Instituto Nacional de Investigación y Tecnología Agraria y Alimentaria (INIA)
Available online at www.inia.es/forestsystems
http://dx.doi.org/10.5424/fs/2013221-02142

Forest Systems 2013 22(1), 15-24
ISSN: 2171-5068
eISSN: 2171-9845

Modelling diameter distributions of *Quercus suber* L. stands in "Los Alcornocales" Natural Park (Cádiz-Málaga, Spain) by using the two parameter Weibull functions

A. Calzado Carretero* and E. Torres Alvarez

Departamento de Ciencias Agroforestales. Universidad de Huelva. Spain

Abstract

Aim of study: The aim of this work was to model diameter distributions of *Quercus suber* stands. The ultimate goal was to construct models enabling the development of more affordable forest inventory methods. This is the first study of this type on cork oak forests in the area.

Area of study: The area of study is "Los Alcornocales" Natural Park (Cádiz-Málaga, Spain).

Material and methods: The diameter distributions of 100 permanent plots were modelled with the two-parameter Weibull function. Distribution parameters were fitted with the non-linear regression, maximum likelihood, moment and percentile-based methods. Goodness of fit with the different methods was compared in terms of number of plots rejected by the Kolmogorov-Smirnov test, bias, mean square error and mean absolute error. The scale and shape parameters in the Weibull function were related to the stand variables by using the parameter prediction model.

Main results: The best fitting was obtained with the non-linear regression approach, using as initial values those obtained by maximum likelihood method, the percentage of rejections by the Kolmogorov-Smirnov test was 2% of the total number of cases. The scale parameter "*b*" was successfully modelled in terms of the quadratic mean diameter under cork ($R^2_{adj} = 0.99$). The shape parameter "*c*" was modelled by using maximum diameter, minimum diameter and plot elevation ($R^2_{adj} = 0.40$).

Research highlights: The proposed model diameter distribution can be a highly useful tool for the inventorying and management of cork oak forests.

Key words: maximum likelihood method; moment method; non linear regression approach; parameter prediction model; percentile method; scale parameter; shape parameter.

Introduction

Cork oak (*Quercus suber* L.) is one of the major forest species in Spain, where it occupies an overall 475,000 ha (24% of the surface it covers worldwide). Spain is the second world producer of cork (the main product of *Q. suber*), with 24% of the world output (Pereira, 2007). Cork oak stands additionally possess ecological, social and economic significance by virtue of their many uses, which include grazing, masting, browsing, hunting, erosion control, climate regulation, biodiversity maintenance, wildlife habitation, carbon storage, landscape enrichment and recreation.

Reconciling the large variety of uses of *Q. suber* stands makes management of these forests rather complex. Growth models are especially useful for educated decision-making in this context. A number of models have been developed to estimate cork weight for individual trees or stands in Spain (Vázquez-Piqué and Pereira, 2008). However, none of these models has so far analyzed diameter distributions in this species.

Most *Q. suber* woodlands are natural stands that have received no systematic, sustained silvicultural treatment. As a result, only some specific stands exhibit well-defined stand structure (even-aged stand, uneven-aged stand or two-aged stand) (Montero and López, 2008). Cork oak woodlands in the Aljibe massif are very heterogeneous, with large differences in tree spatial distribution, species, density, diameter distribution and stand vertical structure (Torres and Montero, 2000). The silvicultural practices of cork oak forests in Cortes de la Frontera (Málaga, Spain) have recently been revised in depth. As a result, a growing trend to developing carefully designed management practices for increasingly small masses has emerged. In this context forest stand

* Corresponding author: carrete@uhu.es
Received: 12-09-11. Accepted: 23-01-13.

management was chosen as a suitable method for these areas (De Benito, 2008).

The associated inventories should provide detailed information about the dasometric characteristics of each stand, determination of which requires heavy and expensive sampling. Palahí *et al.* (2006a) developed an effective method to reduce the complexity and cost of sampling for major species in Catalonia —*Q. suber* included—. The method involves measuring stand variables and using diameter distribution models in subsequent calculations. Other authors have also noted the usefulness of diameter distribution models for forest inventories (Kilkki *et al.*, 1989; Maltamo, 1997; Liu *et al.*, 2002; Palahí *et al.*, 2006a).

Broadly speaking, an accurate knowledge of diameter distributions is highly useful to describing and analysing the structure of forest stands; also, it can be useful for estimating age distribution and assessing stand stability in addition to calculating the number of trees in each diameter class with a view to planning silvicultural treatments.

Diameter distributions have so far relied preferentially on the Weibull function on account of its flexibility, the ability to describe a wide range of unimodal distributions and the relatively easy estimation of its parameters (Rubin *et al.*, 2006). We chose to use the two-parameter Weibull function on the grounds of the good modelling results previously obtained for diameter distributions in various species (*e.g.* Maltamo *et al.*, 1995; Álvarez González, 1997; Maltamo, 1997; Nanos and Montero, 2002; Bullock and Burkhart, 2005; Palahí *et al.*, 2006a, 2006b; Gorgoso *et al.*, 2007).

