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On shape coexistence and quantum phase transitions: Lead and Zirconium regions

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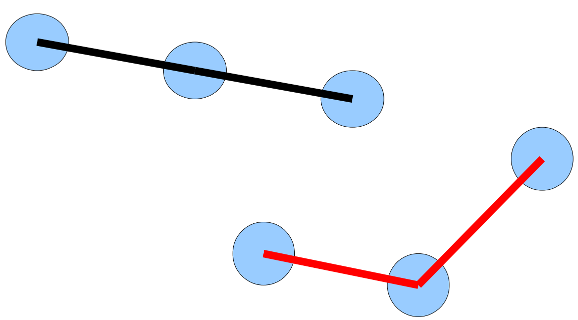
What is shape coexistence?

It appears in quantum systems where eigenstates with very different density distribution coexist.

Therefore, the existence of a geometric interpretation is implicit .

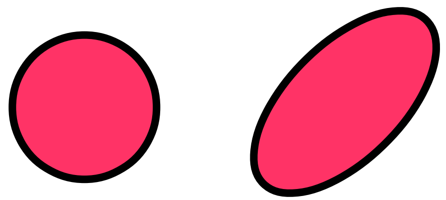
Molecules



$$q_{2,i} = \sqrt{5}\langle 0_i^+ | [\hat{Q} \times \hat{Q}]^{(0)} | 0_i^+ \rangle,$$

$$q_{3,i} = -\sqrt{\frac{35}{2}} \langle 0_i^+ [\hat{Q} \times \hat{Q} \times \hat{Q}]^{(0)} | 0_i^+ \rangle.$$

Nuclei



$$q_2 = q^2,$$

$$q_3 = q^3 \cos 3\delta,$$

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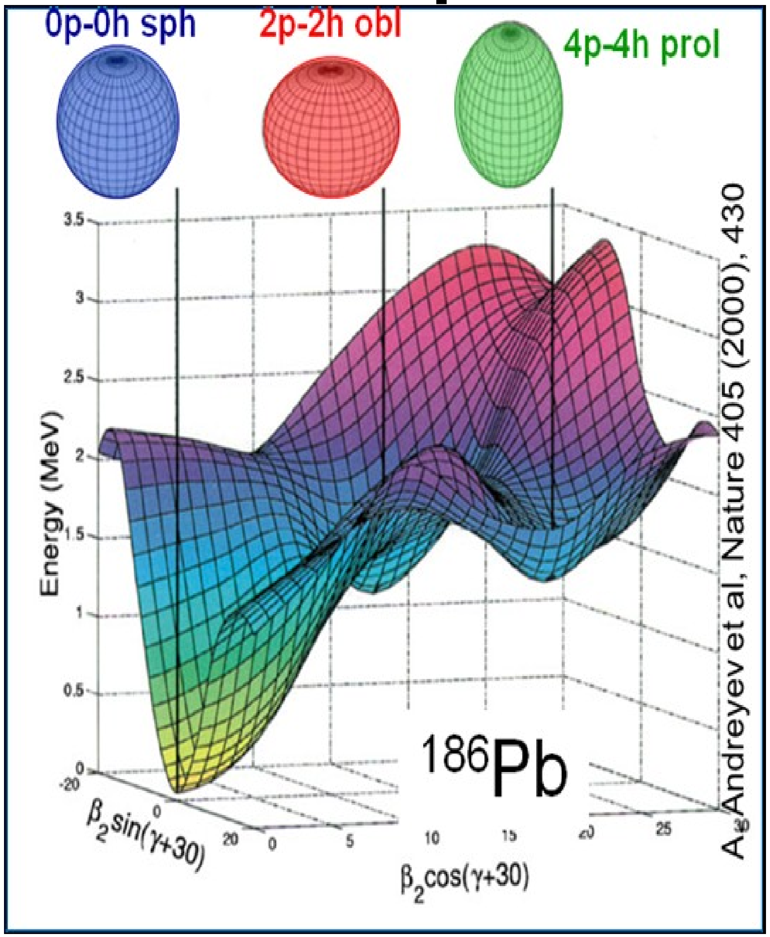
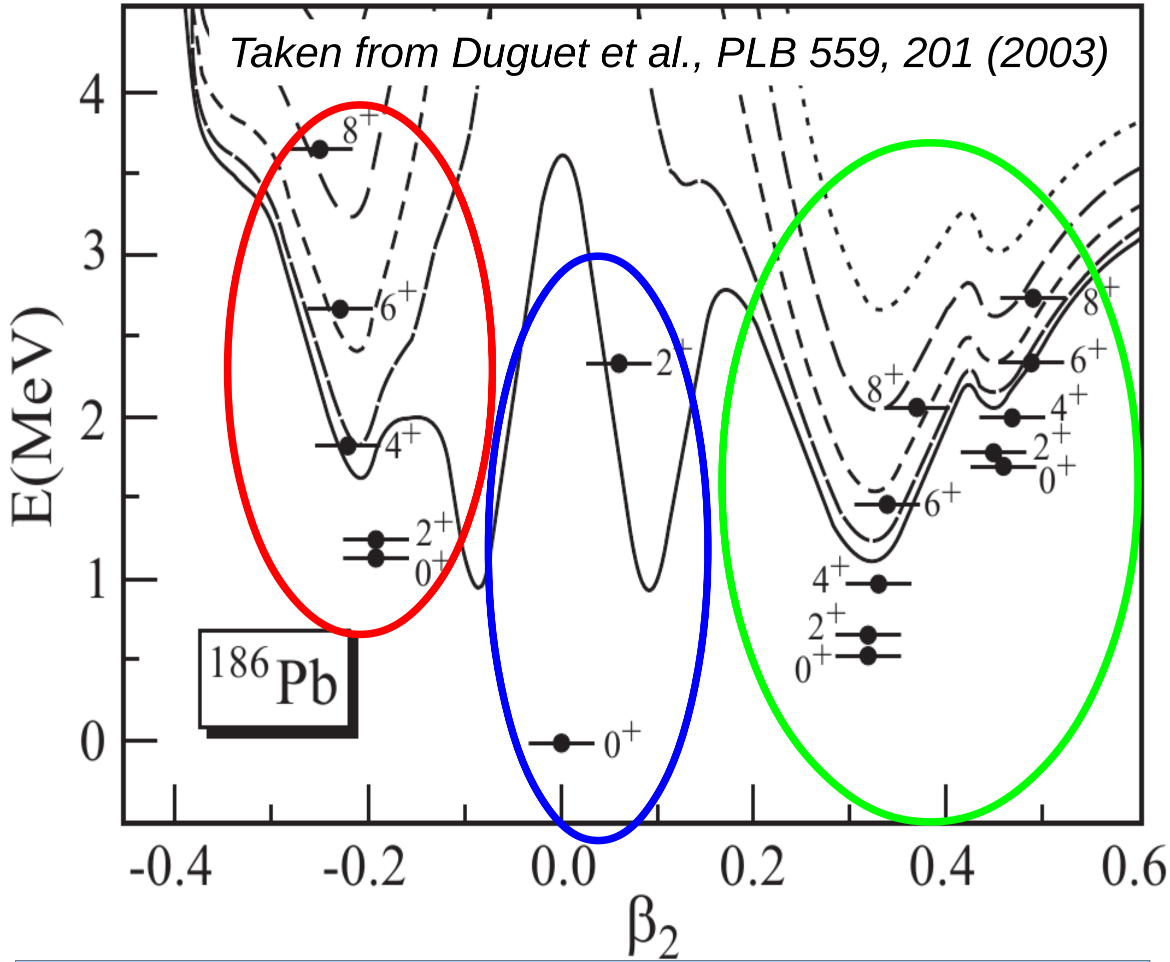
Shape coexistence example

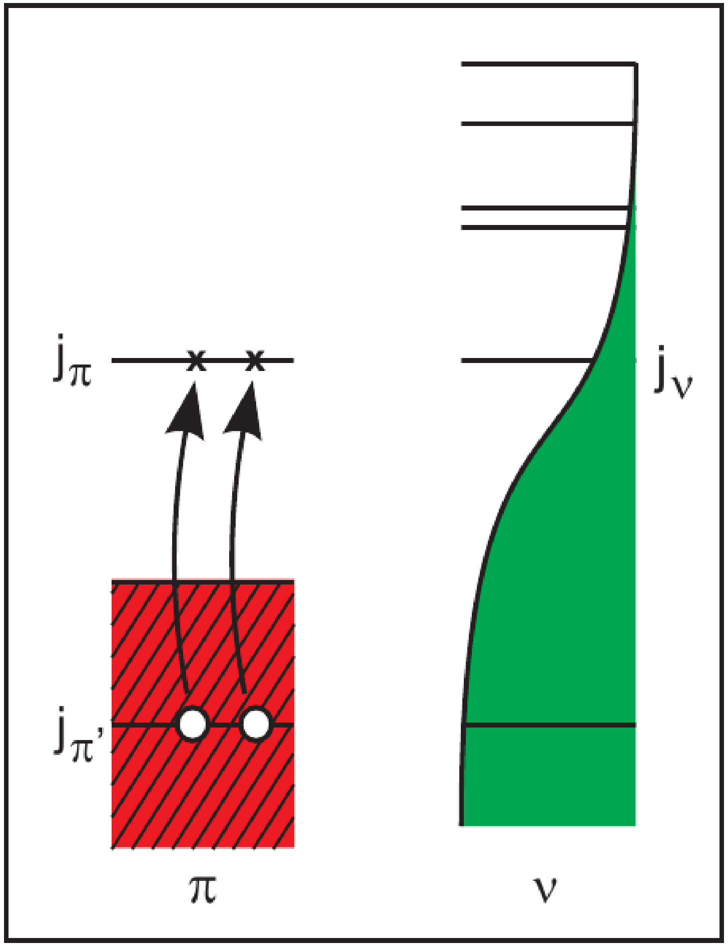


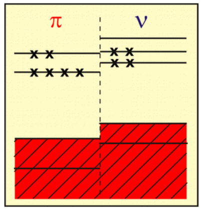


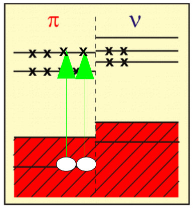
The angular momentum projected mean field plus the Generator Coordinate Method generates different bands with very different deformation.

Shell Model/Interacting Boson Model

- For nuclei near to closed shells, either for neutrons or for protons, it can be energetically favorable to have excitations of 2p-2h, 4p-4h … crossing the energy gap.
- The np-nh excitations have a lower excitation energy than expected due to the correlation energy: pairing and quadrupole correlations.
- Restricted to light and medium-heavy nuclei, at present.



$\phi(J,M) = a(J,M)$ 

$+\ b(J,M)$ 

In heavy nuclei the huge model space imposes some kind of truncation: symmetry dictated truncation (IBM).

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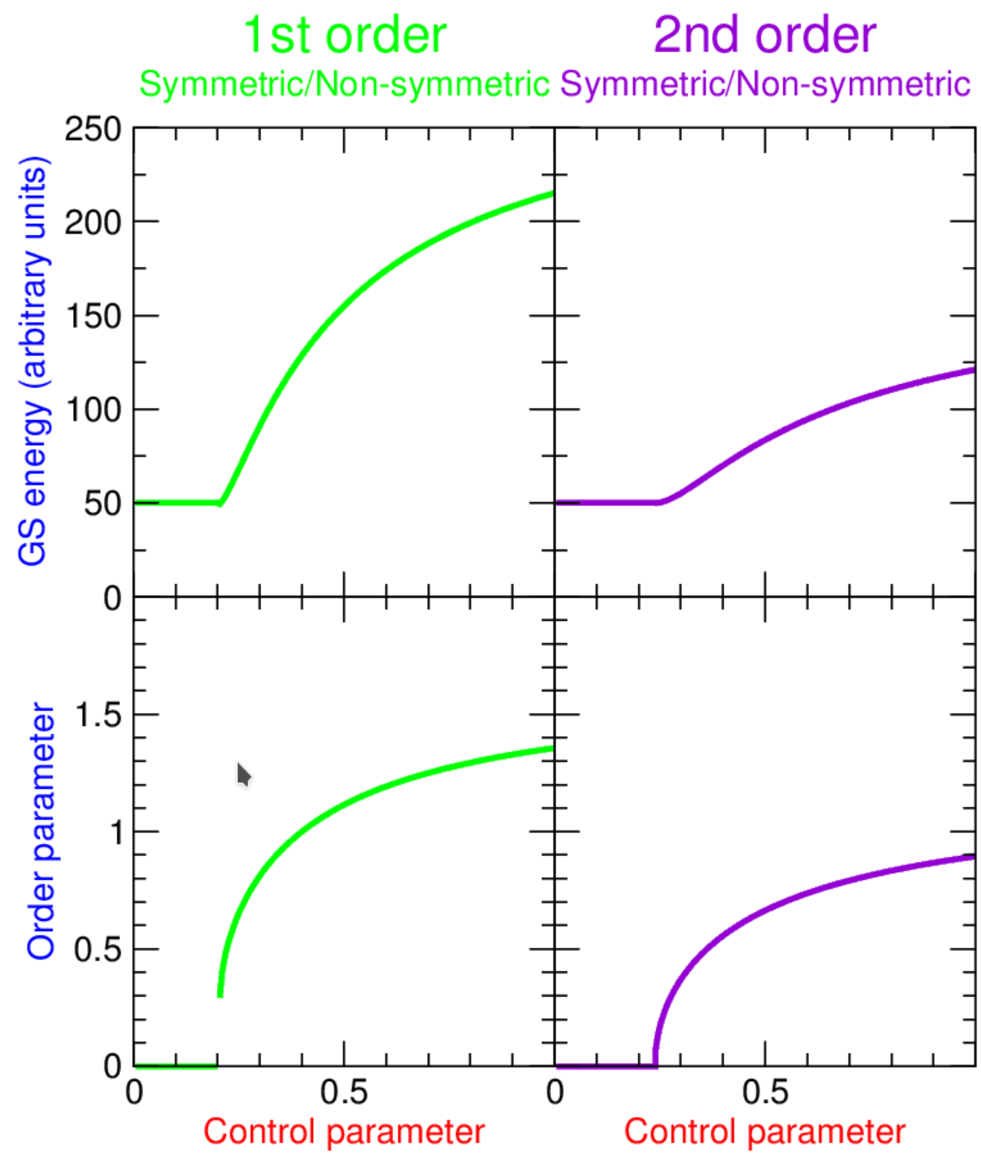
What is a Quantum Phase Transition?

A QPT appears when a quantum system experiences a sudden change in its structure (order parameter) when a parameter that affects the Hamiltonian (control parameter) slightly changes around its critical value. This transitions are assumed to occurs at zero temperature.

$$\hat{H} = (1-\xi)\hat{H}_1 + \xi\hat{H}_2$$



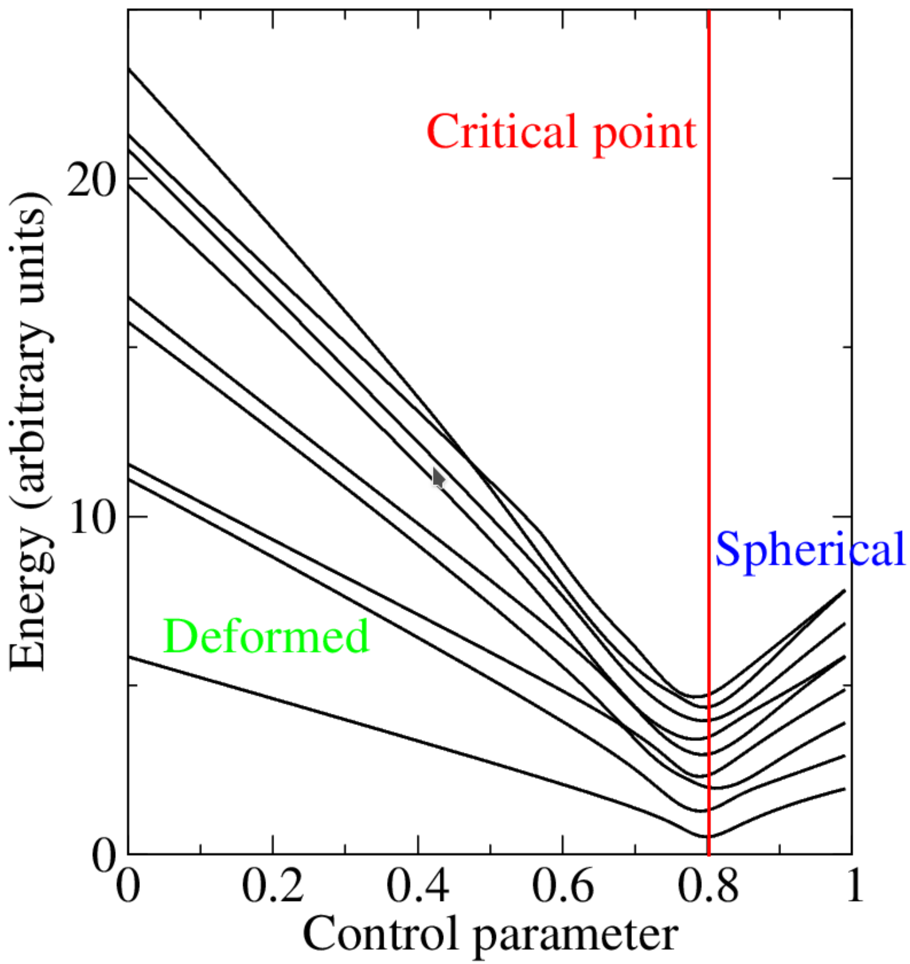
What is a Quantum Phase Transition?

