Key words

Harmonics, series active power filter, state space.

1. Introduction

The harmonic distortion has been a problem that has been present since the electric engineering origin. Nevertheless, it has gone more important in the last decades, due to the electronic loads proliferation because they behave, in general, as non-linear loads, [1].

The APF have been used in the last years to eliminate the harmonic distortion in electrical systems. An APF is a static compensation system based on an electronic converter PWM (Pulse Width Modulation). It may be connected in parallel or in series to the load. The shunt APF is connected in parallel to the load and it goes as a controlled current source. Its systematical use allows the elimination of the current harmonic originated by the named current source harmonic loads. This kind of APF has been the traditionally applied. It is the most studied configuration.

The series APF is especially appropriate to eliminate the voltage harmonic caused by the named voltage source harmonic loads. They are, among others, the frequency converters, commutated power sources or UPSs (Uninterruptible Power Supply). Nevertheless, its correct performance is fixed by the control approach adopted to establish the reference harmonic voltage. Three kinds of compensation strategies have been mainly used. Originally, the first one proposed a reference voltage proportional to the source current harmonics. Later, another strategy has been proposed where the filter voltage is equal to the load voltage harmonic with the opposite sign. Recently, another strategy has been proposed, combination of the both previous. Since the practical point of view, the last one evolves the advantages of the other two and it overcomes their disadvantages, [5-8].

This work is focused on the analysis of the control strategies of the series three-phase active filters. A state model of the system has been developed. Based on this description, an analysis of the compensation strategies earlier referred has been carried out.

The analysis developed has allowed the knowledge of the system dynamic behavior and the stability margins in each situation. As a consequence, an adequate chosen of the final design parameters could be carried out. At last, the system behavior has been contrasted by means of a practical case simulation in the Matlab-Simulink framework.

2. Series Active Filters. Compensation Strategies

Figure 1 show the topology used to eliminate the load voltage harmonics with active filter in the case of voltage source harmonic loads. L_s represent the voltage source inner inductance. In the dc side, the inverter has been connected to two sources whose values are constant. In the AC side, a filter has been connected to eliminate the high frequency components. This filter is composed of an inductance L_R and a capacitor C_R. The transformers connect the compensator to the system.

A previous analysis of the different compensation strategies can be obtained analyzing the system in steady-state by means of its equivalent single-phase circuit. The model used in this previous development is shown in figure 2. It is defined considering harmonics different of the fundamental one. So, the voltage source does not present harmonic components and the only voltage harmonics present in the system are the produced by the load. The load has been modeled by means of a voltage source, V_{LH}. In figure 2, Z_S represents the source impedance corresponding to the harmonic H frequency. The filter is modeled by a controlled source which generates the voltage corresponding to the chosen strategy. Since an ideal point of view, the load has been represented by a voltage source whose frequency corresponds to the harmonic analyzed.

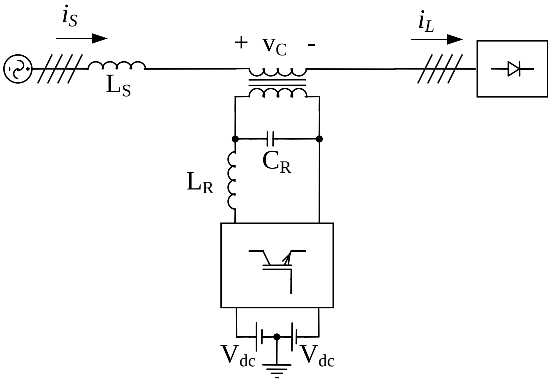

Fig 1. Connection diagram of a series active filter

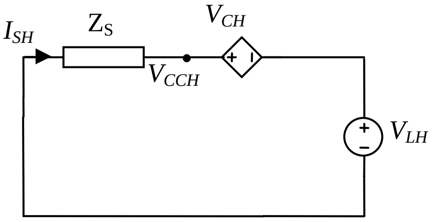

Fig 2. Equivalent single-phase circuit corresponding to harmonic different of the fundamental one

The series active filters control has been carried out by means of three different compensation strategies:

- Control strategy which measures the load voltage to make that the APF generates a voltage with that same harmonic content and the opposite sign.