The aim of this work was thus to model diameter distributions of *Quercus suber* stands in the Aljibe massif by using the two-parameter Weibull function, the parameters "*b*" and "*c*" were related to ecological and stand variables. Four different fitting methods were compared: percentiles, moments, maximum likelihood and non-linear regression. The ultimate goal was to construct models enabling the development of more affordable forest inventory methods and, as an aside, to improve existing models for cork production.

Material and methods

Study area

"Los Alcornocales" is a natural park spanning 167,767 ha in one of the largest cork producing areas in Spain. The local relief is hilly, with steep slopes, 23.5% of the park area having slopes exceeding 34.5% (Junta de Andalucía, 2004). The area has a Mediterranean climate comprising two phytoclimatic subtypes: IV_2 and IV_4 (Allué, 1990). The average annual temperature is 14-18°C and annual precipitation ranges from 700 to 1,200 mm. The most abundant substrate consists of Aljibe sandstones, which form soils of the Cambisol and Luvisol types mainly (FAO), and are accompanied by clays, marls and limestones in Vertisols. Cork oak stands constitute the most extensive plant formation in the area. *Quercus suber* occurs either in pure stands or mixed with *Q. canariensis* in shady zones and valley bottoms, as well as interspersed with trees of *Olea europaea* var. *sylvestris* in drier, transition zones.

Data acquisition

Data were obtained by measuring the 100 circular plots between 2008 and 2009. Plot size ranged from 378 to 5,418 m^2 depending on the particular stand density (in order to achieve a minimum of 30 trees per plot). A total of 3,226 trees were measured. The study area was stratified in accordance with Torres and Montero (2000), who split "Los Alcornocales" into two large formations: sub-sclerophyllous cork oak forests and sclerophyllous cork oak forests. The number of plots in each stratum was calculated in proportion to its surface area. Also, the plots in each formation were chosen in such a way as to ensure that their mean tree diameters would be evenly distributed throughout their potential range and representative of the site qualities and densities of the study area. Table 1 lists the number of plots sampled per stratum and mean diameter range.

The trees in each plot were all subjected to the following measurements: perimeter at breast height over cork, total height and cork thickness in two normal di-

Table 1. Number of plots sampled per stratum and mean diameter range

D_{gu}	Sub-sclerophyllous cork oak forest	Sclerophyllous cor oak forest
< 25	11	4
25-30	17	5
30-35	19	4
35–-40	20	2
> 40	17	1

D_{gu} quadratic mean diameter under cork (cm).

rections. The geographic coordinates for the centre of each plot were measured with a GPS receiver and the position of each tree was georeferenced by measuring its direction and distance to the plot centre.

Preliminary calculations

Prior to analysis, data were subjected to the Grübbs test in order to identify potential outliers (Dagnelie, 2006). Only those trees having an abnormal diameter and lying outside the main stand (*i.e.* those residual trees belonging to a pre-existing stand) were excluded (22 trees from 18 different plots in total). Torres (1995) previously developed cork production models for cork oak forests in the Aljibe massif where he also excluded residual trees and considered the main stand only.

Data were used to calculate the following stand variables: number of trees per hectare (N), mean diameter over and under cork (D_{mo} and D_{mu}), quadratic mean diameter over and under cork (D_{go} and D_{gu}), basal area (G), mean height (H_m) and dominant height (H_o) was calculated from the percentage of the 100 thickest trees per ha (Assmann, 1970). Calculations also included the following percentiles for the frequency distribution of diameters under cock: 1% (P_1), 5% (P_5), 10% (P_{10}), 16.731% (P_{17}), 25% (P_{25}), 50% (P_{50}), 75% (P_{75}), 90% (P_{90}), 95% (P_{95}), 97.366% (P_{97}) and 99% (P_{99}). Table 2 shows the mean values for the main stand variables in the sampled plots.

Estimation of the target parameters

The Weibull cumulative distribution function is obtained by integrating its probability density function. For a random variable x, the function is expressed as follows:

$$F(x)=\int_0^x \left(\frac{c}{b}\right)\cdot\left(\frac{x-a}{b}\right)^{c-1}\cdot e^{-\left(\frac{x-a}{b}\right)^c}dx = 1-e^{-\left(\frac{x-a}{b}\right)^c}$$

with $x \geq a$, $a \geq 0$, $b \geq 0$ and $c \geq 0$. In this equation, $F(x)$ denotes the cumulative relative frequency of trees with a diameter equal to or smaller than the random variable x; "a" is the location parameter that is assumed to be zero in the two-parameter version of the equation; "b" is the scale parameter; and "c" is the shape parameter.

Diameter distributions were fitted for diameters under cork. This variable was preferred to the diameter over cork as it is more representative of tree size. Cork age can vary between sample plots, so diameter over cork may vary widely depending on cork thickness.

Parameters were estimated and compared by using four different methods based on percentiles, moments, maximum likelihood and non-linear regression.