At the critical point:

- The ground state energy is non-analytical.
- Energy gap between the ground and the first excited states goes to zero.

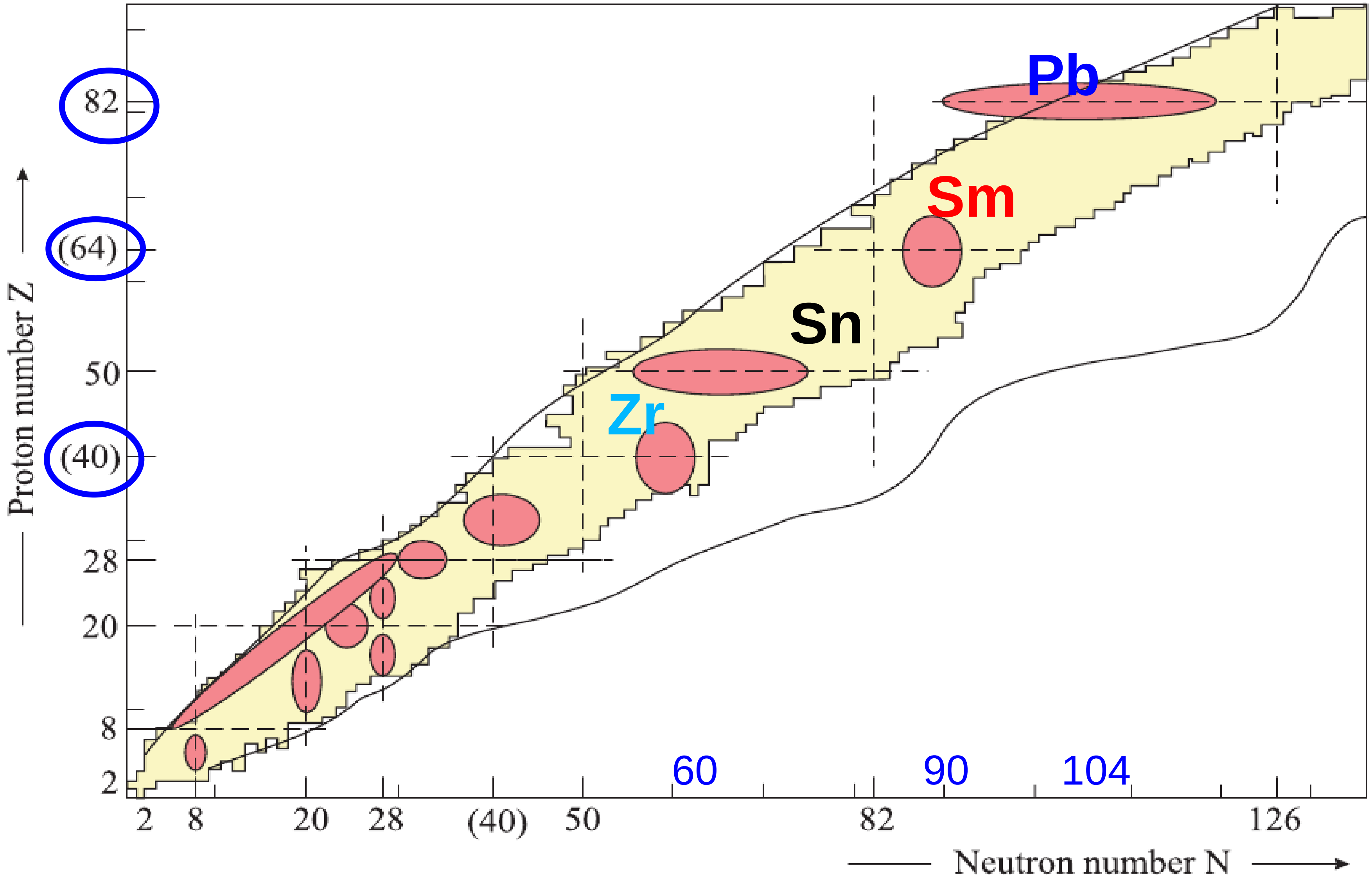
Challenges when dealing with QPTs in atomic nuclei

- It is a finite system, therefore abrupt changes, if any, are smoothed out.
- There is not a true control parameter.
- How can we define an order parameter?
- How can we define the phases of the system?
- The phase transition does not characterize a single nucleus, but it is a property of an entire region.



Low lying 0+ states of an IBM calculation with N=20 between the U(5) and SU(3) limits

Many regions to be understood



K. Heyde and J. L. Wood, Rev. Mod. Phys. 83, 1467 (2011).

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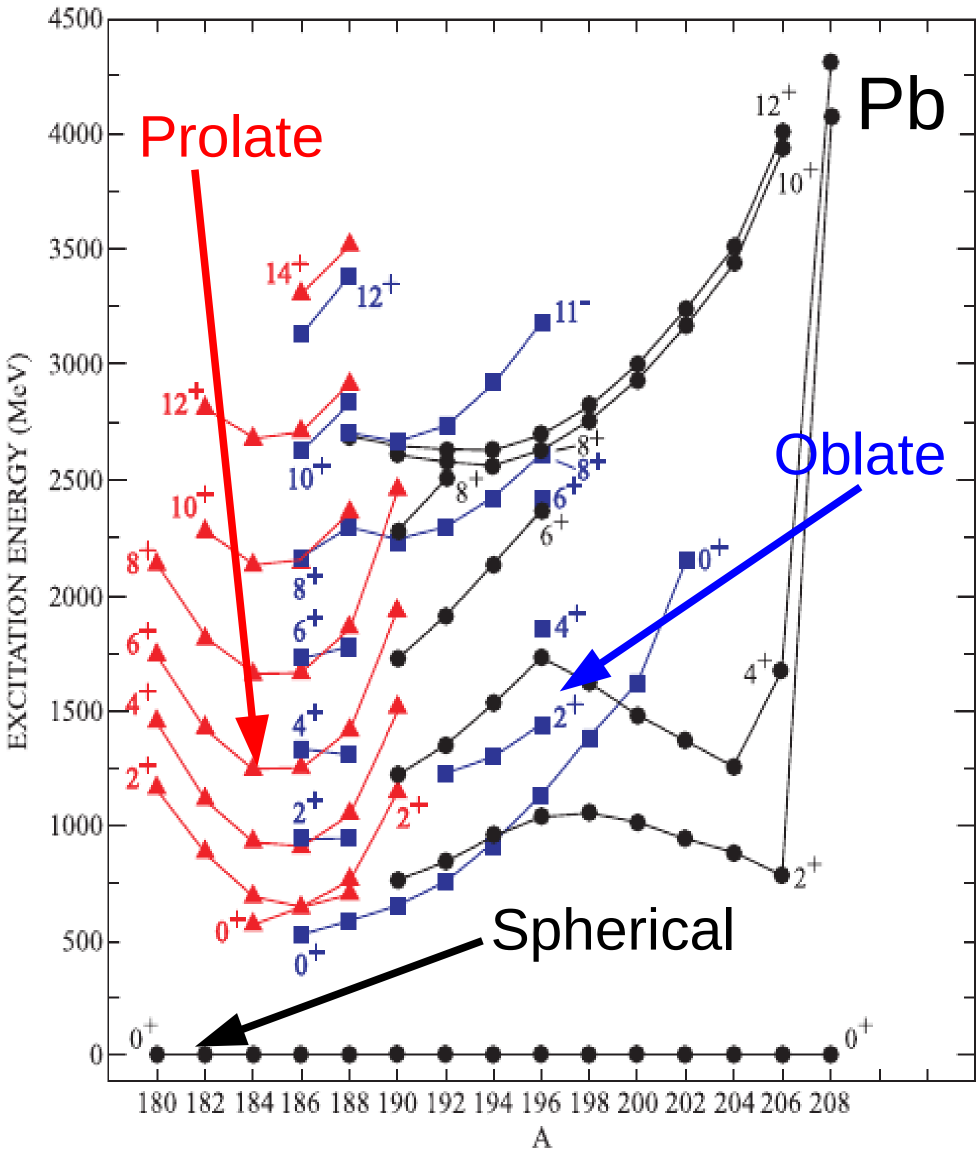
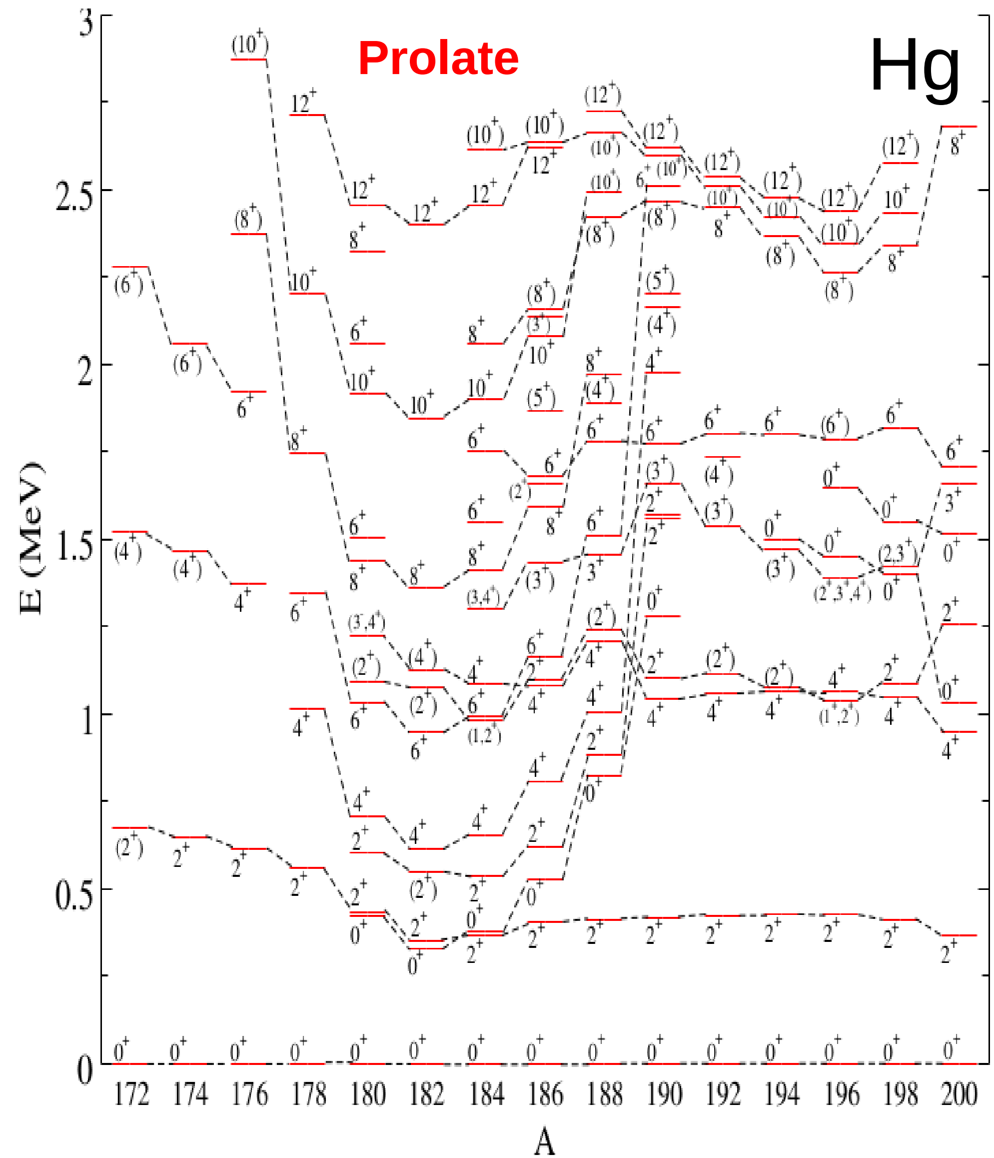
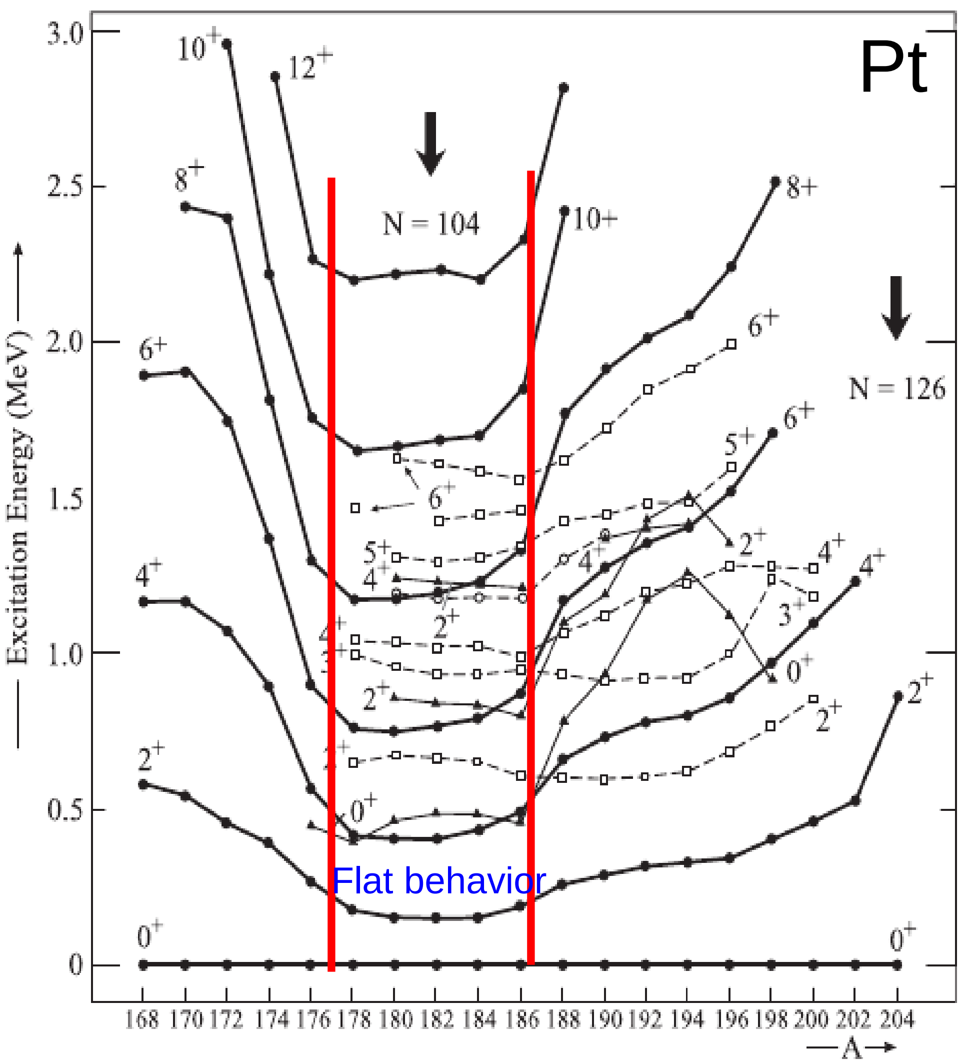
Shape coexistence key indicators



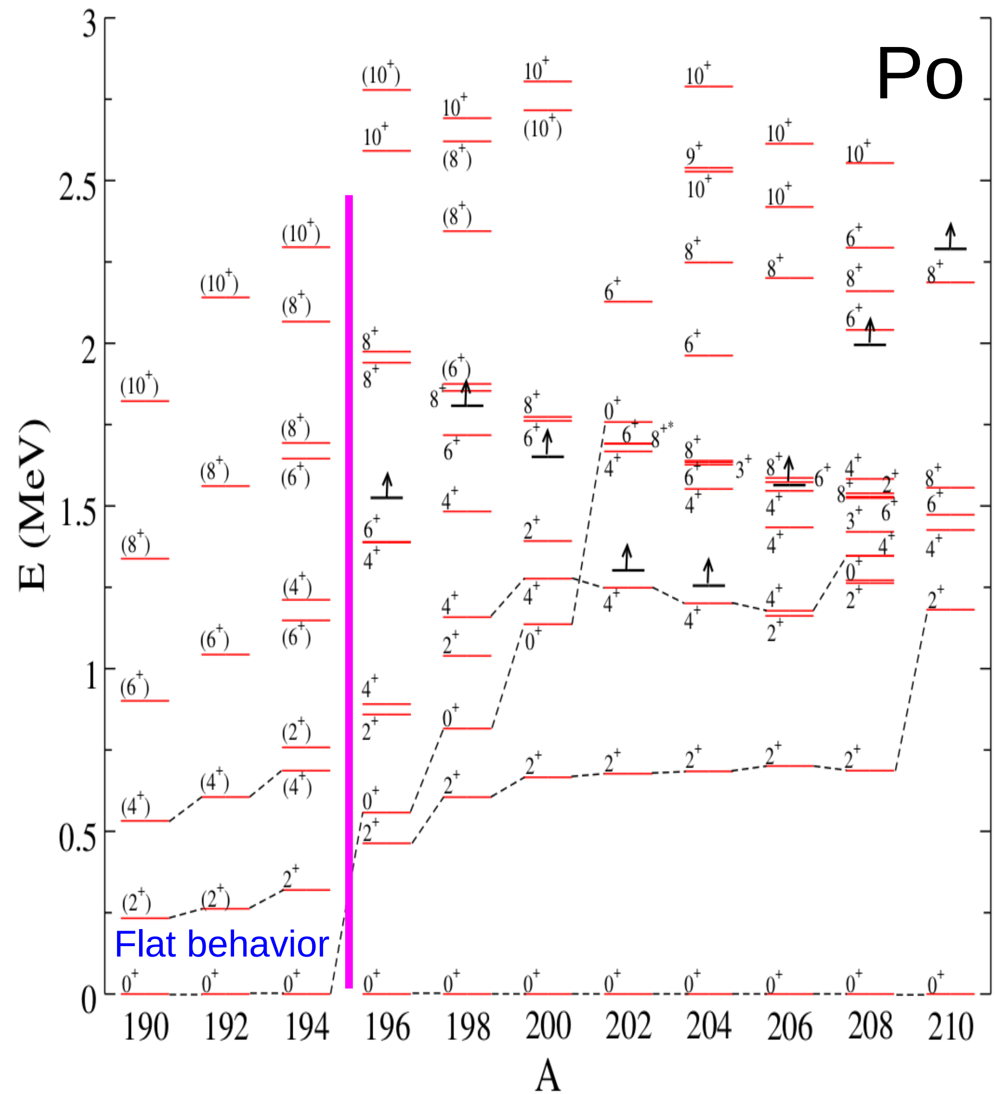


JEGR and K. Heyde, PRC 89, 014306 (2014)

Pt and Po isotopes



JEGR and K. Heyde, NPA 825, 39 (2009),
JEGR, V. Hellemans, and K. Heyde, PRC 84, 014331 (2011).