$$v_{CH} = -v_{LH} \tag{1}$$

Nevertheless, the measurement of the load voltage harmonic generally depends on the instrumentation sensibility k_v. So, the voltage supplied by the APF applying this strategy can be expressed as follows:

$$v_{CH} = -k_v\, v_{LH} \tag{2}$$

According to this expression, the voltage in the point of common coupling is, for a specific harmonic $h \in$H:

$$V_{CCh} = V_{Lh}(1-k_v) \tag{3}$$

- Control strategy based on the source current measurement. It consists of supplying a voltage proportional to the source current harmonics. It is:

$$v_{CH} = \mathrm{k}\, i_{SH} \tag{4}$$

where k is a proportionality constant. The active filter behaves as a resistance whose value corresponding to the fundamental frequency is zero. Its value, corresponding to the different net frequencies, is k ohms. The voltage in the point of common coupling corresponding to the h order harmonic is:

$$V_{CCh} = \frac{Z_{Sh}}{\left(Z_{Sh} + k\right)} V_{Lh} \tag{5}$$

The adequate vale of k is a non solved question. It is because, on the one hand, k can not be very high to insure the system stability; on the other hand, as higher k value is, as better filtering characteristic the filter presents.

- Hybrid strategy where the APF supplies a voltage that combines the previous two strategies:

$$v_{CH} = k\, i_{SH} - k_v\, v_{LH} \tag{6}$$

The voltage in the point of the common coupling is the next:

$$V_{CCH} = \frac{Z_S\left(1-k_v\right)}{\left(Z_S + k\right)} V_{LH} \tag{7}$$

The simultaneous action of the factor (1-k_v) in the numerator and the (Z_S+k) in the denominator, decrease the influence of the mistake in the load harmonic voltages measurement. On the other hand, the simultaneous effect allows the utilization of a smaller k value.

The analysis of the equivalent electric circuit shown in figure 2 corroborates the superiority of the third strategy over the other two. However, the previous analysis has been carried out in a qualitative level. It can only be useful as reference for a more formal analysis in the control theory framework. This question is treated in the next section.

3. Analysis Based on State Variables

To represent the system by means of state variables, the circuit shown in figure 3 is used. It presents the net single-phase model shown in figure 1 corresponding to any order harmonic h$\in$H different from the fundamental one, [2].

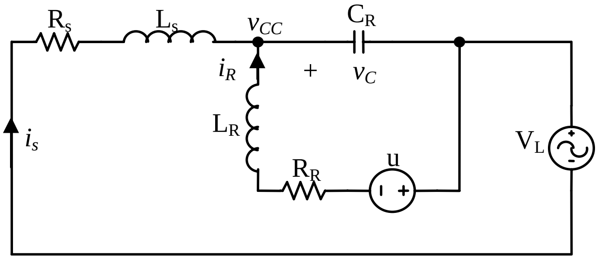

Fig 3. Circuit single-phase model

The elements present in figure 3 are:

R_s, L_s: source resistor and inductance.

C_R, L_R: capacitance and inductance of the filter used to eliminate the high frequency components caused by the converter PWM commutation.
R_R: resistor which models the losses in the active filter, in the passive filter and in the coupling transformer.
V_L: voltage in the non-linear load side

This system is characterized by the next state equation:

$$\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{A}\,\boldsymbol{x} + \boldsymbol{B_1}\,\boldsymbol{u} + \boldsymbol{B_2}\,\boldsymbol{v_L}\\ \boldsymbol{y} &= \boldsymbol{C}\,\boldsymbol{x} + \boldsymbol{D_1}\,\boldsymbol{u} + \boldsymbol{D_2}\,\boldsymbol{v_L}\end{aligned} \qquad (8)$$

where the state vector is:

$$\boldsymbol{x} = \begin{bmatrix} i_S & i_R & v_C \end{bmatrix}^T \qquad (9)$$

The A matrix is expressed as:

$$\boldsymbol{A} = \begin{bmatrix} -\dfrac{R_s}{L_s} & -\dfrac{1}{L_s} & -\dfrac{1}{L_s} \\ -\dfrac{1}{L_R} & -\dfrac{R_R}{L_R} & -\dfrac{1}{L_R} \\ \dfrac{1}{C} & \dfrac{1}{C} & 0 \end{bmatrix} \qquad (10)$$

B_1 is the vector with the next value:

$$\boldsymbol{B_1} = \begin{bmatrix} 0 & \dfrac{1}{L_R} & 0 \end{bmatrix}^T \qquad (11)$$

And the B_2 vector is defined as follows:

$$\boldsymbol{B_2} = \begin{bmatrix} -\dfrac{1}{L_s} & 0 & 0 \end{bmatrix}^T \qquad (12)$$

The voltage in the point of the common coupling, v_{CC}, has been established as the output variable. So, the matrix C is:

$$\boldsymbol{C} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \qquad (13)$$

And finally $\boldsymbol{D_1}$=[0] and $\boldsymbol{D_2}$=[1]. From now, each compensation strategy will be analyzed according to the state model (8).