Percentile method

The method of percentiles was previously employed for a similar purpose by several authors (Zarnoch and Dell, 1985; Shiver, 1988; Nanang, 1998; García Güemes *et al.*, 2002; Zhang *et al.*, 2003; Gorgoso *et al.*, 2007). The parameters in the Weibull function were calculated from the following equations:

$$\ln \hat{b} = \frac{\ln x_r - \dfrac{\ln x_t \cdot \ln(-\ln(1-r))}{\ln(-\ln(1-t))}}{1-\dfrac{\ln(-\ln(1-r))}{\ln(-\ln(1-t))}}$$

$$\hat{c} = \frac{\ln\left[\dfrac{\ln(1-r)}{\ln(1-t)}\right]}{\ln\left(\dfrac{x_r}{x_t}\right)}$$

where x_r and x_t are the sample percentiles with $0 < r < 1$ and $0 < t < 1$. Following Dubey (1967), Bailey and Dell (1973) proposed using two different pairs of percentiles to estimate the parameters, namely: P_{17} and P_{97} or P_{40} and P_{82}. Later on, Shiver (1988) proposed using the 16.731 and 97.366 percentiles.

Table 2. Summary of main stand variables measured in the sampled plots

Variable	Mean	Maximum	Minimum	Standard deviation
Density (trees/ha)	269.9	858.9	55.4	146.3
Basal area (m^2/ha)	24.5	50.7	5.6	7.7
Quadratic mean diameter (cm)	32.7	50.4	18.1	7.4
Mean height (m)	9.5	15.6	5.6	2.1
Dominant height (m)	10.7	16.8	6.8	2.1

Moment method

The moment method, which was previously used to estimate the parameters in the Weibull function by some authors (Shifley and Lentz, 1985; Shiver, 1988; Lindsay *et al.*, 1996; Nanang, 1998; Río and Montero, 2001; Zhang *et al.*, 2003; Gove, 2003; Merganic and Sterba, 2006; Gorgoso *et al.*, 2007), relies on first- and second-order sample moments for calculation. The expressions relating parameters "*b*" and "*c*" in the Weibull function to the sample mean and variance are as follows:

$$b = \frac{\bar{d}}{\Gamma\left[1+\frac{1}{c}\right]}$$

$$\sigma^2 = \frac{\bar{d}^2}{\Gamma^2\left(1+\frac{1}{c}\right)} \cdot \left[\Gamma\left(1+\frac{2}{c}\right) - \Gamma^2\left(1+\frac{1}{c}\right)\right]$$

where $\bar{d}$ is the mean diameter of the distribution, σ^2 the variance and Γ the gamma function. Substituting the calculated mean and variance into the previous expressions provides two equations in two unknowns that allow "*b*" and "*c*" to be readily estimated.

Maximum likelihood method

The maximum likelihood method has also been widely used to estimate the parameters in the Weibull distribution function (Maltamo *et al.*, 1995; Maltamo, 1997; Álvarez González and Ruiz González, 1998; Maltamo *et al.*, 2000; Nanos and Montero, 2002; Gove, 2003; Cao, 2004; Newton *et al.*, 2005; Palahí *et al.*, 2006*a*, 2006b; Gorgoso *et al.*, 2007). This method allows parameters "*b*" and "*c*" to be calculated from the following equations:

$$\frac{\sum_{i=1}^{n} (x_i)^{\hat{c}} \ln(x_i)}{\sum_{i=1}^{n} (x_i)^{\hat{c}}} - \frac{1}{\hat{c}} = \frac{1}{n}\sum_{i=1}^{n} \ln(x_i)$$

$$\hat{b} = \left[\frac{1}{n}\sum_{i=1}^{n} (x_i)^{\hat{c}}\right]^{\frac{1}{\hat{c}}}$$

where n is the number of observations and x_i the diameter for each individual tree in the sample.

Non-linear regression method

This method, which has been successfully used by authors such as Álvarez González (1997), García Güemes *et al.* (2002) and Gorgoso *et al.* (2007), requires appropriate initial values as input data. Such values can be obtained by using the maximum likelihood method (Álvarez González, 1997) or the percentile method (García Güemes *et al.*, 2002; Gorgoso *et al.*, 2007). In this work, parameter convergence was obtained with the Marquardt iterative algorithm (Marquardt, 1963) in SPSS, using the maximum likelihood values as input data.

Goodness of fit

Goodness of fit was assessed via the Kolmogorov-Smirnov test at the 20% significance level. Mean absolute error (MAE), mean square error (MSE) and bias were calculated, which were obtained from the following equations:

$$MAE = \frac{\sum_{i=1}^{n} |y_i - \hat{y}_i|}{n}$$

$$MSE = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}$$

$$Bias = \frac{(y_i - \hat{y}_i)}{n}$$

where $\hat{y}_i$ is the cumulative frequency obtained by applying the model to each plot datum and y_i is the cumulative frequency for each observation in each plot.

Modelling distribution parameters in terms of stand variables

Prior to modelling, the presence or absence of significant differences in Weibull parameters between the two strata (sclerophyllous an sub-sclerophyllous cork oak forest) was confirmed with an analysis of variance.