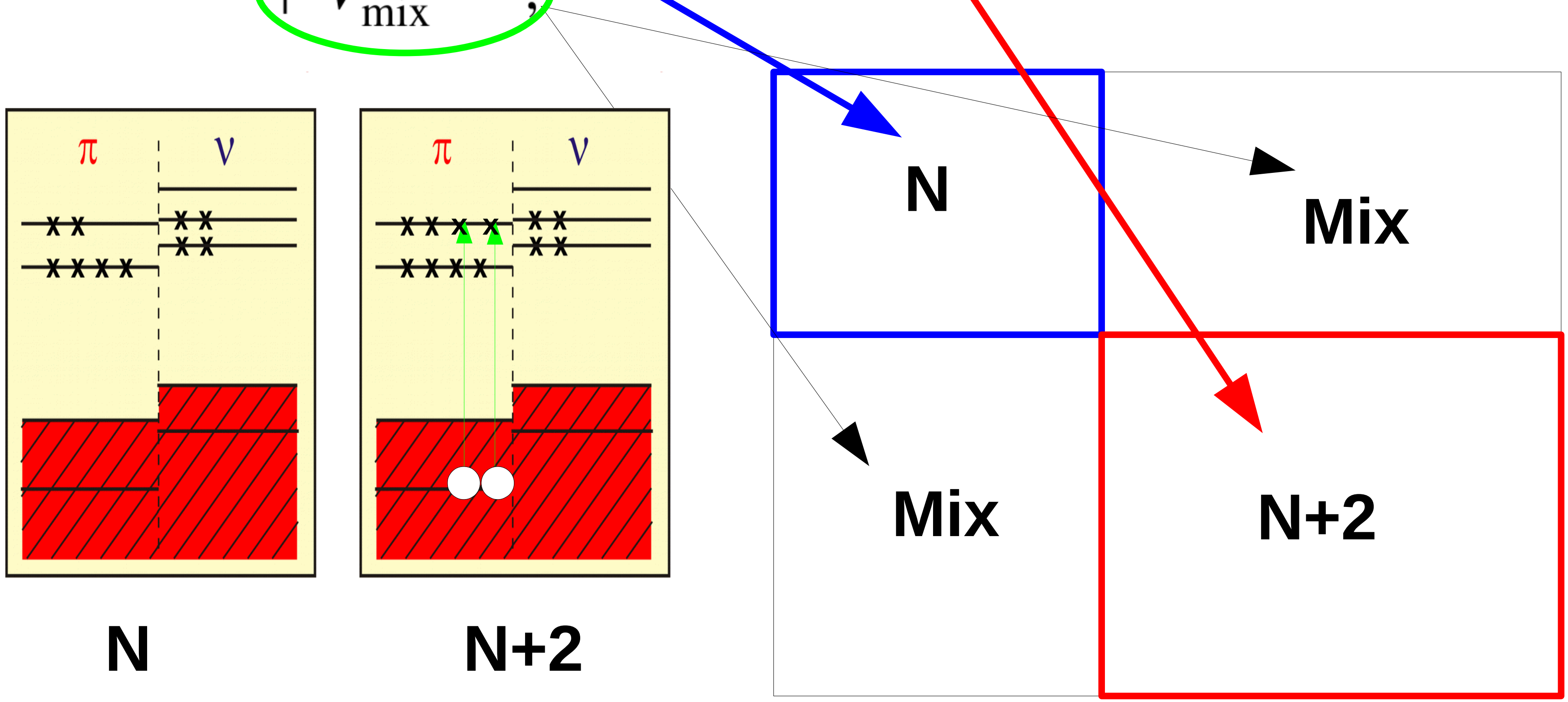


JEGR and K. Heyde, PRC 92, 034309 (2015).

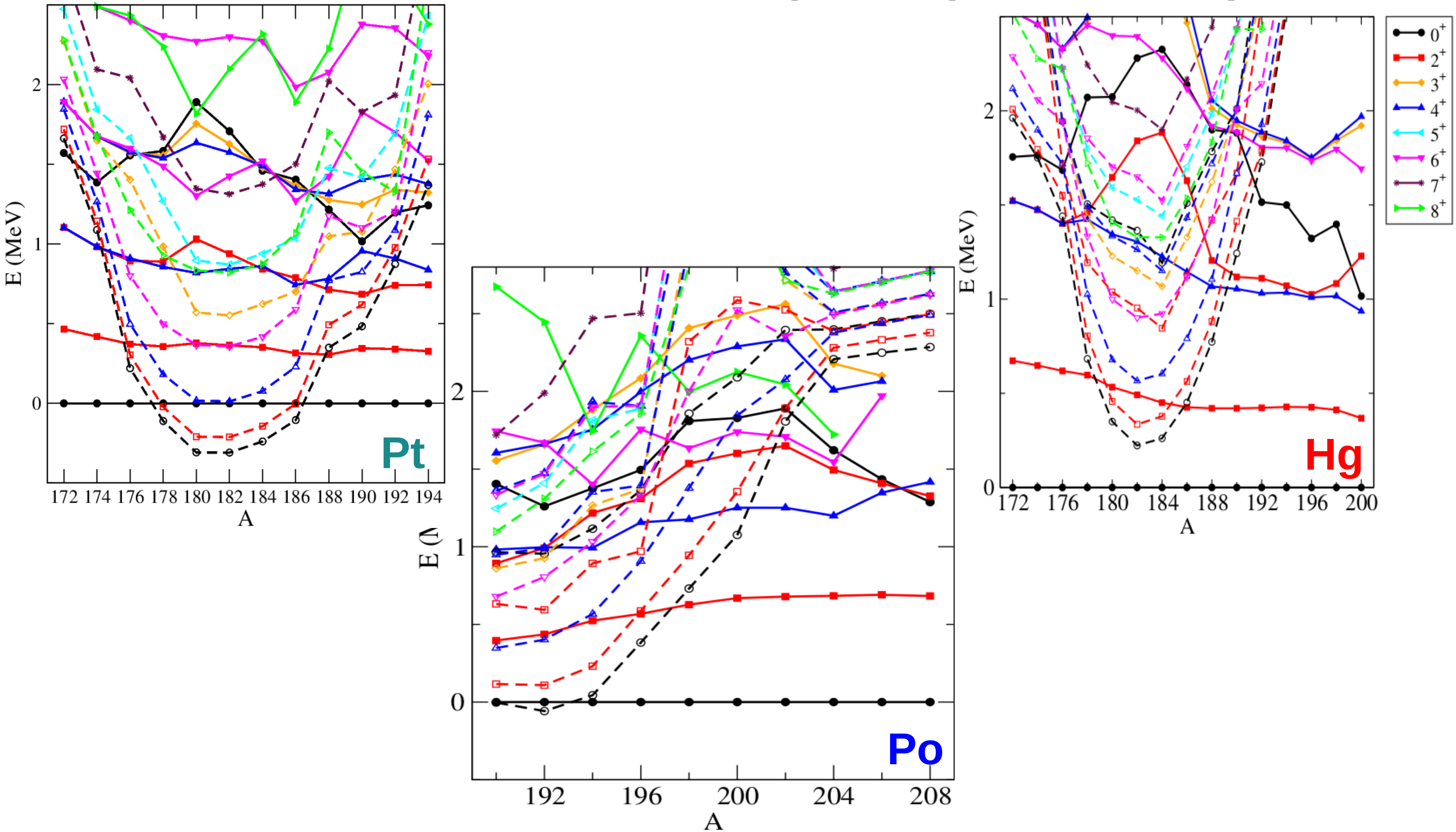
Interacting Boson Model

(configuration mixing)

$$\hat{H} = \hat{P}_N^\dagger \hat{H}_{\text{ECQF}}^N \hat{P}_N + \hat{P}_{N+2}^\dagger \left(\hat{H}_{\text{ECQF}}^{N+2} + \Delta^{N+2}\right) \hat{P}_{N+2} + \hat{V}_{\text{mix}}^{N,N+2},$$



Unperturbed energies (IBM-CM)



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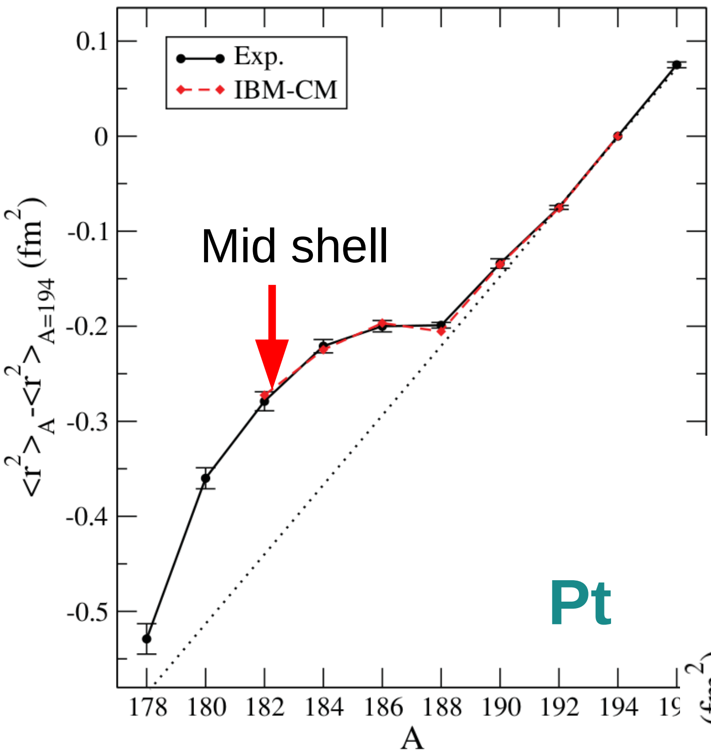
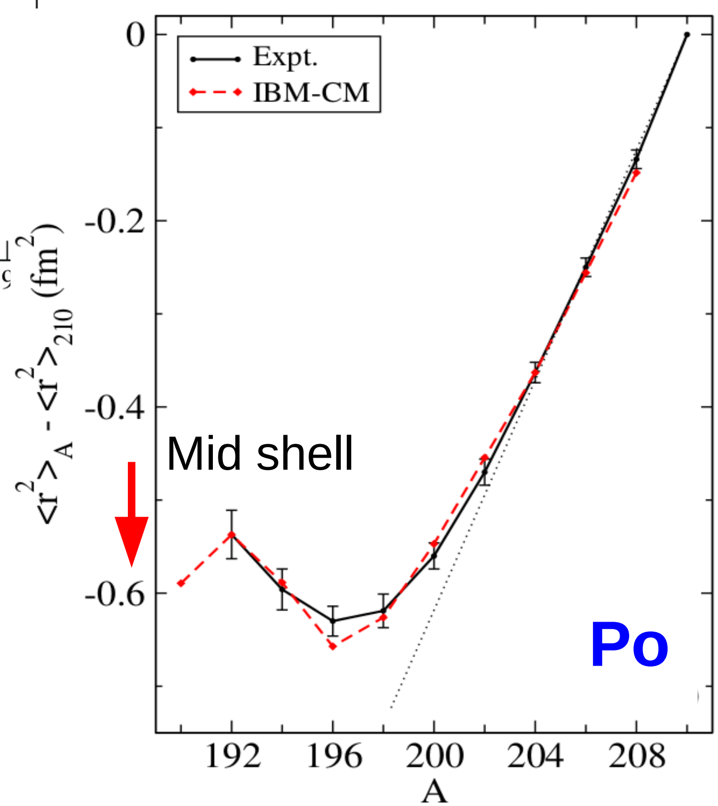
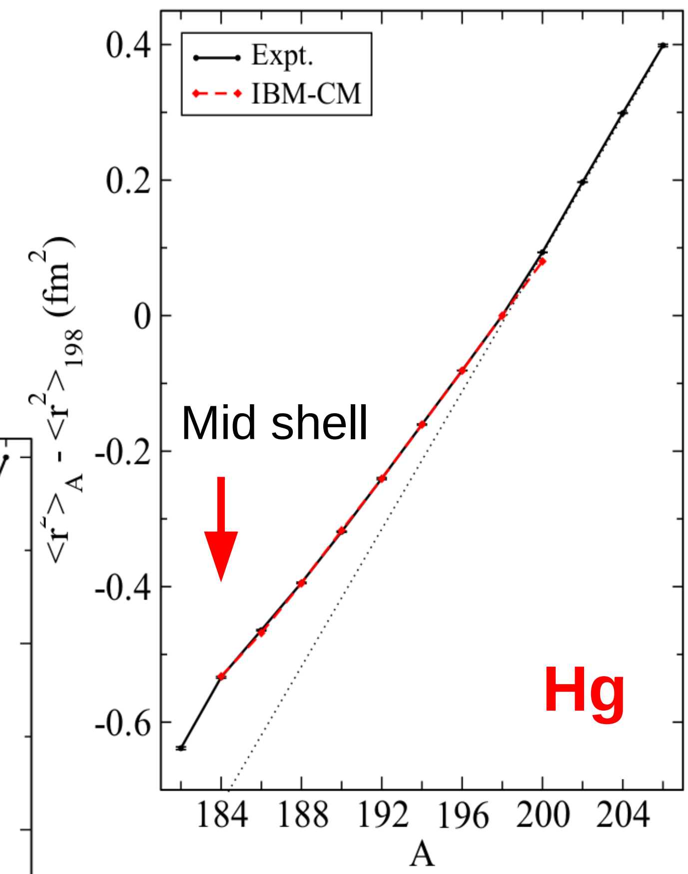
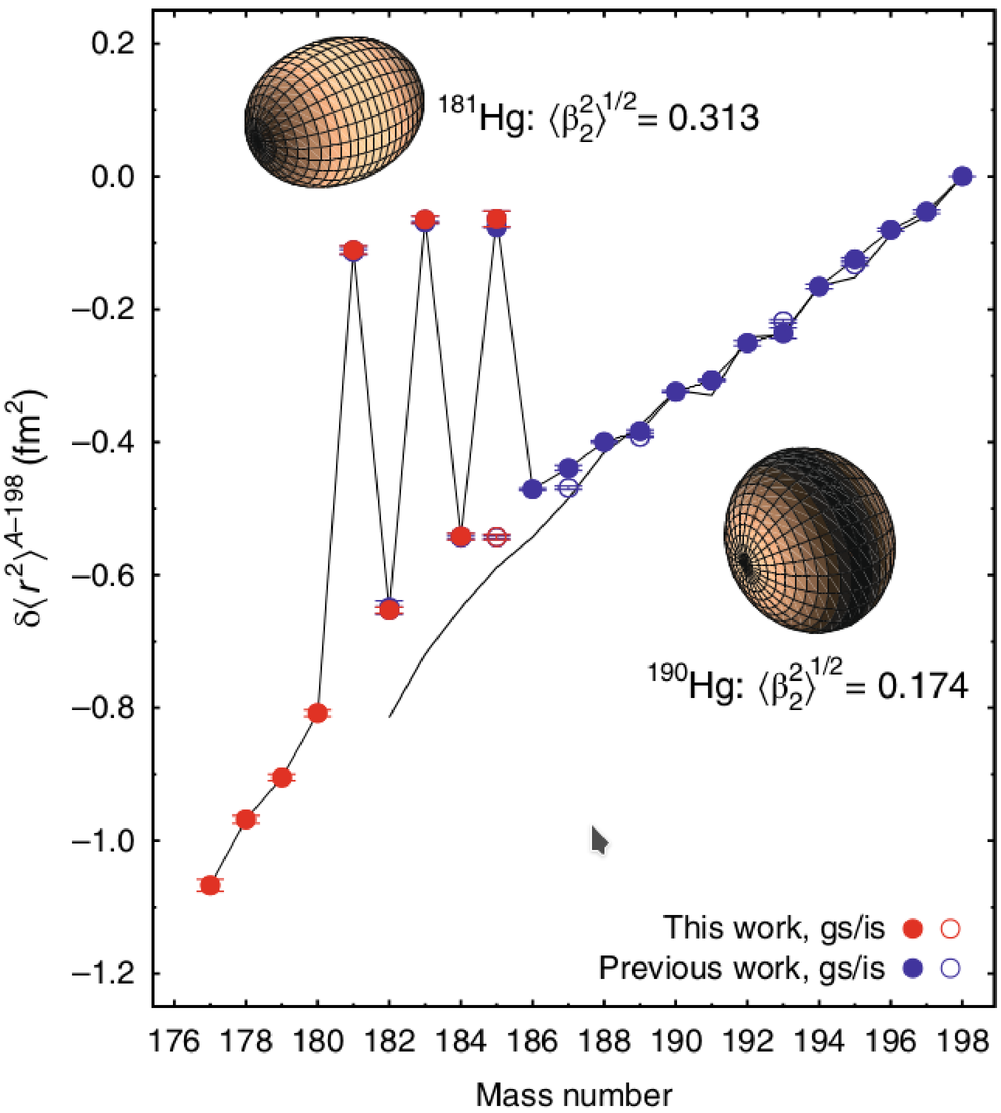
Radii in shape coexistence regions