When the first control strategy proposed is applied, the voltage u has the value corresponding to the expression (2). So the system state equation is:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\,\boldsymbol{x} + \left(\boldsymbol{B_2} - \boldsymbol{B_1}\right)\boldsymbol{v_L} \qquad (14)$$

The state model represented by the equation (14) shows up that the first compensation strategy does not modify the original system dynamic, i.e., the system poles location is not changed. With respect to the PCC voltage, the control based on the load voltage harmonic measurement is an open control. So, the stability is not interfered. Besides, its compensation characteristics only depend on the sensibility of the instruments used to measure the load harmonics and on the converter PWM control accuracy. The control gain k does not influence the control with this strategy. It may be an advantage. Actually, it presents the disadvantages corresponding to an open control.

When the second strategy is adopted, the control signal u is:

$$u = k\, i_S \qquad (15)$$

And in matrix format:

$$\boldsymbol{u} = \boldsymbol{K}\,\boldsymbol{x} \qquad (16)$$

Where the matrix $\boldsymbol{K}$ has the next expression:

$$\boldsymbol{K} = \begin{bmatrix} k & 0 & 0 \end{bmatrix} \qquad (17)$$

The state equation is:

$$\dot{\boldsymbol{x}} = \left(\boldsymbol{A} + \boldsymbol{B_1}\,\boldsymbol{K}\right)\boldsymbol{x} + \boldsymbol{B_2}\,\boldsymbol{v_L} \qquad (18)$$

This control modifies the system matrix because it is closed by one of its state variables. So, a state feedback control is carried out.

The third strategy combines the two earlier. In this case, the state model is characterized by the next expression:

$$\dot{\boldsymbol{x}} = \left(\boldsymbol{A} + \boldsymbol{B_1}\,\boldsymbol{K}\right)\boldsymbol{x} + \left(\boldsymbol{B_2} - \boldsymbol{B_1}\right)\boldsymbol{v_L} \qquad (19)$$

Now, the system new matrix has the same form as the model corresponding to the strategy based on the source current harmonic measurement. The load voltage harmonics take part in the same way as the first compensation strategy. The model presented by the equation (19) is effectively a combination of the previous strategies.

4. Application to a Practical Case

To corroborate the behavior of the different strategies, the figure 1 circuit has been considered. The elements values are the indicated in table I. The stability analysis and the control design are carried out from the state model presented in section 3.

TABLE I. Passive elements values

L_s	R_s	L_R	R_R	C_R
0.1 mH	1 Ω	5 mH	1 Ω	50 µF

For the values presented in table 1 and when the control signal u is null, the system has a low frequency gain of -6 dB, with three poles and three zeros, placed as shown in figure 4. Figure 5 shows the poles and zeros map obtained applying the first control strategy.

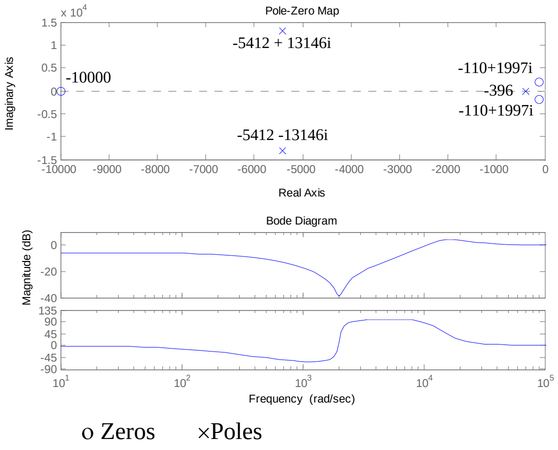

Figure 4. Poles and zeros map and Bode diagram, when control sign is null

As it was introduced in the previous analysis, this strategy does not modify the system matrix. So, the poles will be located in the same place as in Fig. 4.