The Pearson correlation coefficient between the Weibull function parameters obtained with the non-linear regression method and the following variables was calculated and analysed: plot elevation (Z); number of trees per hectare (N); basal area (G); mean diameter over and under cork (D_{mo} and D_{mu}), quadratic mean diameter over and under cork (D_{go} and D_{gu}); mean height (H_m); dominant height (H_o); the percentiles P_1, P_5, P_{10}, P_{25}, P_{50}, P_{75}, P_{90}, P_{95} and P_{99}; maximum and minimum diameter (D_{max} and D_{min}); the ratios of P_{25} to the quadratic mean diameters over and under cork (P_{25}/D_{go} and P_{25}/D_{gu}); and their respective natural logarithms [L (P_{25}/D_{go}) and L (P_{25}/D_{gu})].

Table 3. Mean values of bias, mean absolute error, mean square error (both in number of trees per one) and number of plots rejected by the Kolmogorov-Smirnov test ($\alpha = 0.20$) for the four fitting methods applied to the Weibull distribution function

Fitting method	Bias	MAE	MSE	KS
Percentile	0.1187	0.0635	0.0075	34
Moment	0.0308	0.0481	0.0039	9
Maximum likelihood	0.0256	0.0469	0.0036	9
Non-linear regression	–0.0031	0.0325	0.0017	2

MAE: mean absolute error. MSE: mean square error. KS: number of plots rejected by the Kolmogorov-Smirnov test.

Parameter prediction method

This method related the parameters in the Weibull function to stand variables via straightforward linear models. Parameter "*b*" was modelled by stepwise regression, using the input variables mentioned in the preceding paragraph. The shape parameter, "*c*", was modelled in two different ways; one (Method 1) involved stepwise regression with the same input variables as with parameter "*b*" and the other (Method 2) relating "*c*" to the ratio of P_{25} to the quadratic mean diameter, a procedure which provided good results in previous studies (Álvarez González, 1997; Gorgoso *et al.*, 2007). In order to facilitate application of the resulting equation, a second model predicting P_{25} in terms of stand variables was also fitted.

As with correlations, only the data for the plots not rejected by the Kolmogorov-Smirnov test were considered in the analysis.

Validation of the model

The ensuing model was verified by cross-validation. The fitting procedure was applied as many times as plots were studied, with omission of the data for one plot in each run. Calculations included the Kolmogorov-Smirnov test, MSE, MAE and bias between the observations for each plot and the predicted values provided by the model with exclusion of the plot concerned.

Results

Estimation of the target parameters

Table 3 shows the errors, bias and number of fits rejected by the Kolmogorov-Smirnov test in the fitting of "*b*" and "*c*" in the Weibull function with the percentile, moment, maximum likelihood and non-linear regression methods. Table 4 gives the means and standard deviations for the shape and scale parameters.

Based on the results of the Kolmogorov-Smirnov test, the non-linear regression method is the most suitable for fitting diameter distributions of *Q. suber* stands; in fact, only 2% of the fits were rejected. The moment and maximum likelihood methods provided worse results, which, however, were similar to each other. Finally, the percentile method was the worst performer, with 34% of rejections.

The non-linear regression method provided the lowest mean square error (MSE), mean absolute error (MAE) and bias; by contrast, the percentile method had the poorest results in these respects. The moment and maximum likelihood methods performed similarly except for a slightly higher bias with the former. Based on the foregoing, the non-linear regression method provided the best fits.

Modelling distribution parameters in terms of stand variables

An analysis of variance of the two parameters in the Weibull function revealed the absence of significant differences ($p > 0.05$) between the two strata (sclerophyllous and sub-sclerophyllous cork oak forest).

Table 5 shows the correlations found between the Weibull parameters as calculated with the non-linear regression method and stand variables. As can be seen, the highest correlation for parameter "*c*" was that with the ratio of P_{25} to the quadratic mean diameter under

Table 4. Mean and standard deviation of the scale and shape parameters in the Weibull function

Parameter	Mean	Standard deviation
b (scale)	33.3	7.3
c (shape)	5.9	1.5

cork: in addition, "*c*" exhibited moderately high correlation ($p < 0.01$) with the diameter distribution percentiles from P_1 to P_{50}, and also with the minimum diameter. There was also significant correlation between "*c*" and plot elevation. On the other hand, the highest correlation for "*b*" was that with the quadratic mean diameter under cork. This parameter was additionally correlated, with a high significance, with all other variables except the basal area.