Radii in shape coexistence regions



nature physics — LETTERS
https://doi.org/10.1038/s41567-018-0292-8

Characterization of the shape-staggering effect in mercury nuclei

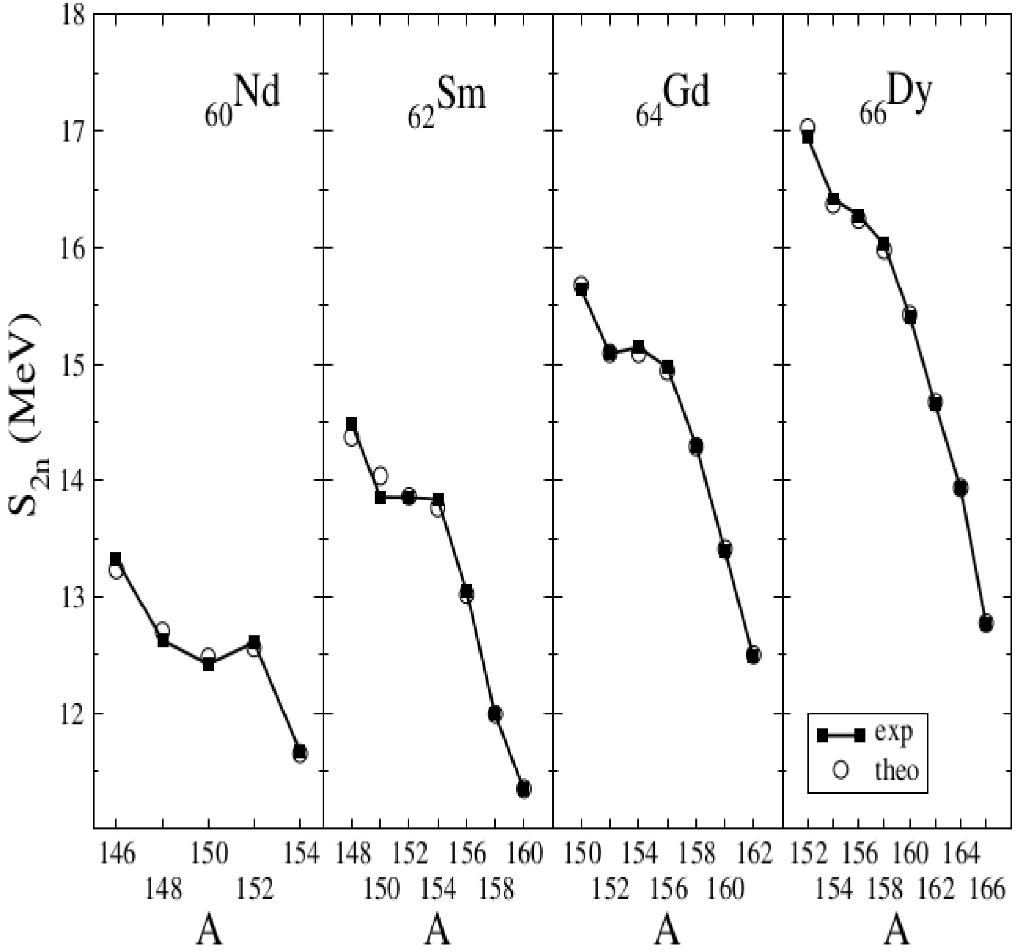
B. A. Marsh[1*], T. Day Goodacre[1,2,18], S. Sels[3,18], Y. Tsunoda[4], B. Andel[5], A. N. Andreyev[6,7], N. A. Althubiti[2], D. Atanasov[8], A. E. Barzakh[9], J. Billowes[2], K. Blaum[8], T. E. Cocolios[2,3], J. G. Cubiss[6], J. Dobaczewski[6], G. J. Farooq-Smith[2,3], D. V. Fedorov[9], V. N. Fedosseev[1], K. T. Flanagan[2], L. P. Gaffney[3,10], L. Ghys[3], M. Huyse[3], S. Kreim[8], D. Lunney[11], K. M. Lynch[1], V. Manea[8], Y. Martinez Palenzuela[3], P. L. Molkanov[9], T. Otsuka[3,4,12,13,14], A. Pastore[6], M. Rosenbusch[13,15], R. E. Rossel[1], S. Rothe[1,2], L. Schweikhard[15], M. D. Seliverstov[9], P. Spagnoletti[10], C. Van Beveren[3], P. Van Duppen[3], M. Veinhard[1], E. Verstraelen[3], A. Welker[16], K. Wendt[17], F. Wienholtz[15], R. N. Wolf[8], A. Zadvornaya[3] and K. Zuber[16]

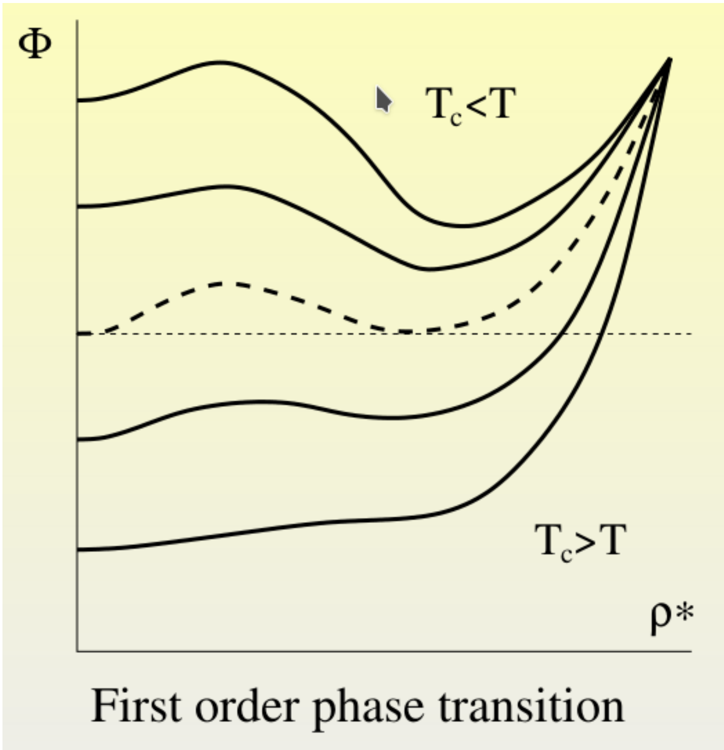
October 2018

The shape staggering effect manifests characteristic features of a quantum phase transition[15–17]: in a given nucleus, different phases—a near spherical and a strongly deformed nuclear shape—appear at almost the same energy without mixing. By making small changes in the control parameter, which in this case is the neutron number, the system alternates between the two phases. In the case of the mer-

Clear cut example of a delicate balance between monopole, quadrupole and pairing interaction.

QPT key indicators:
2n separation energies

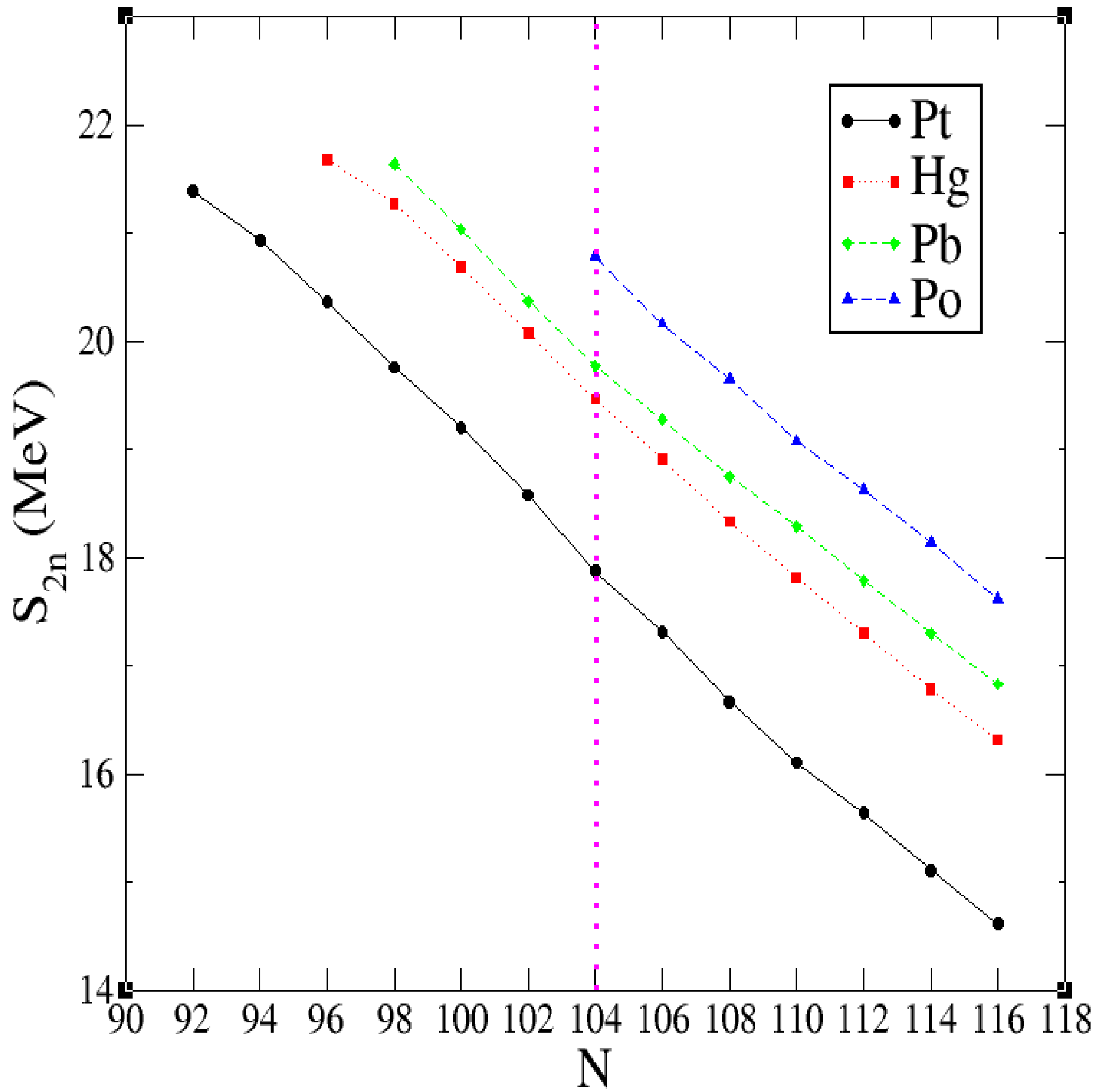




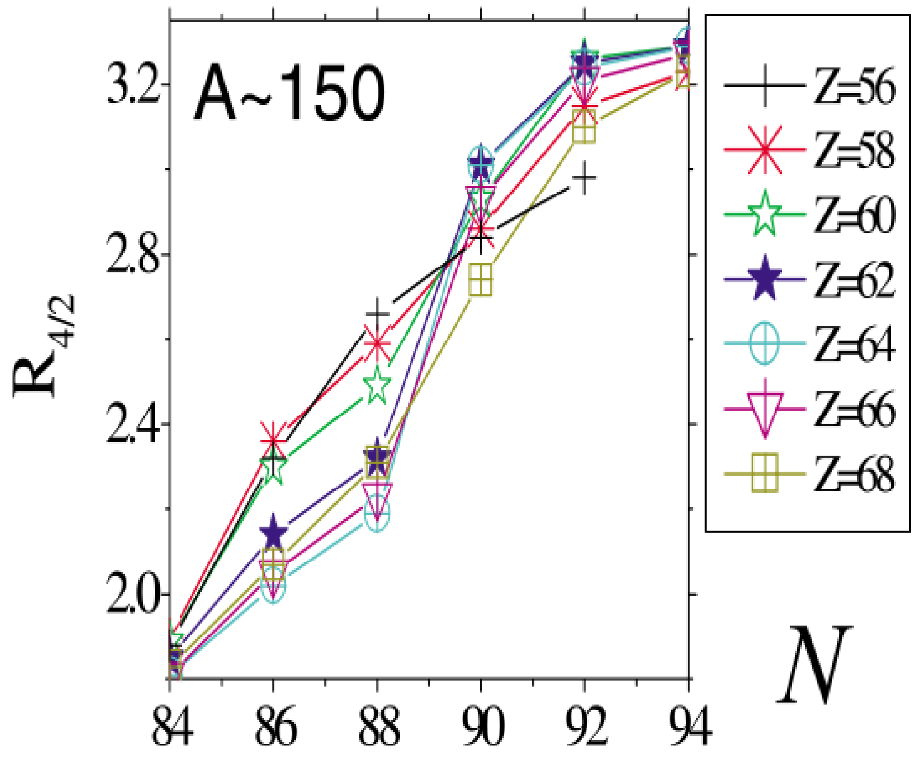
Discontinuity in the first derivative of the energy

No indication of QPT in lead region

The linear trend is observed all the way, therefore, the structure of the ground state remains stable.