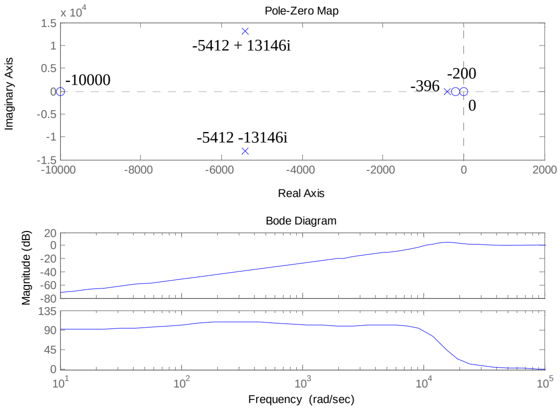

Figure 5. Poles and zeros map and Bode diagram, when u=-v_L

It is not the case of the zeros. So, if the main objective is the elimination of the harmonics in the source side, a low gain of the interesting harmonic frequencies will be necessary. These frequencies are those that include the most significant harmonics. For a typical nonlinear load, they are below 3500 rad/s. In the Bode diagram it is observed how the gain varies from -72 dB with low frequencies, up to -14 dB with 3500 rad/s.

In the second strategy, the control signal is proportional to the source current harmonics. For the determination of the proportionality constant, the gain criteria have been considered. So, the PCC voltage gain, in decibels, and the load voltage at source voltage harmonic frequencies, is sufficiently small. In this case, the *k* gain is fixed to -40 dB. The polynomial characteristic of the system depends on *k* according to the equation (20).

$$\varphi(A)=\det\left|s\,I-A\right|=s^{3}+11220\,s^{2}+ \\ +\left(206.4\cdot10^{6}+2\cdot10^{5}\,k\right)s+8\cdot10^{10}+4\cdot10^{10}\,k \qquad (20)$$

If the zeros values and the gain of the original system are considered, it results k=98. In the Bode diagram presented in figure 6, it is possible to observe that the objective of gain -40 dB is satisfied. Nevertheless, this k value will destabilize the system. Figure 6 confirms this consideration. There is a pair of poles in the right semi-plane of poles and zeros map.

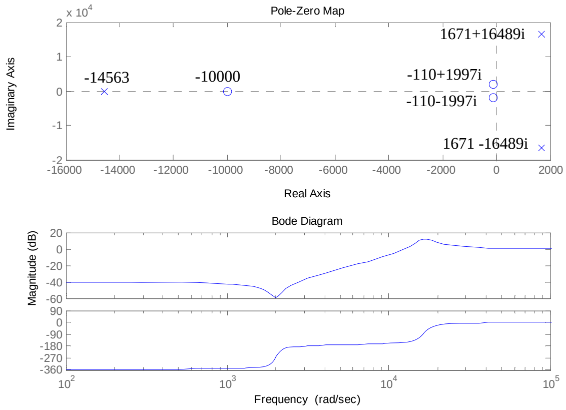

Figure 6. Poles and zeros map and Bode diagram, when u=98 i_s

In order to know the maximum value of k that the system is destabilizes, the stability criterion of Routh-Hurwitz is applied to the characteristical polynomial of the system. The method shows that a change of sign is produced when k=59. For this value, two complex poles located on the imaginary axis are obtained. A value of k lower than this value will assure the stability of the system. Nevertheless, the poles will moved away of the origin to reduce the instability risk. Another aspect considered to determine the constant k is the speed of the system answer.

The final solution adopted is k=40. The poles of the system are shown in figure 7. The system is stable, and the low frequencies gain is about -32 dB. This gain will go to -25 dB at 3500 rad/s.

The hybrid strategy corresponds to the state model represented in the equation (19). Therefore, the zeros will fit in with the first strategy ones and the poles with the second strategy ones.

The analysis of this strategy is presented in figure 8, where a value of k=40 is considered. In the frequency range studied, the gain is reduced. This reduction is more significant to low frequencies, up -98 dB. This allows the reduction of the k value, and the corresponding control signal generated. If k=10 is considered, the gain at low frequencies becomes -81 dB.