Parameter prediction method

Following fitting with the non-linear regression method, the parameters in the Weibull function were processed with the parameter prediction method. Stepwise regression of the data led to the following equation:

$$b = 1.02 + 0.257 D_{gu} \; (R^2_{adj} = 0.99)$$

Table 5. Correlations between the principal stand variables and the parameters of the Weibull function as obtained with the non-linear regression method

Variable	*c*	*b*
Z	0.271*	0.246*
N	–0.086	–0.723**
G	0.118	–0.023
D_{mo}	0.226*	0.942**
D_{mu}	0.259*	0.979**
D_{go}	0.210*	0.970**
D_{gu}	0.235*	0.996**
H_m	0.251*	0.701**
H_o	0.247*	0.627**
P_1	0.533**	0.694**
P_5	0.542**	0.808**
P_{10}	0.476**	0.902**
P_{25}	0.436**	0.968**
P_{50}	0.309**	0.993**
P_{75}	0.156	0.993**
P_{90}	0.059	0.956**
P95	0.016	0.922**
P_{99}	0.001	0.883**
D_{max}	0.003	0.869**
D_{min}	0.524**	0.734**
P_{25}/D_{gu}	0.843**	0.306**
P_{25}/D_{go}	0.702**	0.516**
$L(P_{25}/D_{gu})$	0.823**	0.303**
$L(P_{25}/D_{go})$	0.570**	0.446**

** Significant at the $p < 0.01$ level. * Significant at the $p < 0.05$ level. Z: Plot elevation (m), N: density (trees/ha). G: basal area (m^2/ha), D_{mo}: mean diameter over cork (cm), D_{mu}: mean diameter under cork (cm), D_{go}: quadratic mean diameter over cork (cm), D_{gu}: quadratic mean diameter under cork (cm), H_m: mean height (m). H_O: dominant height (m). P_1: 1% percentile (cm). P_5: 5% percentile (cm). P_{10}: 10% percentile (cm). P_{25}: 25% percentile (cm). P_{50}: 50% percentile (cm). P_{75}: 75% percentile (cm). P_{90}: 90% percentile (cm). P_{95}: 95% percentile (cm). P_{99}: 99% percentile (cm). D_{max}: maximum diameter (cm). D_{min}: minimum diameter (cm). $L(P_{25}/D_{gu})$: ln (P_{25}/D_{gu}). $L(P_{25}/D_{go})$: ln (P_{25}/D_{go}).

Method 1

Parameter "*c*" was modelled as a function of the maximum diameter, minimum diameter and plot elevation (*Z*). The ensuing equation was

$$c = 5.262 + 0.151 D_{\min} - 0.055 D_{max} + \\ + 1.58 \times 10^{-6} Z^2 \; (R^2_{adj} = 0.40)$$

Method 2

The equation relating parameter "*c*" to the ratio of P_{25} to the quadratic mean diameter was:

$$c = 10.463 \left(\frac{P_{25}}{D_{gu}} \right)^{3.1074} \qquad \left(R^2_{adj} = 0.77 \right)$$

Using this equation required fitting a second model capable of predicting P_{25} in terms of stand variables:

$$P_{25} = 18.285 - 0.038N + 0.433G + 0.915H_m \; (R^2_{adj} = 0.789)$$

Validation of the model

Table 6 shows the validation results obtained with the two methods used. Method 1, which calculated "*c*" as a function of the maximum and minimum diameters, and plot elevation, provided better results in terms of bias, errors and number of rejections in the Kolmogorov-Smirnov test than did Method 2, which calculated "*c*" as a function of P_{25} and the quadratic mean diameter. The parameter "*b*" was modelled as a function of the quadratic mean diameter under cork in Method 1 and 2. Fig. 1 shows the absolute histogram of observed

Table 6. Bias, mean absolute error, mean square error (both in number of trees per one) and number of plots rejected by the Kolmogorov-Smirnov test ($\alpha = 0.20$) for the two methods derived by cross-validation

Method	Bias	MAE	MSE	KS
1	–0.0017	0.0469	0.0037	14
2	–0.0116	0.0784	0.0126	33

MAE: mean absolute error. MSE: mean square error. KS: number of plots rejected by the Kolmogorov-Smirnov test.

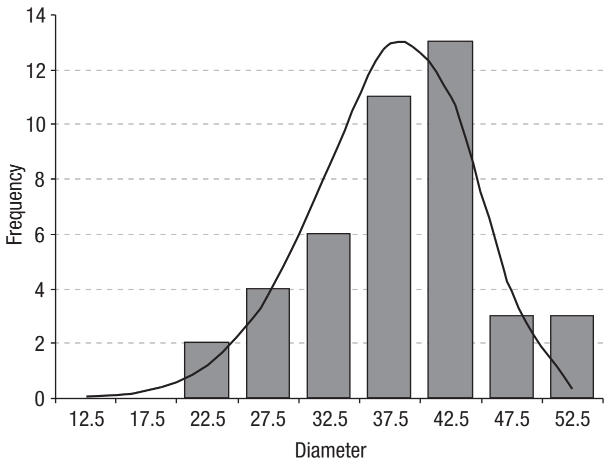

Figure 1. Absolute histogram of observed diameter distribution and distribution fitted by Method 1 of a plot with average results in terms of godness of fit.

diameter distribution and distribution fitted by Method 1 of a plot with average results in terms of godness of fit.

Discussion

Judging by the plot rejection results of the Kolmogorov-Smirnov test, errors and bias, the most accurate fitting method was non-linear regression using the values from the maximum likelihood method as input data. Non-linear regression was previously used to estimate the parameters in the Weibull distribution function and proved an advantageous choice over alternative methods (Álvarez González and Ruiz González, 1998; García Güemes *et al.*, 2002; Gorgoso *et al.*, 2007). Thus, the non-linear regression method has been found to provide better results than the maximum likelihood method in stands of *Pinus pinaster* (Álvarez González and Ruiz González, 1998); the percentile method in *Pinus pinea* (García Güemes *et al.*, 2002), and both the percentile, moment and maximum likelihood methods in *Betula alba* (Gorgoso *et al.*, 2007).