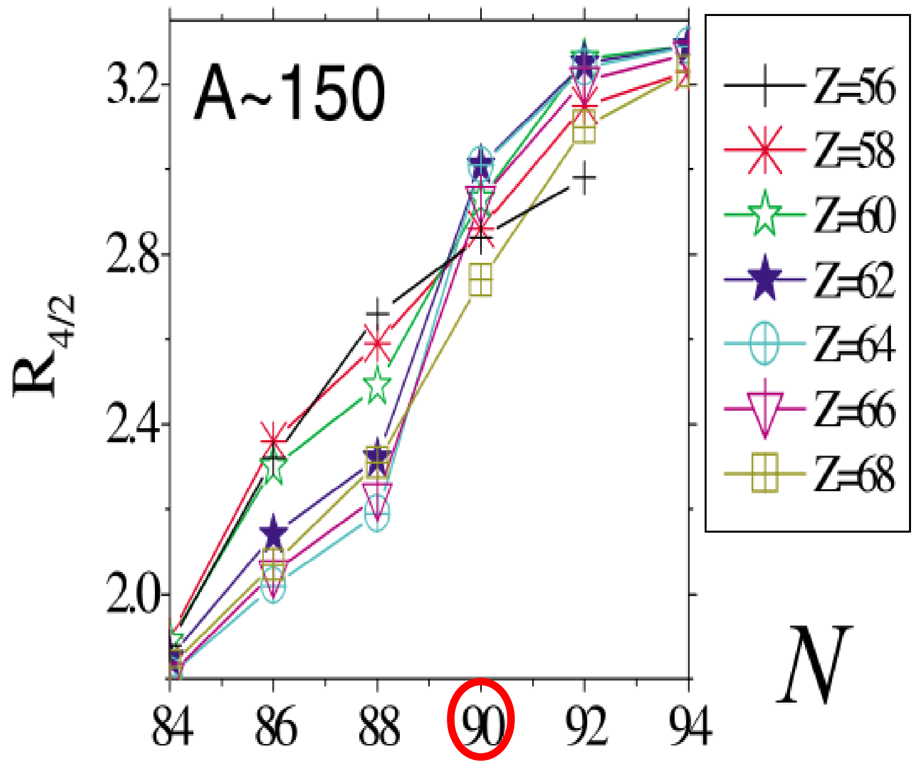
QPT key indicators: energy ratios



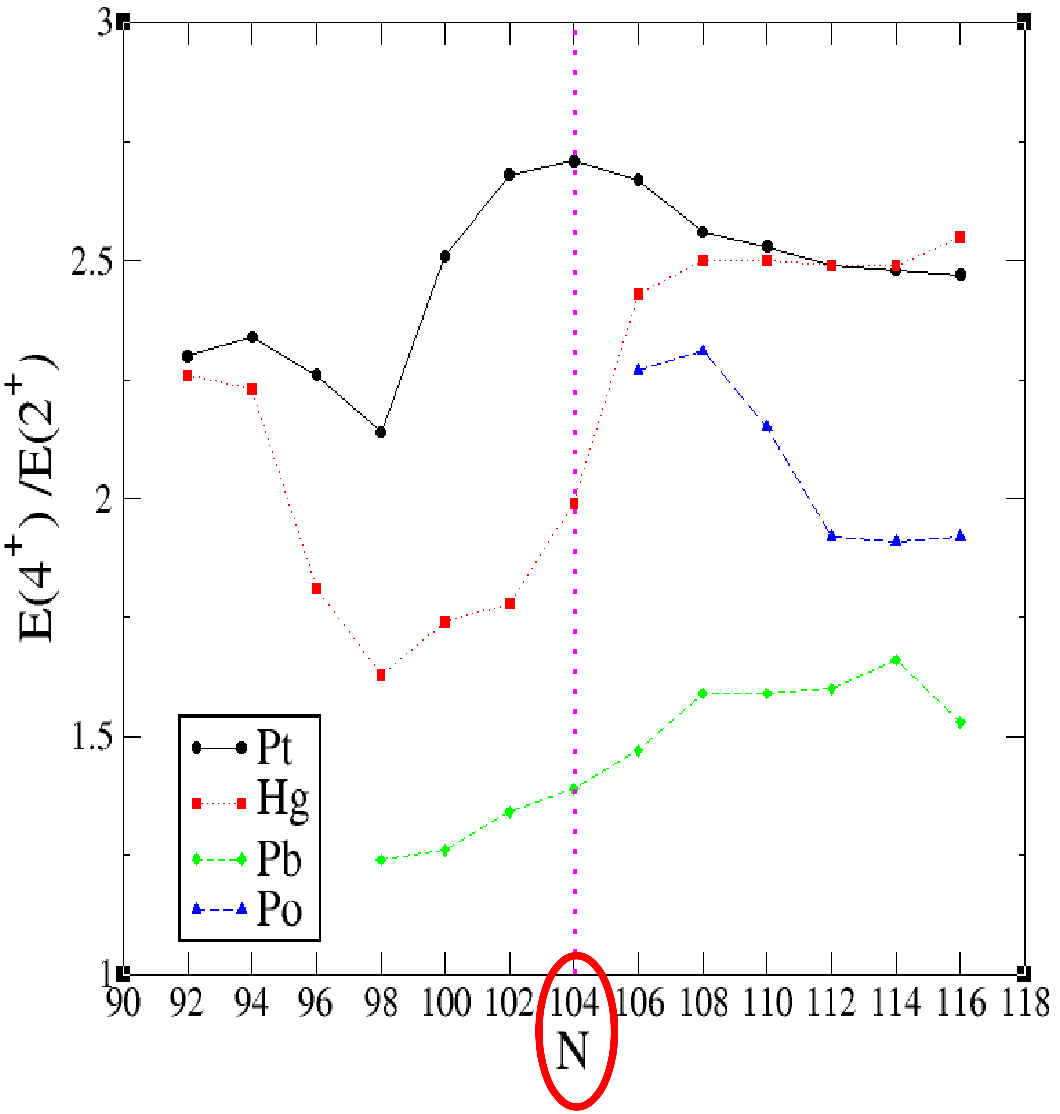
P. Cejnar, J. Jolie, and R.F. Casten,
RPM 82, 2155 (2010)

E(4+)/E(2+) can be used as an order parameter and, therefore is a key observable to find where QPT develop

No clear indication of a QPT in lead region



P. Cejnar, J. Jolie, and R.F. Casten, RPM 82, 2155 (2010)



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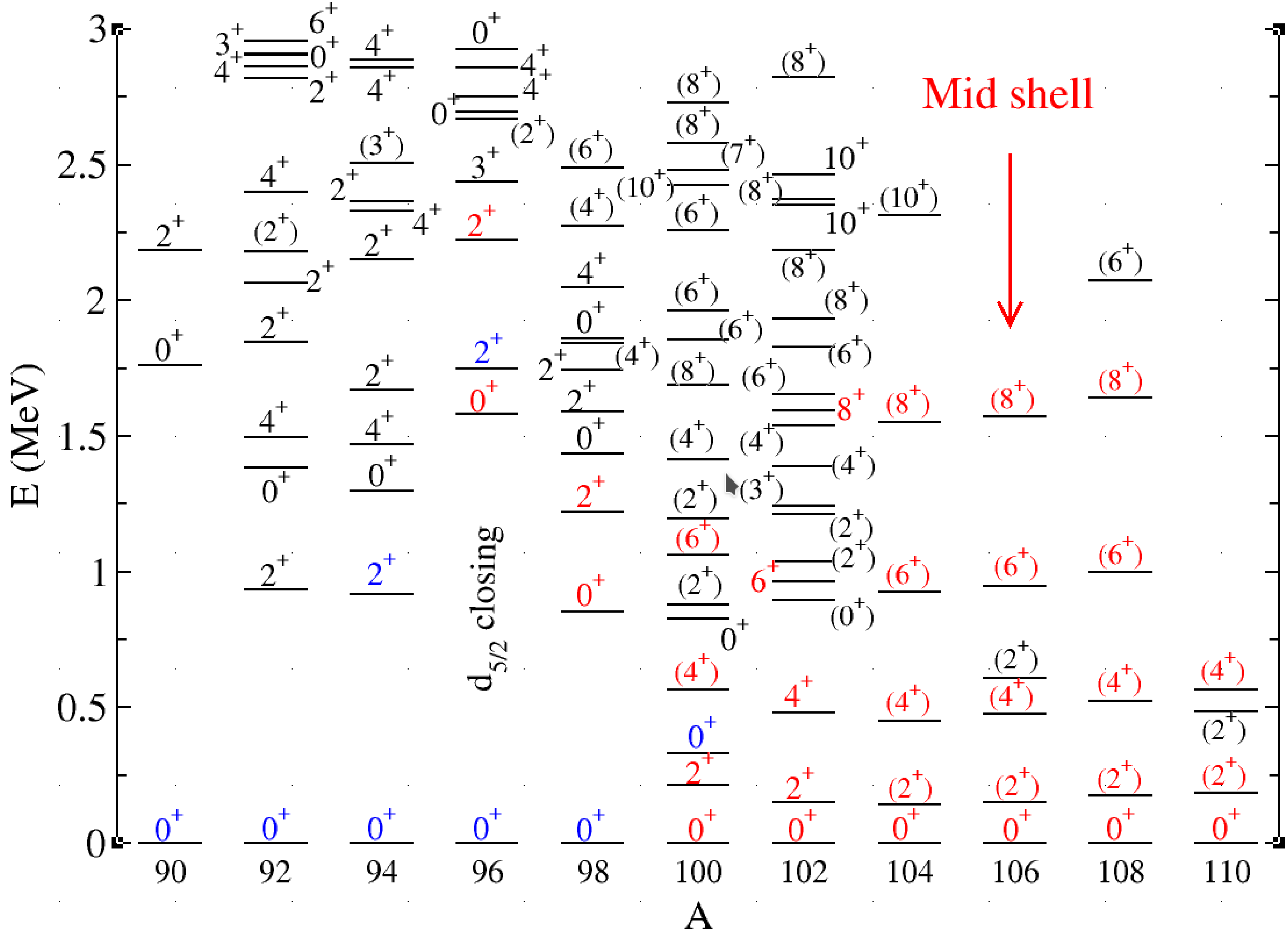
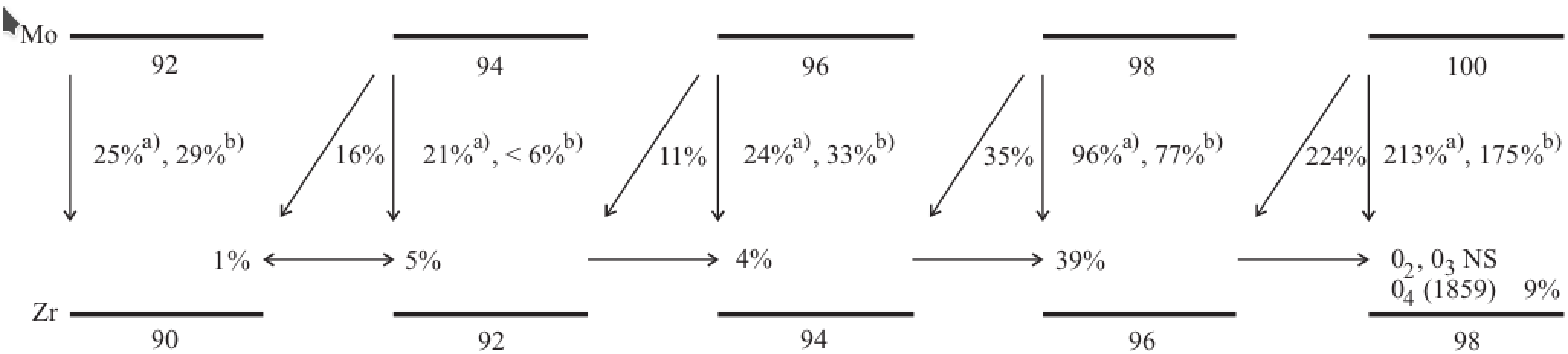
$_{40}$Zr spectra



Multiparticle transfer strengths



K. Heyde and J. L. Wood, Rev. Mod. Phys. 83, 1467 (2011).

"… reveal that pairing collectivity for the 0^+ ground states and first excited states for some of these isotopes exhibits a "coexisting" character, i.e., transfer strength goes strongly to more than one "condensate"..."

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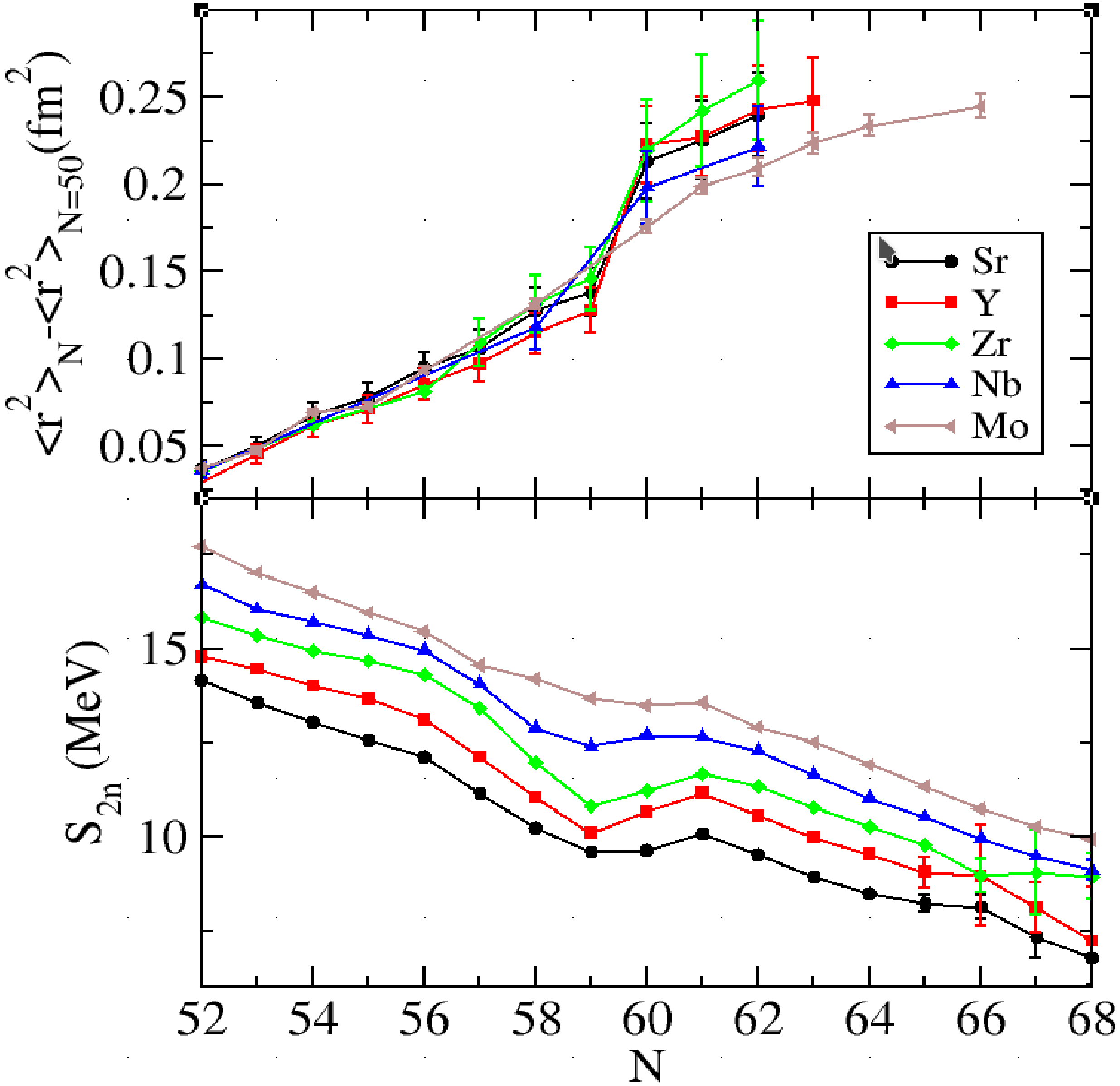
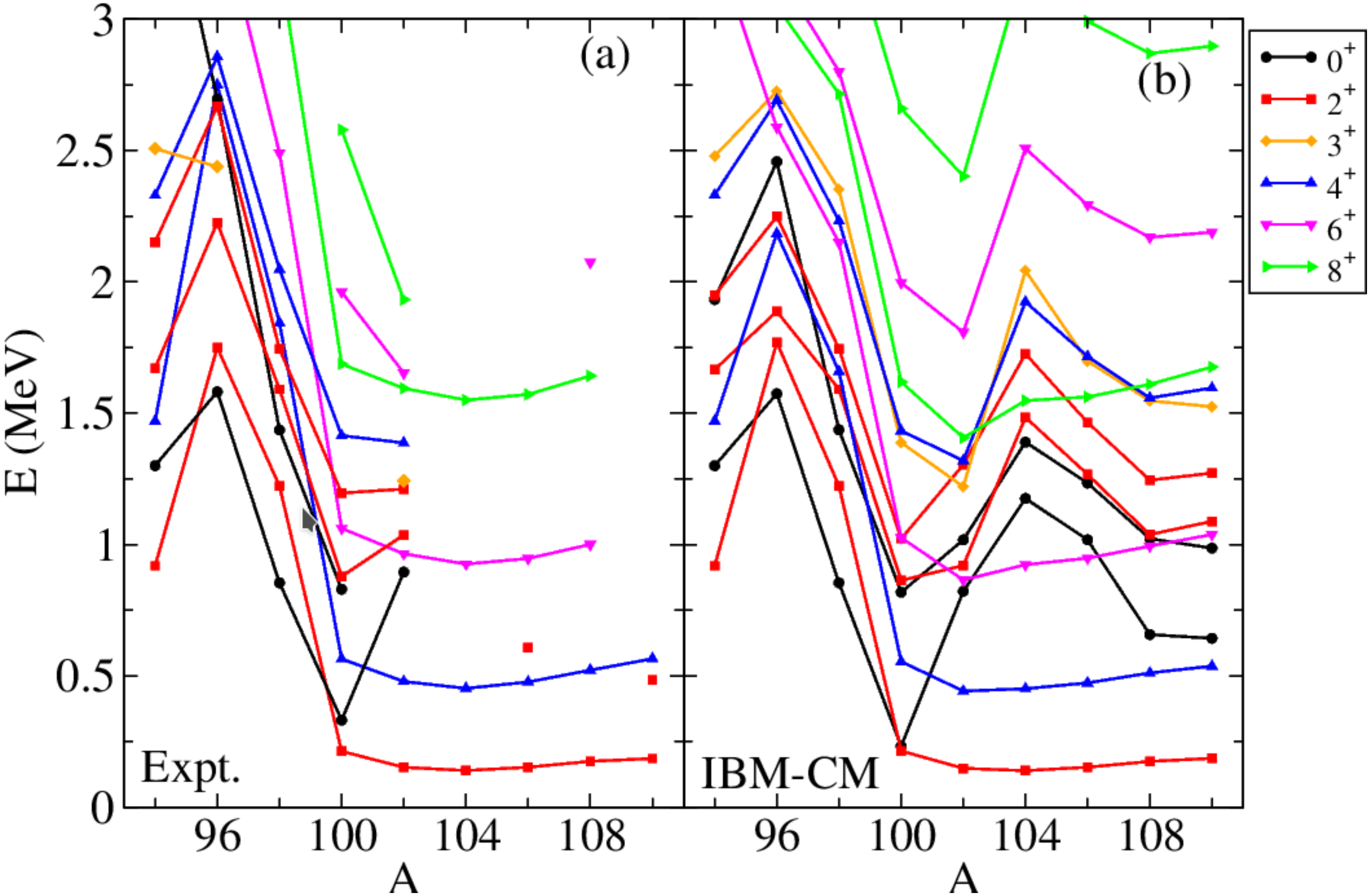
Shape Coexistence or Quantum Phase Transition?