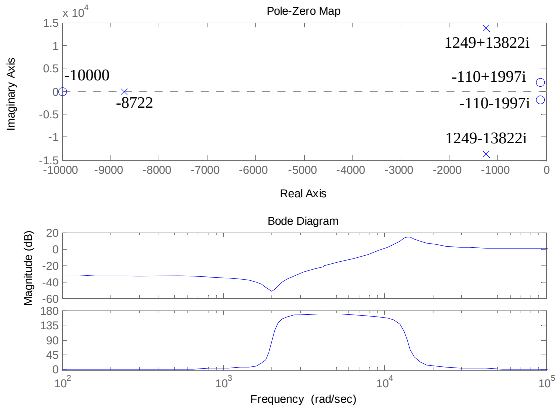

Figure 7. Poles and zeros map and Bode diagram, when u=40 i_s

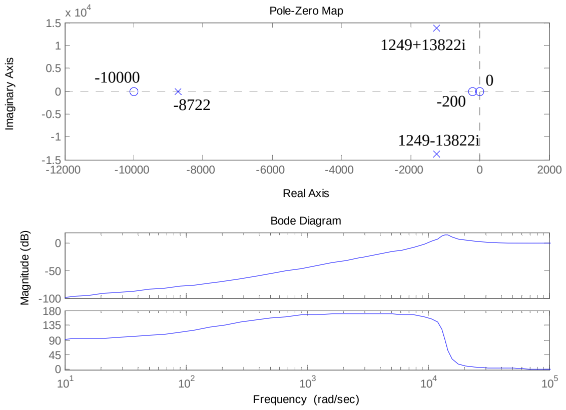

Figure 8. Poles and zeros map and Bode diagram, when u=40 i_s-v_L

The first strategy is the best to eliminate the output harmonic. With the second one, it is possible to modify the system dynamic, and to reduce the errors in the filter voltage generation. With the hybrid control, smaller values of k allow to obtain the proposed objective, avoiding instability problems of the system.

The three strategies have been applied, at simulation level, in a three-phase electrical system. To do that, Simulink and SymPowerSystem blockset have been used. The HVS load used includes three single-phase rectifiers with a 1000 μF capacitor and a 5 Ω resistor in the dc side. The rest of the parameters are shown in table 1. In figure 9, the voltage waveform in the point of common coupling is presented. In this case, the active filter is not connected. This signal total harmonic distortion (THD) is 10,7%. The harmonic present are the odd order, being most significant those of the order 3, 5, 7 and 9.

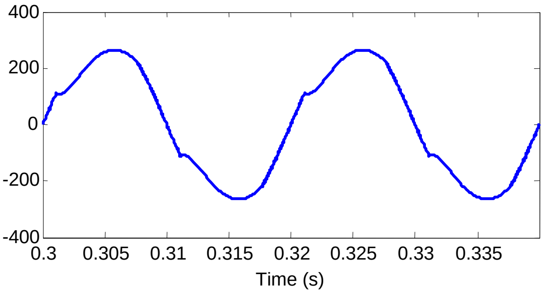

Figure 9. Voltage waveform in the point of common coupling when the active filter is not connected.

When the active filter is connected, the first strategy reduces the voltage THD in the source side to 3.83 %. Figure 10 shows this voltage waveform.

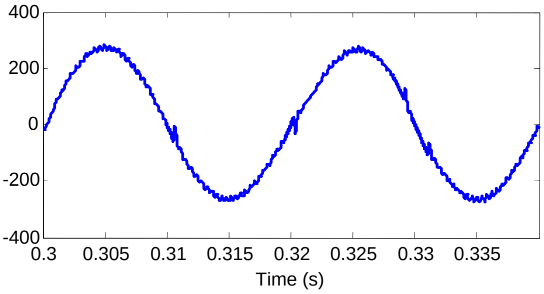

Figure 10. Voltage waveform in the point of common coupling when the APF strategy control is $u=-v_L$

Figure 11 presents the voltage waveform obtained in the source side when the second strategy is applied with k=40. In this case, the voltage THD is 1.82 %. The reduction of the gain with high frequencies in this case allows the THD decrement.

With respect to the hybrid strategy, the constant k value has been diminished up to 10. The result is the waveform presented in figure 12.

The obtained THD is 1,67 %, a value lower than the obtained with the previous strategy with a small k value.