The regression relating the scale parameter with the quadratic mean diameter under cork accounted for a very high proportion of the variability in "*b*" ($R^2_{adj} = 0.99$). These results are consistent with previously works as regards both the favourable impact of using the mean square diameter as input variable (Rennols *et al.*, 1985; Kilkki *et al.*, 1989; Lejeune, 1994; Álvarez González, 1997; Torres Rojo *et al.*, 2000; García Güemes *et al.*, 2002; Gorgoso *et al.*, 2007), and the high resulting correlation (Álvarez González, 1997; Torres Rojo *et al.*, 2000; García Güemes *et al.*, 2002; Gorgoso *et al.*, 2007).

Modelling the shape parameter with Method 1 involved using the maximum diameter, minimum diameter and plot elevation as predictors; by contrast, modelling with Method 2 involved the quadratic mean diameter and P_{25}. Álvarez González (1997) used the same independent variables as in our Method 2 for *Pinus pinaster*, and so did Gorgoso *et al.* (2007) for *Betula alba*. As in these studies, our P_{25} was explained by the mean tree height and number of trees per hectare; in our case, however, basal area was also influential. The low correlation for *Quercus suber* is comparable to that previously observed by other authors (Maltamo *et al.* 1995; García Güemes *et al.* 2002) and may have resulted from the special characteristics of cork oak forests in Los Arconocales Natural Park. In fact, such characteristics are highly variable, largely as a consequence of the absence of systematic, sustained use of appropriate silvicultural practices (Montero and López, 2008).

Although stand age was previously used as a variable to predict parameter "*c*" (Maltamo *et al.* 1995; Nanang 1998), incorporating it into our model for *Quercus suber* would have been rather complicated since the traditionally growth ring counting method has very often proved inaccurate for estimation age owing to their ill-definition in stripped cork oaks (Pereira, 2007).

A proportion of 95% of the plots had "*c*" values in excess of 3.6 (the mean was 5.9). The diameter distribution was mound-shaped and most of the plots exhibit negative asymmetry. These results are consistent with the general characteristics of cork oak forests in the study area, which consist of largely even-aged stands —but also, occasionally, two-aged stands— of highly variable age occurring together in each compartment. Low thinning and/or low competition generally cause negative asymmetry (García Güemes *et al.*, 2002). Los Alcornocales is under very conservative silvicultural practices including highly restricted felling and targeted at the lower quality trees, which are usually the thinnest ones, in order to preserve those with better production characteristics. In this work, "*c*" was positively correlated with tree mean diameter; as a result, an increase in diameter was invariably accompanied by one in negative asymmetry. This is consistent with previous results of White and Harper (1970), who found overmature populations to frequently exhibit a negative bias in their distributions.

The input variables used to model parameter b, and those used to model parameter "*c*" (Method 2), were those typically used in whole-stand models. These two equations were used to complete the cork yield tables developed until then (Montero and Cañellas, 1999) by including a new tool (viz. the diameter distribution, which provides a knowledge of the stand structure at different growth stages). However, using these equations in the cork production model entails updating it by incorporating the mean height and circumference under cork —instead of that over cork— as input variables.

Forest stand management in Cortes de la Frontera (Málaga) therefore seems to be a suitable choice. With this approach, the inventory should provide appropriate information about each individual stand. Application of traditional inventory methods leads to higher costs owing to small size of the stands. Using inventories with a reduced number of measured variables essentially relying on relascopic measurements would be more cost-effective. Thus, by measuring the number of trees per hectare and the basal area with a Bitterlich relascope, as well as the maximum and minimum diameter in each plot and the cork thickness of the mean tree, would allow the diameter distribution to be reproduced by the proposed models for parameters "*b*" and "*c*" (Method 1). Applying the models for these two parameters in Method 2 would entail measuring not only the basal area and number of trees per hectare, but also the height and cork thickness of the mean tree. These two methods would allow one to reproduce the diameter distribution in terms of rapid measurements. It should be noted that, although measuring the number of trees per hectare and basal area with a Bitterlich relascope involves measuring diameters in a sample of trees in the plot and provides an approximation to the diameter distribution, the relative small number of trees measured would be inadequate to accurately reproduce the diameter distribution. Also, the number of trees per hectare could be determined ocularly classifying trees into broad diameter groups (Bitterlich, 1984). Thus, Palahí *et al.* (2006a) proposed cheaper forest inventory method based on relascopic measurements of the number of trees per hectare and basal area for the major forest species in Catalonia —cork oak included.