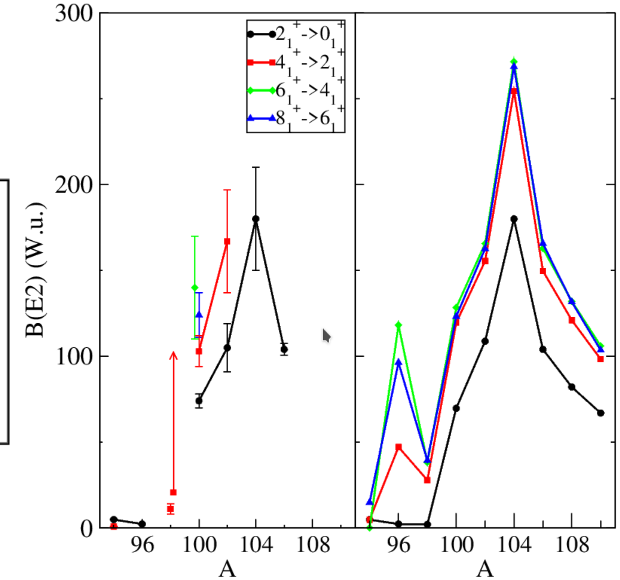


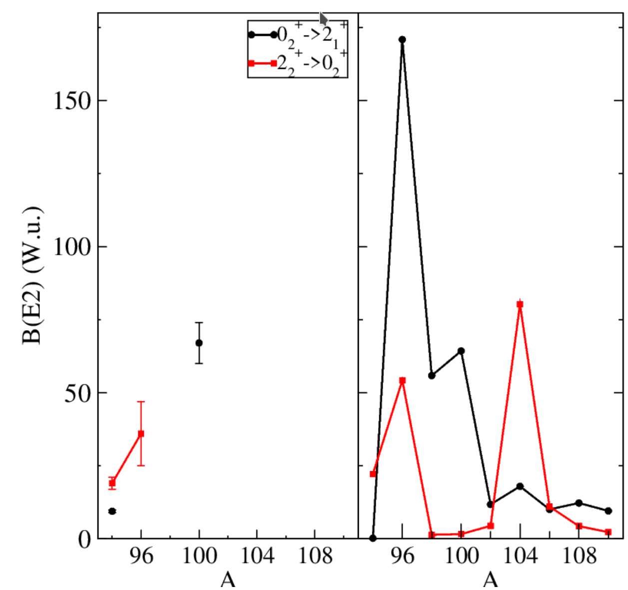
The IBM calculation

Excitation energies



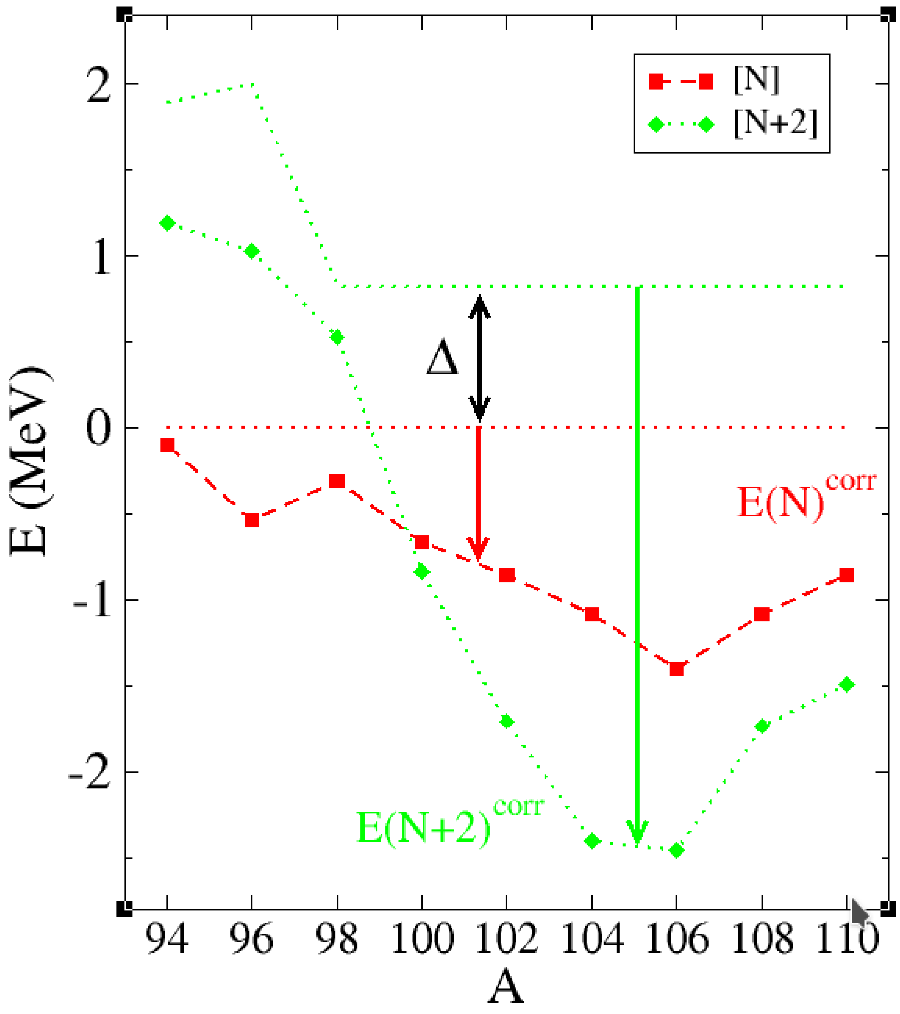
BE2's

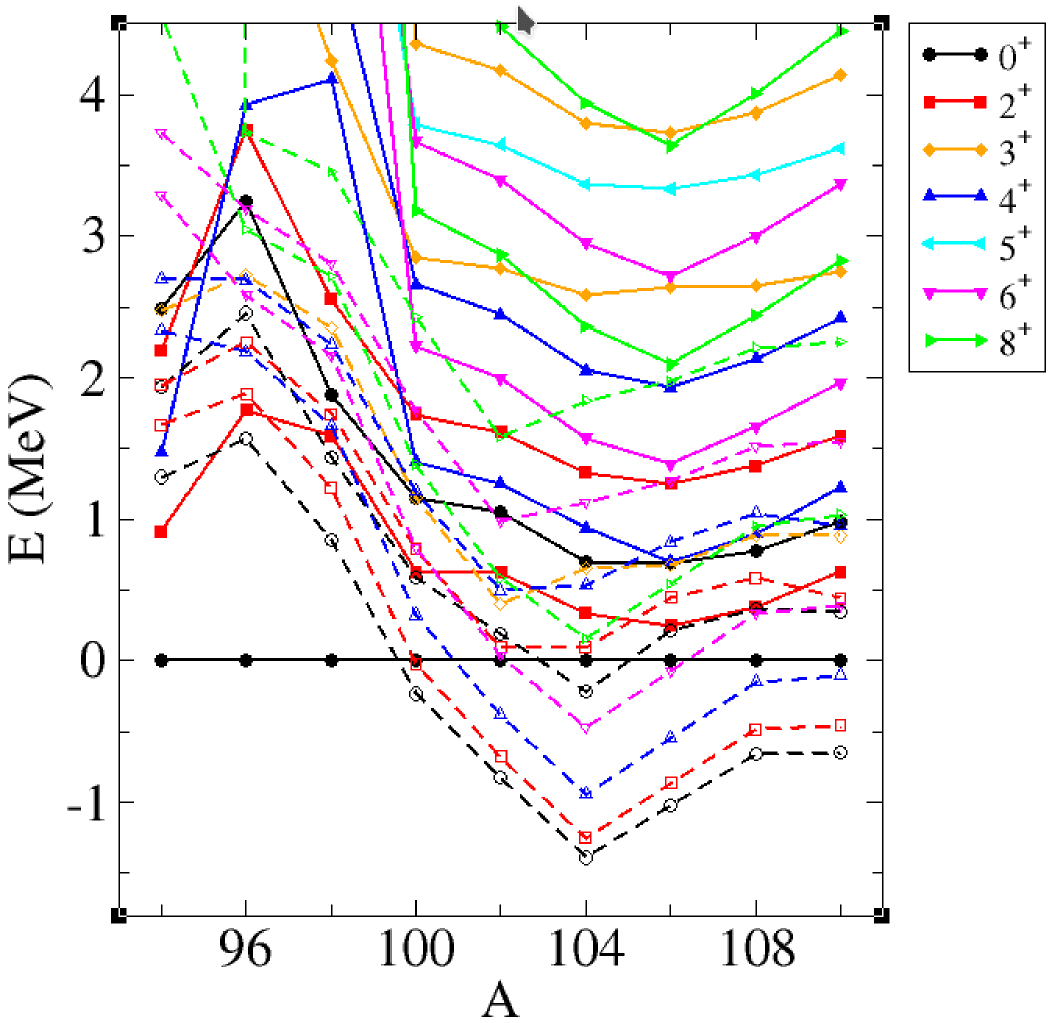




See also Noam Gabrielov's talk, today before lunch

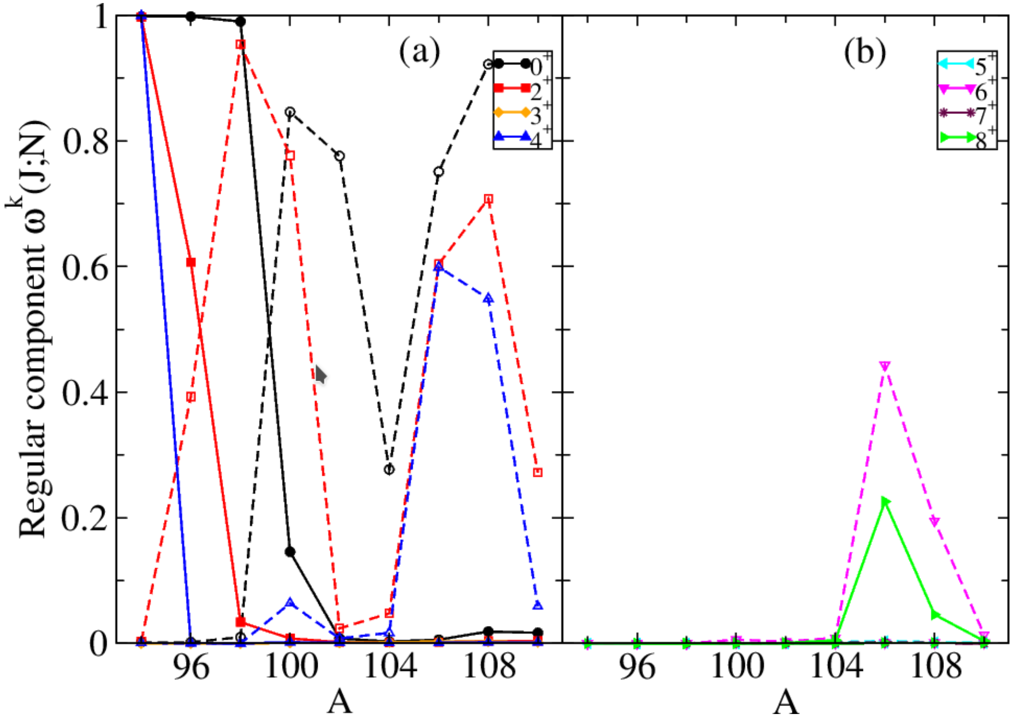
Unperturbed energies



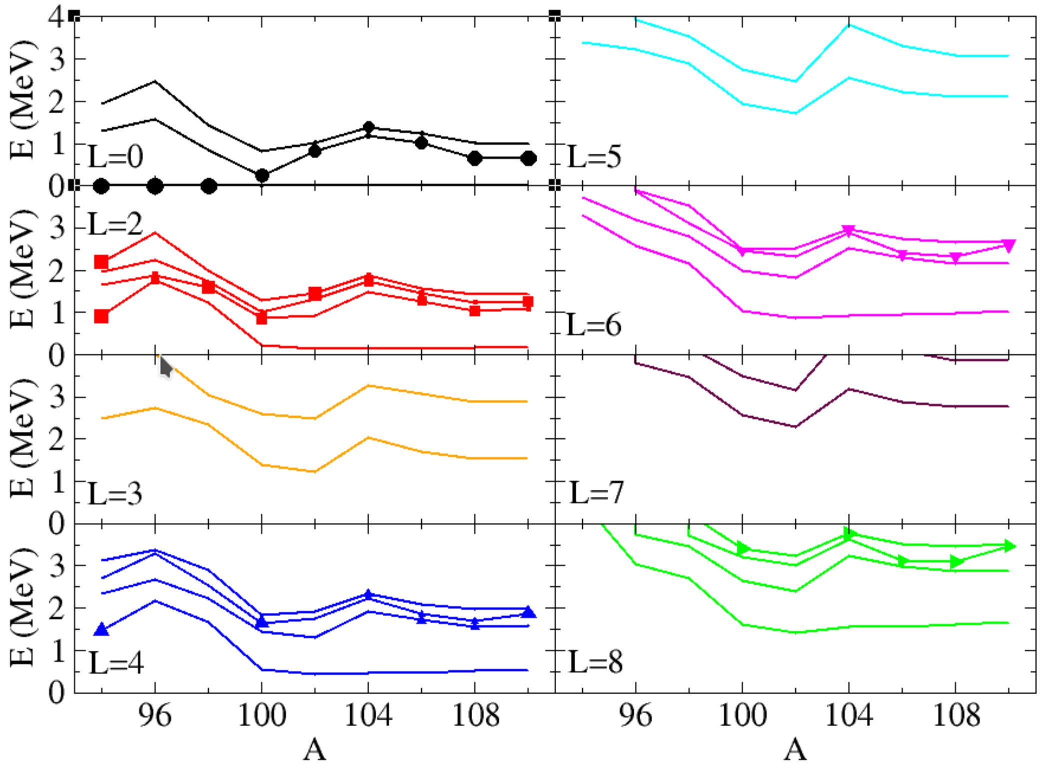


The wave function

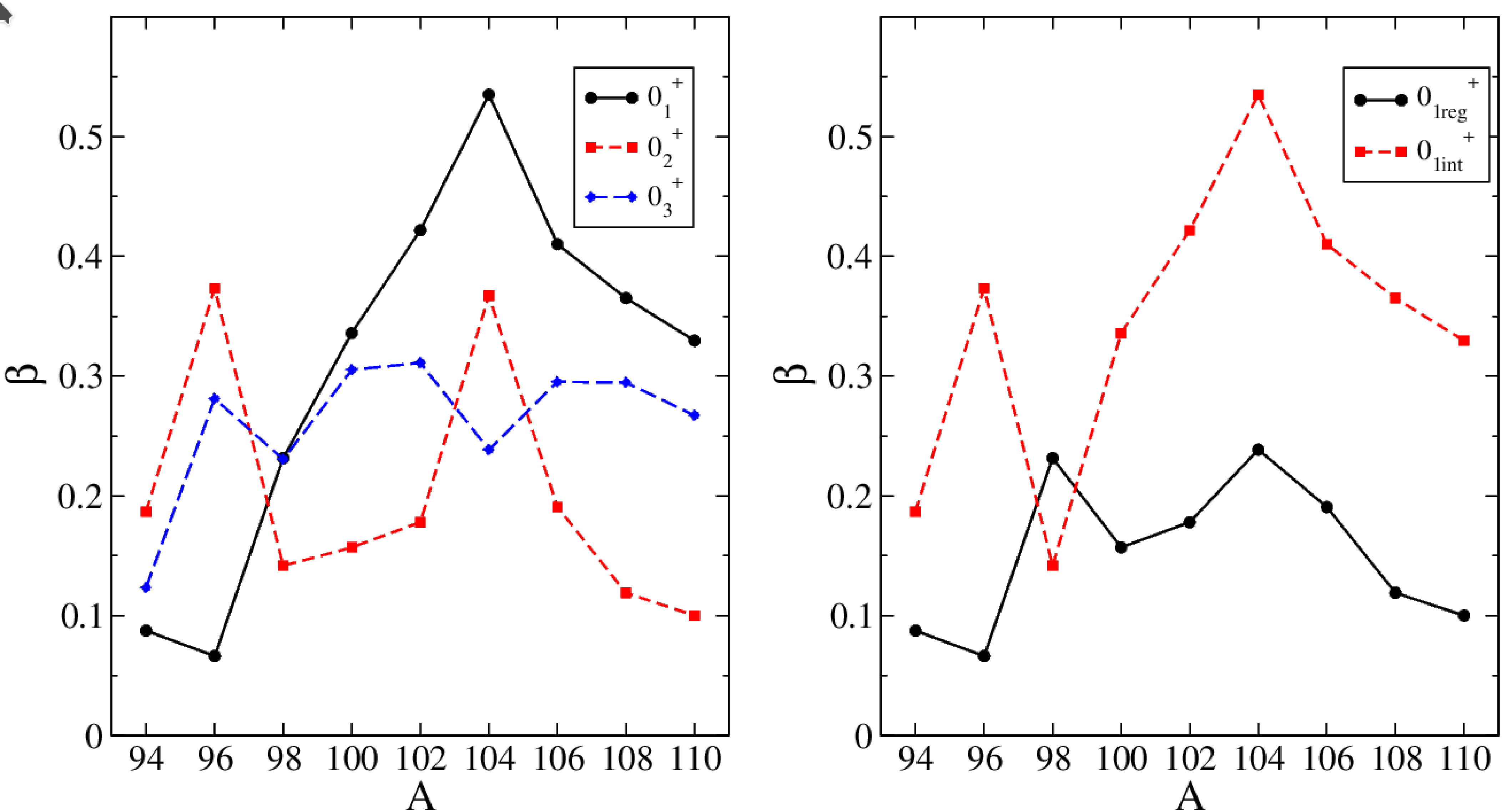
Regular component of selected states



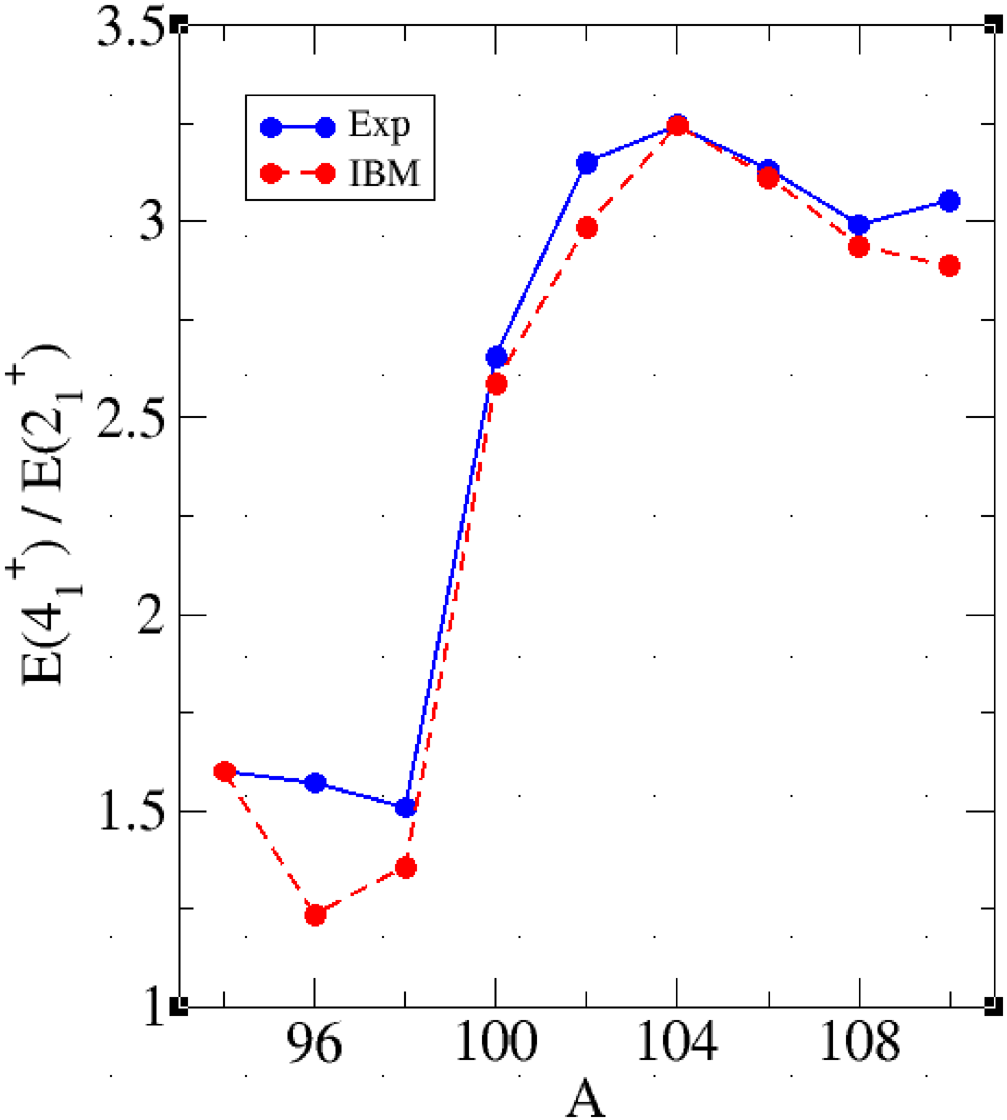