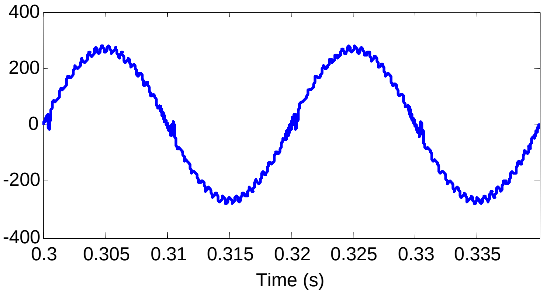

Figure 11. Voltage waveform in the point of common coupling when the APF strategy control is $u=40i_S$

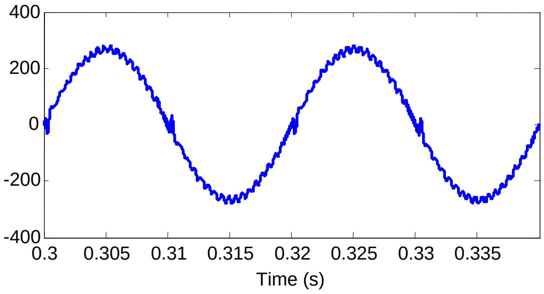

Figure 12. Voltage waveform in the point of common coupling when the APF strategy control is $u=10i_S-v_L$

5. Conclusions

The harmonics generated by the non-linear loads named voltage-source type of harmonic sources can be compensated by series active power filter. However, their operation characteristics depend decisively on the type of compensation approach adopted to generate the compensation voltage references. In this paper, three compensation strategies have been initially analyzed, from the point of view of the equivalent electrical circuit: the detection of the voltage load, the detection of the source current and a hybrid control.

The electrical analysis demonstrated the superiority of the hybrid control approach with respect to the two previous strategies. Nevertheless, this type of approach does not allow the knowledge of the system stability and it does not establish any criterion for the election of the design parameters. Here, the state model of a compensated system has been found by means of a series active filter. From it, the three types of compensation strategies have been analyzed. In the state space, the terms of the hybrid control have been analyzed. Then, the system dynamic behavior and the design parameters election have been studied. Finally, the results of a practical case have allowed the verification of the proposed scheme validity.

Acknowledgments

This work is part of the projects "A new technique to reduce the harmonic distortion in electrical systems by means of equipment of active compensation", ref. DPI2004-03501, sponsored by the “Comisión

Interministerial de Ciencia y Tecnología, CICYT, del Ministerio de Ciencia y Tecnología" of Spain, and "Design and implementation of a new equipment of active compensation with series connection for the improvement of the electrical waveform quality", ref. P06-TEP-02354, sponsored by the "Consejería de Innovación, Ciencia y Empresa de la Junta de Andalucía", of Andalucía, Spain.

References

[1] H. Akagi, "Active harmonic filters," *Proceedings of the IEEE* Volume 93, Issue 12, Dec. 2005 Page(s):2128 – 2141.

[2] A. Ghosh, A. K. Jindal, A. Joshi, "Design of a capacitor-supported dynamic voltage restorer (DVR) for unbalanced and distorted loads," *IEEE Trans. Power delivery*. Vo, 19, no.1, January 2004, pp. 405-413.

[3] Y. S. Kim, J. S. Kim and S. H. Ko, "Three-Phase Three-Wire Series Active Power Filter, which Compensates for Harmonics and Reactive Power", *IEE proc. Electr. Power* Appl., Vol, 151, no. 3, May 2004, pp. 276-282.

[4] S. P. Litrán, P. Salmerón, J. R. Vázquez and R. S. Herrera, "Different control strategies applied to series active filters", in Proc. *ICREPQ*, May, 2007.

[5] F. Z. Peng and D. J. Adams, "Harmonics sources and filtering approaches," in Proc. *Industry Aplications Conference*, October 1999, Vol, 1, pp. 448-455.

[6] F. Z. Peng, H. Akagi, A. Nabae, "A novel harmonic power filter," in Proc. *IEEE/PESC*, April, 1988, pp. 1151-1159.

[7] F. Z. Peng, H. Akagi, A. Nabae, "A new approach to harmonic compensation in power systems-a combined system of shunt passive and series active filters," *IEEE Trans. Industry Applications.* Vol, 26, no. 6, Nov/Dec 1990, pp. 983-990.

[8] Z. Wang, Q. Wang, W. Yao and J. Liu, "A series active power filter adopting hybrid control approach," *IEEE Trans. Power Electronics*, vol. 16, no. 3, May 2001, pp. 301-310.