The significance of a knowledge of diameter distributions in a cork oak forests lies not only in its usefulness towards predicting cork production, but also in that of the information it provides about the structure of the stand, which is essential to assessing stability and choosing the most suitable management forest. Also, knowing the diameter distribution enables the application of available individual-tree models for cork oak. Thus, the growth model of Sánchez-González *et al.* (2006) would allow one to predict the future diameter distribution of a stand in terms of site index and stand density. Also, knowledge of diameter distribution would allow one to estimate cork production by using the model of Ribeiro y Tomé (2002), which includes the circumference under cork as input variable; however, this model should be used with caution since it was developed for cork oak forests in central-southern Portugal. Also applicable here would be the model of Montero *et al.* (1996), which, however, require using an equation relating the circumference to the stripping height as input variables.

In summary, the two-parameter Weibull function proved effective towards fitting diameter distributions of cork oak forests in Los Arconocales Natural Park. The non-linear regression method was more accurate than the moment, percentile and maximum likelihood-based methods. The independent variable providing the best fit of parameter "*b*" was the mean square diameter under cork, whereas those for parameter "*c*" (in terms of the Kolmogorov-Smirnov test) were the maximum plot diameter, minimum diameter and plot elevation. This work has substantially increased our knowledge of diameter distributions of cork oak forests in Los Arconocales Natural Park. In fact, this is the first study of this type on cork oak in the area. The proposed model diameter distribution can be a highly useful tool for the inventorying and management of cork oak forests.

Acknowledgements

This study has been conducted within the research project SUBERPRO (AGL2005-03813) funded by the Ministerio de Ciencia y Tecnología of Spain. The authors would like to thank all the colleagues and friends who work on this subject.

References

Allué JL, 1990. Atlas Fitoclimático de España. Taxonomías. INIA, Ministerio de Agricultura Pesca y Alimentación. Madrid, Spain. 221 pp.

Álvarez González JG, 1997. Análisis y caracterización de las distribuciones diamétricas de *Pinus pinaster* Ait. en Galicia. PhD Thesis. ETSIM-UPM. Spain.

Álvarez González JG, Ruiz González AD, 1998. Análisis y modelización de las distribuciones diamétricas de *Pinus* *pinaster* Ait. en Galicia. Invest Agr: Sist Recur For 7(1-2): 123-137.

Assmann E, 1970. The principles of forest yield study. Pergamon Press. New York. 506 pp.

Bailey RL, Dell TR, 1973. Quantifying diameter distributions with the Weibull function. For Sci 19(2): 97-104.

Bitterlich W, 1984. The relascope idea. Commonwealth Agricultural Bureaux. England. 242 pp.

Bullock BP, Burkhart HE, 2005. Juvenile diameter distributions of loblolly pine characterized by the two-parameter Weibull function. New For 29: 233-244.

Cao QV, 2004. Predicting parameters of a Weibull function for modeling diameter distribution. For Sci 50(5): 682-685.

Dagnelie P, 2006. Statistique théorique et appliquée. De Boeck & Larcier. Brussels, Belgium. 511 pp.

De Benito N, 2008. Evolución histórica del pensamiento dasocrático en ordenación de alcornocales. El caso de los montes de Cortes de la Frontera (Málaga). En: Cuadernos de la Sociedad Española de Ciencias Forestales nº 27, SECF. Madrid, Spain. pp: 73-78.

Dubey SD, 1967. Some percentile estimators for Weibull parameters. Technometrics 9: 119-129.

García Güemes C, Cañadas N, Montero G, 2002. Modelización de la distribución diamétrica de las masas de *Pinus pinea* L. de Valladolid (España) mediante la función Weibull. Invest Agr: Sist Recur For 11(2): 263-282.

Gorgoso JJ, Álvarez González JG, Rojo A, Grandas-Arias JA, 2007. Modelling diameter distributions of *Betula alba* L. stands in northwest Spain with the two-parameter Weibull function. Invest Agr: Sist Recur For 16(2): 113-123.

Gove JH, 2003. Moment and maximum likelihood estimators for Weibull distributions under length and area-biased sampling. Environ Ecol Stat 10: 455-467.

Junta de Andalucía, 2004. Plan de ordenación de los recursos naturales del Parque Natural de los Alcornocales. Junta de Andalucía. Sevilla, Spain. 92 pp.

Kilkki P, Maltamo M, Mykkänen R, Päivinen R, 1989. Use of the Weibull function in estimating the basal area dbh-distribution. Silva Fenn 23(4): 311-318.

Lejeune P, 1994. Construction d'un modèle de répartition des arbres par classes de grosseur pour des plantations d'épicéa commum (*Picea abies* L Karst) en Ardenne belge. Ann Sci For 51: 53-65.

Lindsay SR, Wood GR, Woollons RC, 1996. Stand table modelling through the Weibull distribution and usage of skewness information. For Ecol Manag 81(1-3): 19-23.

Liu Ch, Zhang L, Davis CJ, Solomon DS, Gove JH, 2002. A finite mixture model for characterizing the diameter distributions of mixed-species forest stands. For Sci 48(4): 653-661.

Maltamo M, 1997. Comparing basal area diameter distributions estimated by tree species and for the entire growing stock in a mixed stand. Silva Fenn 31(1); 53-65.