Distribution of the regular component

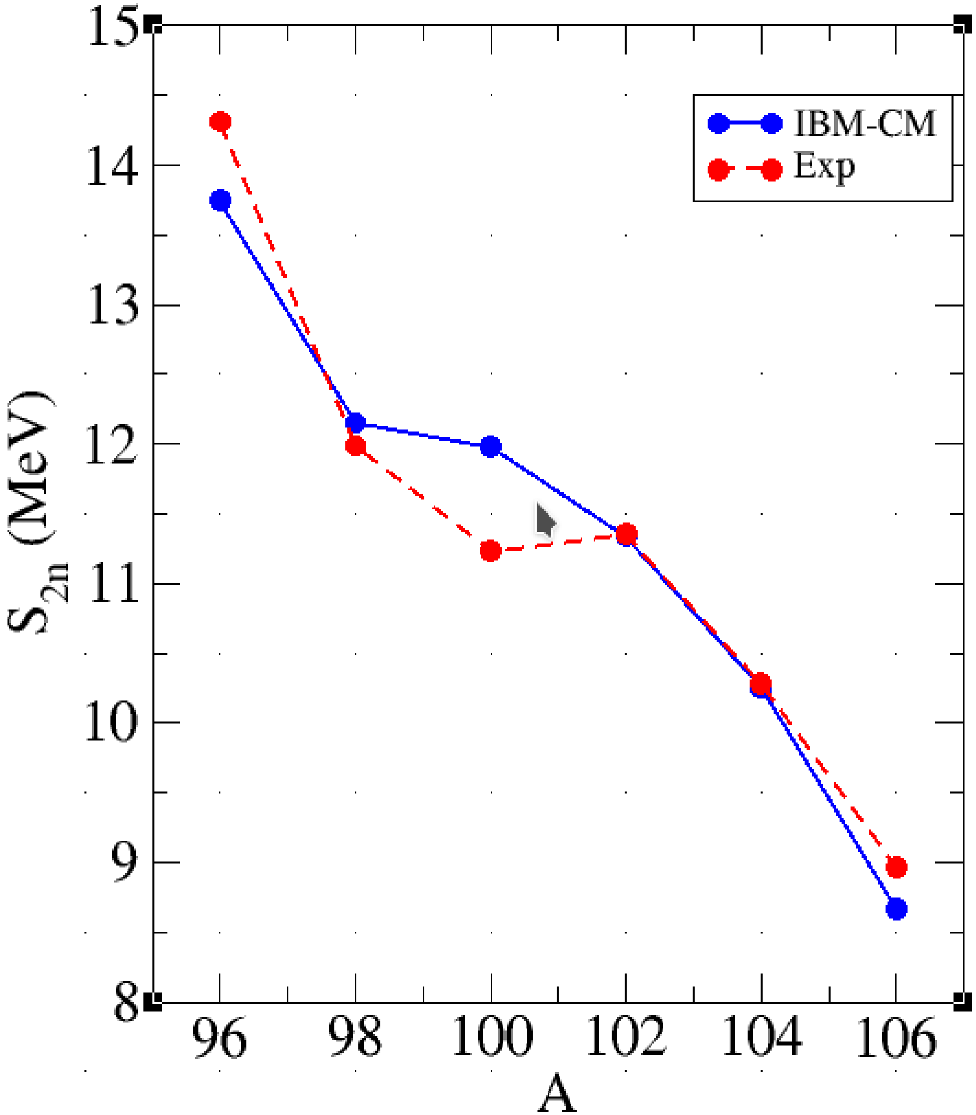


Deformation from quadrupole shape invariants



Hints pointing to a Quantum Phase Transition

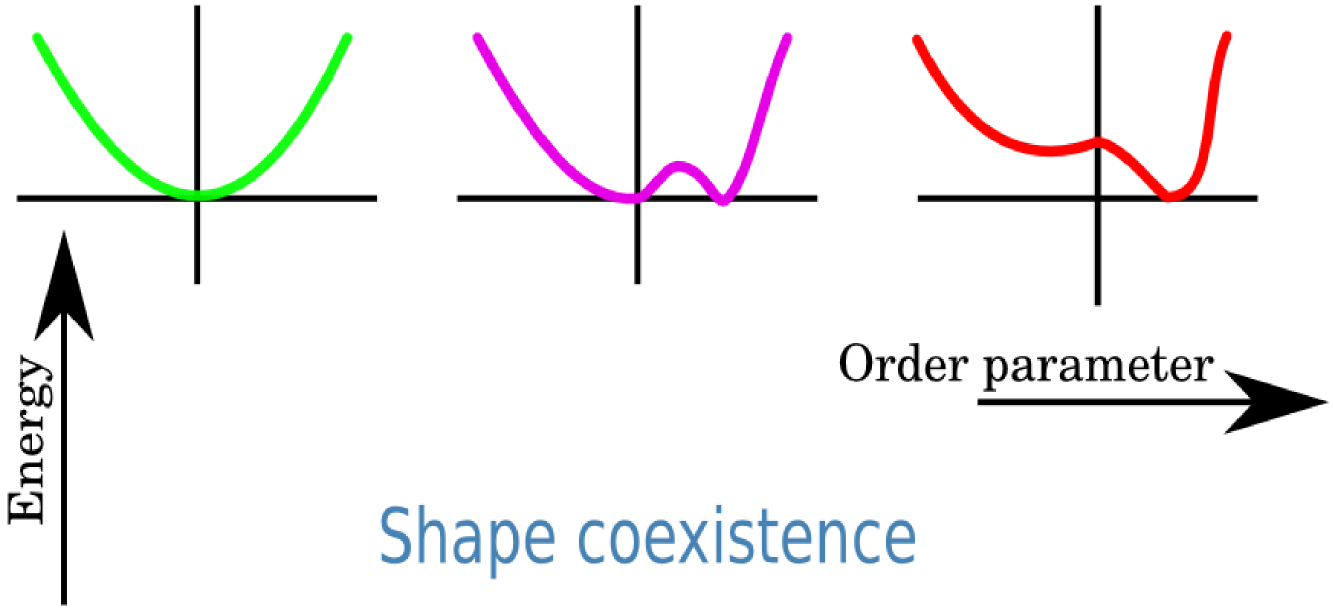


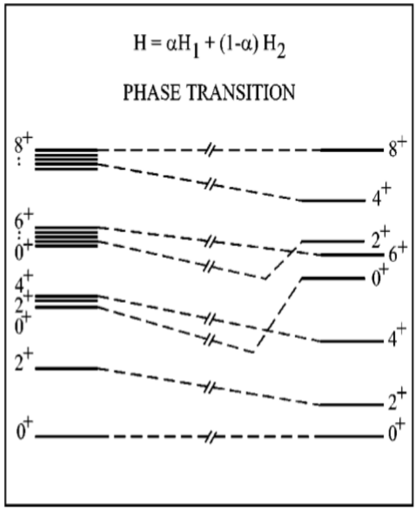


JEGR and K. Heyde, to be published

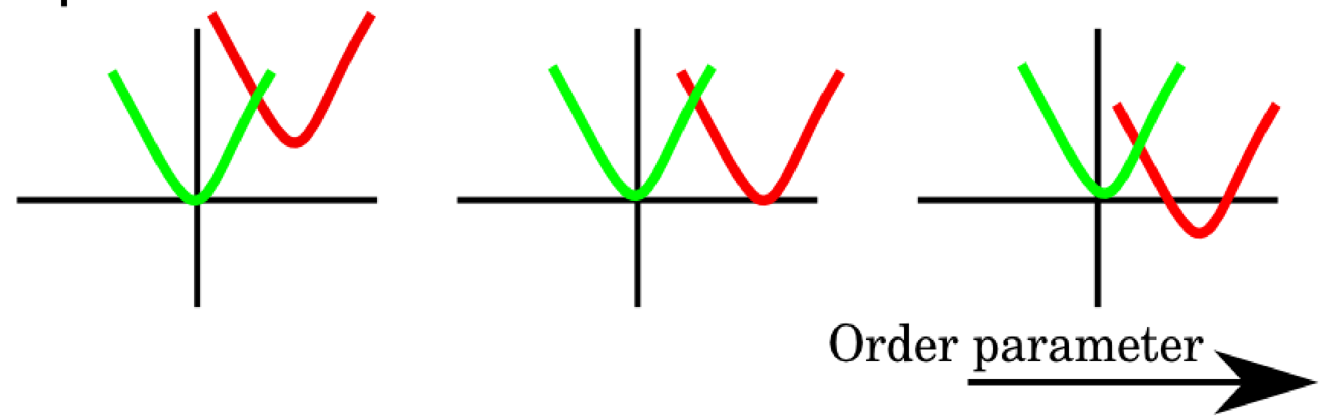
A schematic view

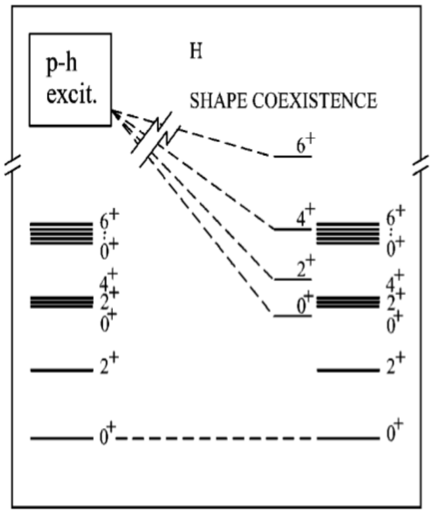
Phase transition





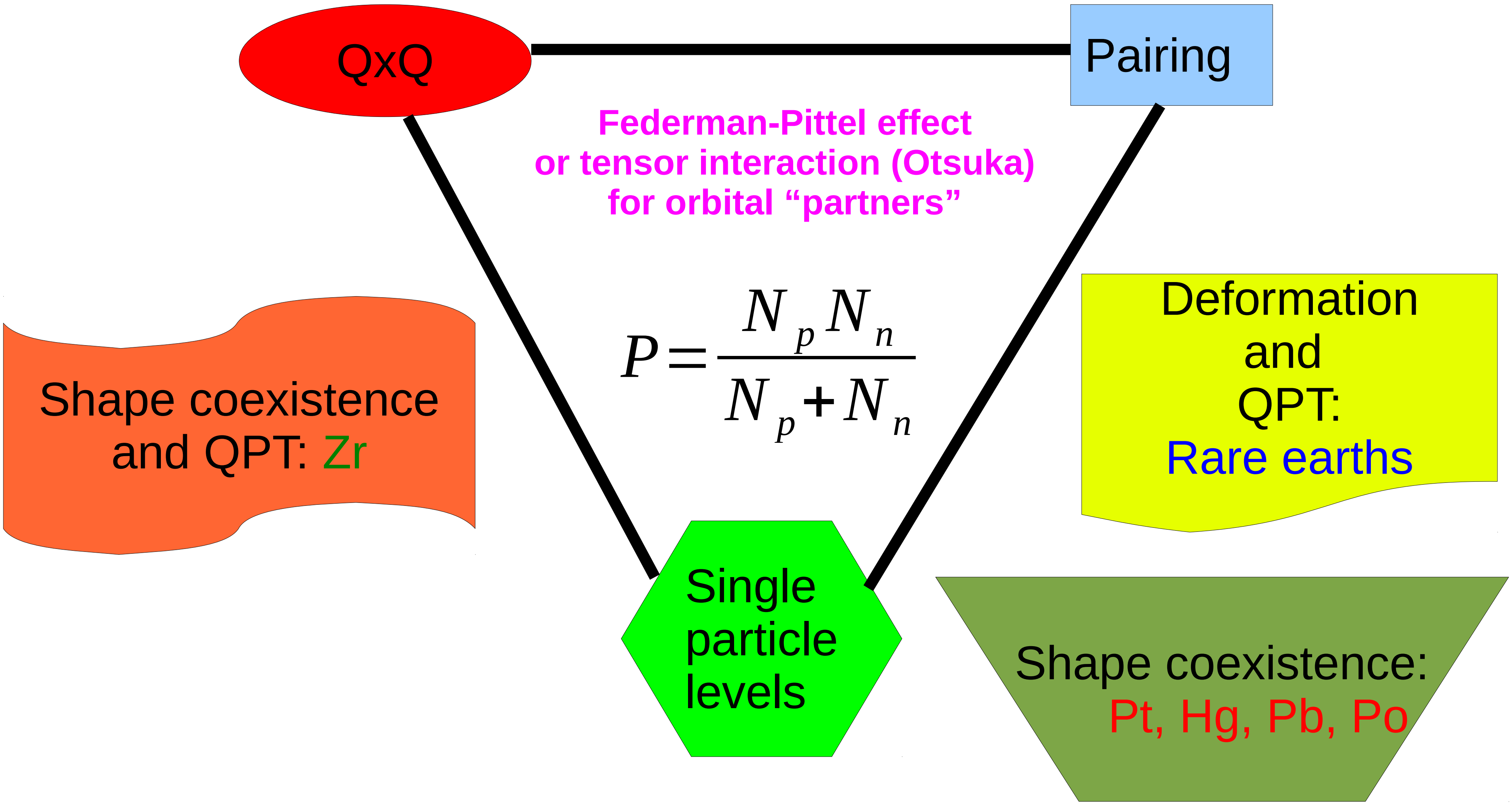
Shape coexistence





PRC 69, 054304 (2004)

Competition between interactions

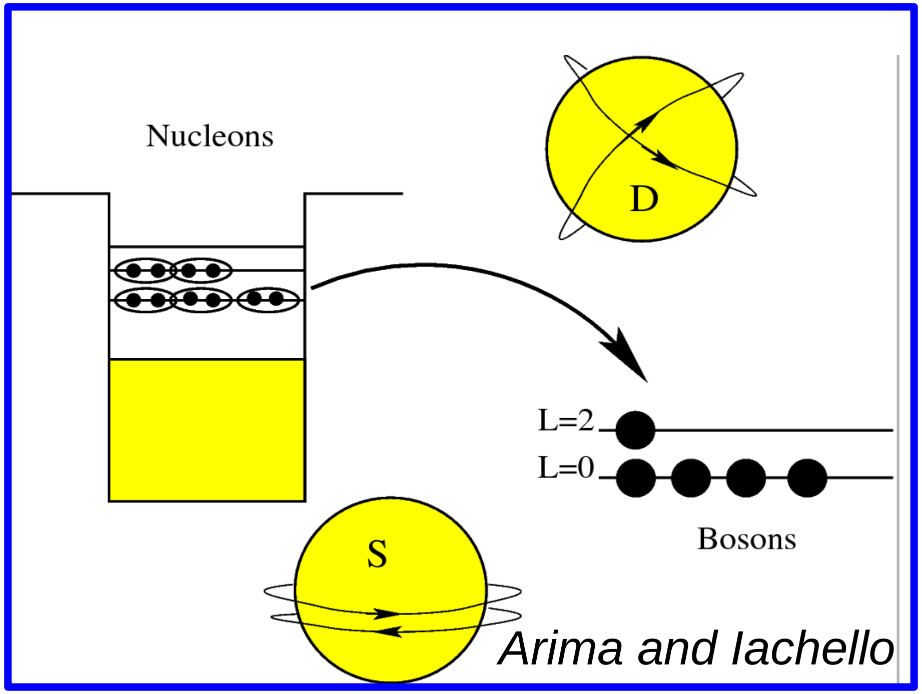


Some open questions

- Are both descriptions compatible?
- Can a Quantum Phase Transition be described in terms of the onset of intruder configurations?
- Is shape coexistence always present “before” a Quantum Phase Transition sets in, or are they fully disconnected?

Thank you

Interacting Boson Model (IBM)



Nucleons couple preferably in pairs with angular momentum either equal to 0 (S) or equal to 2 (D). Those pairs are then described by means of bosons: s and d.

$$s^\dagger, d_m^\dagger (m = 0, \pm 1, \pm 2)$$

$$s, d_m (m = 0, \pm 1, \pm 2)$$

$$\hat{H}_{ECQF} = \epsilon \hat{n}_d + \kappa \hat{Q} \cdot \hat{Q} + \kappa' \hat{L} \cdot \hat{L}$$

How to fix the parameters (IBM)

Least squares fit to the experimental data, including excitation energies and absolute B(E2) transitions.

$$\chi^2 = \frac{1}{N_{data} - N_{par}} \sum_{i=1}^{N_{data}} \frac{(X_i(data) - X_i(IBM))^2}{\sigma_i^2}$$

Error (keV)	States
$\sigma = 0.1$	2_1^+
$\sigma = 1$	$4_1^+, 0_2^+, 2_2^+$
$\sigma = 10$	$2_3^+, 3_1^+, 4_2^+, 6_1^+, 8_1^+$
$\sigma = 100$	$2_4^+, 3_1^+, 4_3^+, 6_2^+$

+ all the known B(E2) transitions

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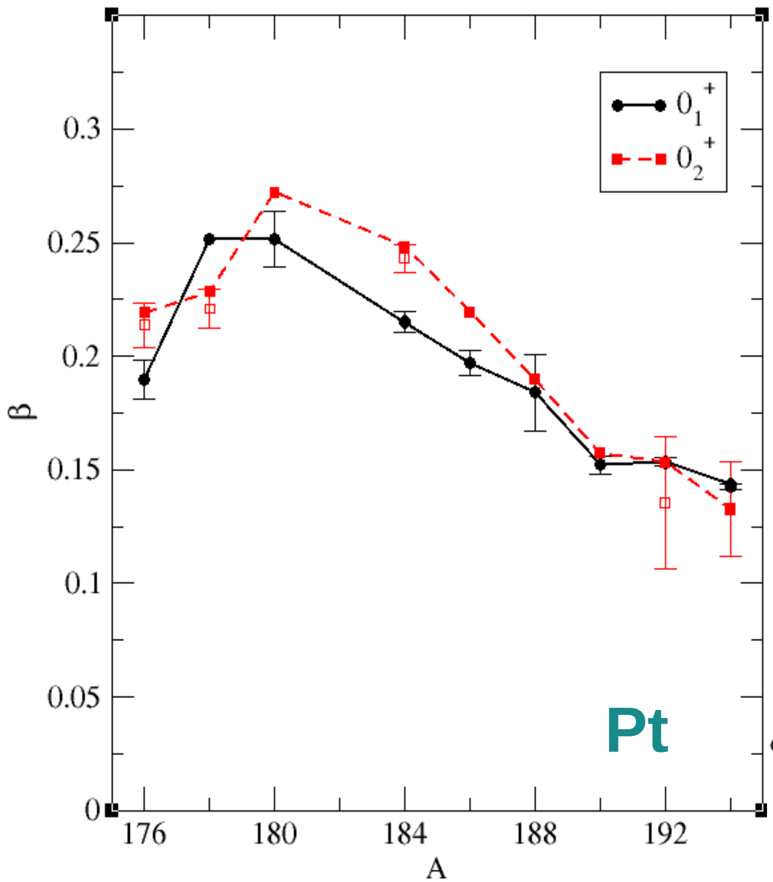
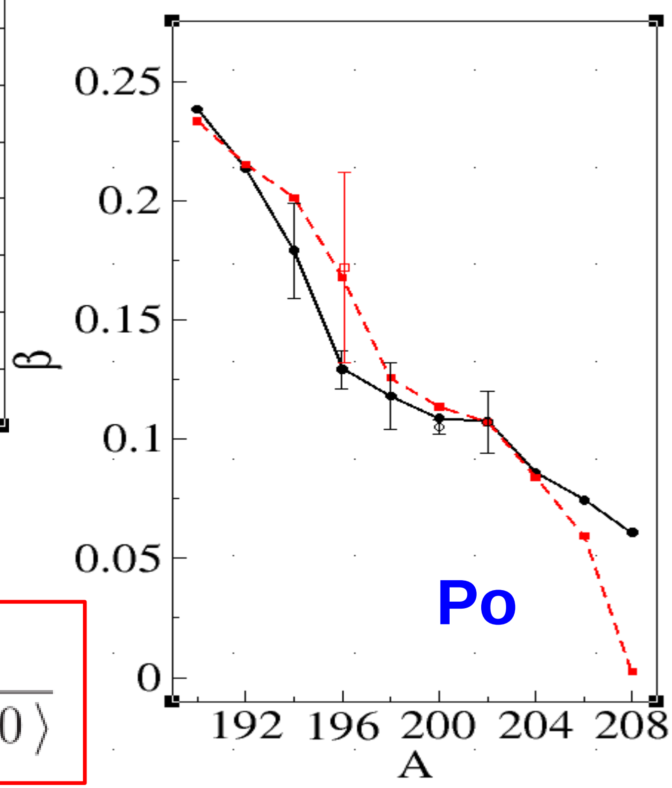
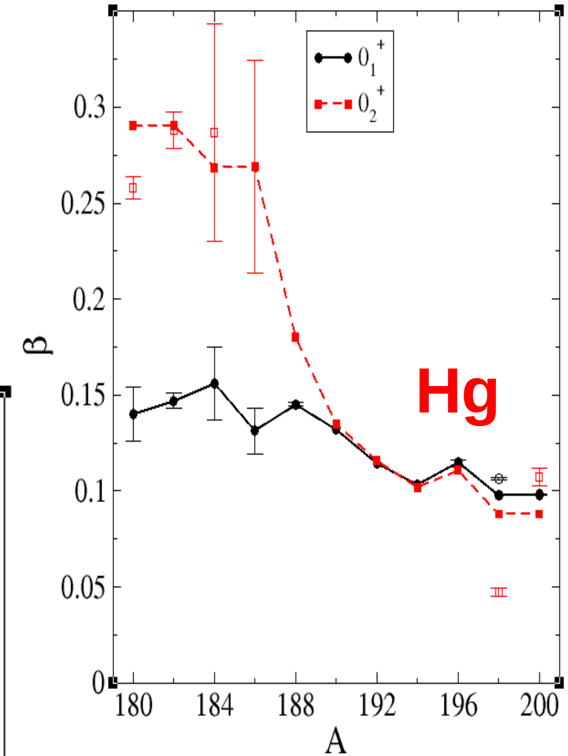
Deformation







Beta's from B(E2) values

$$\beta = \frac{4\pi\sqrt{B(E2; J \rightarrow J-2)}}{3 Z e r_0^2 A^{2/3} \langle J\,0\,2\,0 | J-2\,0 \rangle}$$

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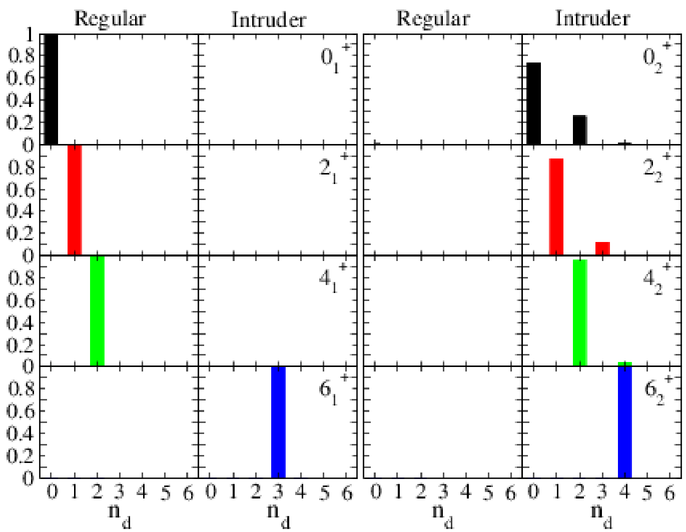
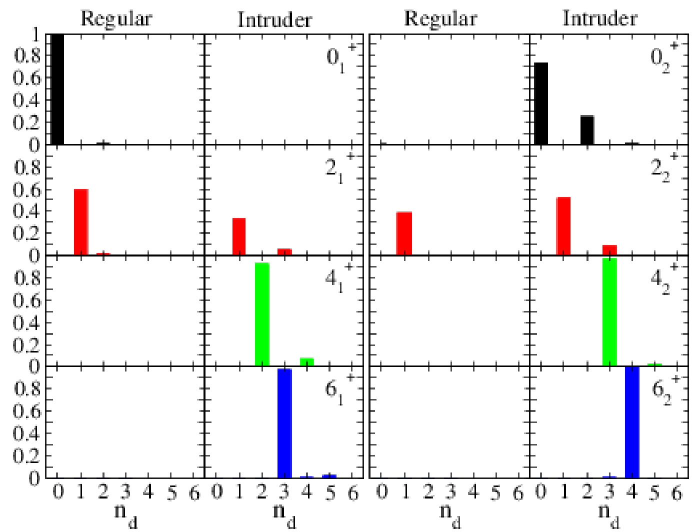
U(5) components

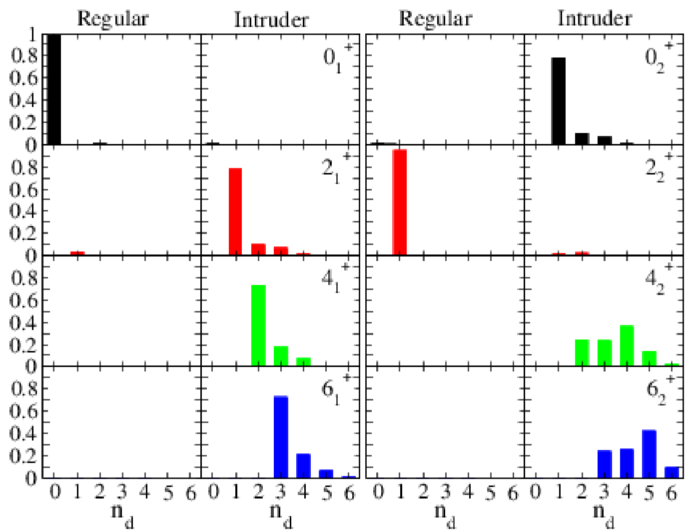
^{94}Zr



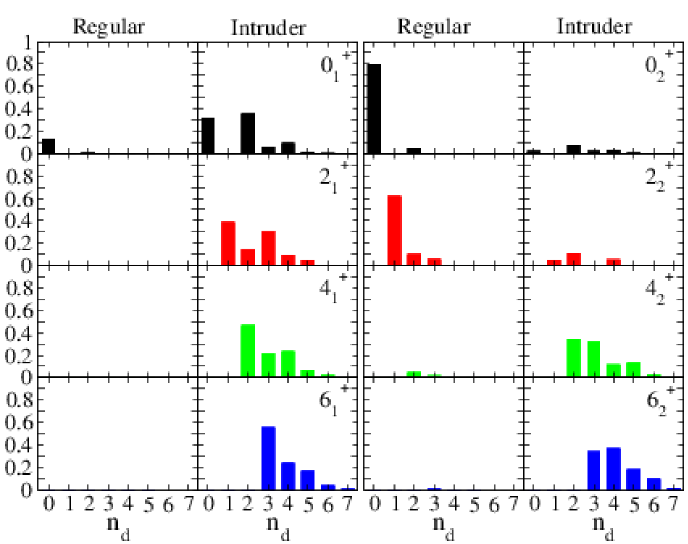
^{96}Zr



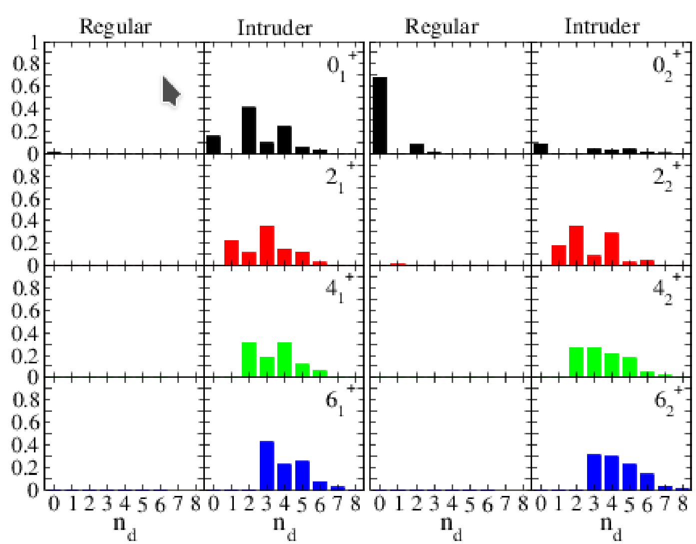
^{98}Zr



^{100}Zr



^{102}Zr



^{104}Zr