Maltamo M, Puumalainen J, Päivinen R, 1995. Comparison of Beta and Weibull functions for modelling basal area diameter distribution in stands of *Pinus sylvestris* and *Picea abies*. Scan J For Res 10: 284-295.

Maltamo M, Kangas A, Uuttera J, Torniainen T, Saramäki J, 2000. Comparison of percentile based prediction methods and the Weibull distribution in describing the diameter distribution of heterogeneous Scots pine stands. For Ecol Manag 133(3): 263-274.

Marquardt DW, 1963. An algorithm for least squares estimation of nonlinear parameters. Journal of the Society for Industrial and Applied Mathematics 2: 431-441.

Merganic J, Sterba H, 2006. Characterisation of diameter distribution using the Weibull function: method of moments. Eur J Forest Res 125: 427-439.

Montero G, Torres E, Cañellas I, Ortega C, 1996. Modelos para la estimación de la producción de corcho en alcornocales. Invest Agr: Sist Recur For 5(1): 97-127.

Montero G, Cañellas I, 1999. Manual de reforestación y cultivo de alcornoque (*Quercus suber* L.). Instituto Nacional de Investigación y Tecnología Agraria y Alimentaria (INIA). Ministerio de Agricultura, Pesca y Alimentación. Madrid, Spain. 103 pp.

Montero G, López E, 2008. Selvicultura de *Quercus suber* L. En: Compendio de Selvicultura Aplicada en España, Fundación Conde del Valle de Salazar. Madrid, Spain. pp: 779-829.

Nanang DM, 1998. Suitability of the Normal, Log-normal and Weibull distributions for fitting diameter distributions of neem plantations in Northern Ghana. For Ecol Manag 103(1): 1-7.

Nanos N, Montero G, 2002. Spatial prediction of diameter distributions models. For Ecol Manag 161(1-3): 147-158.

Newton PF, Lei Y, Zhang SY, 2005. Stand-level diameter distribution yield model for black spruce plantations. For Ecol Manag 209(3): 181-192.

Palahí M, Pukkala T, Trasobares A, 2006a. Calibrating predicted tree diameter distributions in Catalonia, Spain. Silva Fenn 40(3): 487-500.

Palahí M, Pukkala T, Trasobares A, 2006b. Modelling the diameter distribution of *Pinus sylvestris*, *Pinus nigra* and *Pinus halepensis* forest stands in Catalonia using the truncated Weibull function. Forestry 79(5): 553-562.

Pereira H, 2007. The cork oak. In: Cork. Elsevier Science BV. Amsterdam, Netherlands. pp: 103-125.

Rennols K, Geary DN, Rollinson TJ, 1985. Characterizing diameter distributions by the use of the Weibull distribution. Forestry 58(1): 57-66.

Ribeiro F, Tome M, 2002. Cork weight prediction at tree level. For Ecol Manag 171(3), 231-241.

Río M, Montero G, 2001. Modelo de simulación de claras en masas de *Pinus sylvestris* L. Monografías INIA: Forestal no. 3. Ministerio de Ciencia y Tecnología. Madrid, Spain. 113 pp.

Rubin BD, Manion PD, Faber-langendoen D, 2006. Diameter distributions and structural sustainability in forests. For Ecol Manag 222(1-3): 427-438.

Sánchez-González M, Del Río M, Cañellas I, Montero G, 2006. Distance independent tree diameter growth model for cork oak stands. For Ecol Manag 225(1-3): 262-270.

Shifley S, Lentz E, 1985. Quick estimation of the three-parameter Weibull to describe tree size distributions. For Ecol Manag 13: 195-203.

Shiver BD, 1988. Sample sizes and estimation methods for the Weibull distribution for unthinned slash pine plantation diameter distributions. For Sci 34(3): 809-814.

Torres Rojo JM, Magaña Torres OS, Acosta Mireles M, 2000. Methodology to improve the prediction for diameter distribution parameters. Agrociencia 34: 627-637.

Torres E, 1995. Estudio de los principales problemas selvícolas de los alcornocales del Macizo del Aljibe (Cádiz y Málaga). PhD Thesis. ETSIM-UPM, Spain.

Torres E, Montero G, 2000. Los alcornocales del macizo del Algibe y sierras del Campo del Gibraltar: clasificación ecológica y caracterización selvícola y productiva. Ministerio de Agricultura, Pesca y Alimentación. Madrid, Spain. 228 pp.

Vázquez Piqué J, Pereira H, 2008. ¿Qué hay que tener en cuenta para elaborar modelos de producción de corcho? Invest Agr: Sist Recur For 17(3): 199-215.

White J, Harper JL, 1970. Correlated changes in plant size and number in plant populations. J Ecol 58: 467-485.

Zarnoch SJ, Dell TR, 1985. An evaluation of percentile and maximum likelihood estimators of Weibull parameters. For Sci 31(1): 260-268.

Zhang L, Packard KC, Liu CH, 2003. A comparison of estimation methods for fitting Weibull and Johnson's S_B distributions to mixed spruce-fir stands in northeastern North America. Can J For Res 33(7): 1340-1347.