Instantaneous Reactive Power Theory to N Wire Systems

R. S. Herrera, P. Salmerón, J. R. Vázquez and S. P. Litrán
Department of Electrical Engineering
Escuela Politécnica Superior, University of Huelva
Huelva, Spain
Email: reyes.sanchez@die.uhu.es; patricio@uhu.es; vazquez@uhu.es; salvador@uhu.es

Abstract—**The control strategy derived from the instantaneous reactive power theory is one of the most commonly used in the Active Power Filters (APFs). For the last decades other formulations have been developed in order to achieve compensation objectives different to the proposed in the original one. Nevertheless, all of them can be only applied to three-phase systems, i. e.: in those formulations frameworks, they can not be used to obtain the control strategy for a polyphase system. This paper presents a new approach which can be applied to n wire systems. The powers and currents expressions derived from the formulations presented up now can be obtain applying this new approach. In this paper, the original p-q and the modified p-q formulation expressions are obtain in the new approach framework.**

Key words: Power Quality, Active Power Filter, Instantaneous Reactive Power Theory, Control Strategy

I. INTRODUCTION

Along the last decades, several formulations of the instantaneous reactive power theory have been developed, [1]-[7]. Some of them treat the zero-sequence axe in a different way of the resting system. The other formulations propose an unified treatment for all the components. Therefore, none of them can be applied to n wire systems with n>3.

In this paper a new formulation of the instantaneous reactive theory which can be applied to the n wire systems with n>2 is proposed. In this new formulation, denominated tensorial, the current vector is decomposed in two orthogonal components: the instantaneous active current which supply all the instantaneous real power and the instantaneous reactive power which supplies all the instantaneous imaginary power.

This is a global formulation which involves all the previously published to be applied to three-phase systems. The obtention of each of them depends on the current vector decomposition.

In the development of this new formulation, some mathematical operations are used, like the dyadic product or the exterior product. On the other hand, the instantaneous imaginary power presents a matrix expression, introduced at first time in the power electric systems study in 1986, [8]-[9].

The instantaneous imaginary power defined in the original formulation, [1], can only be applied to three-phase systems. All those formulations based on a coordinate translation can not be extended to n-phase systems. So, the definition presented in the modified p-q formulation, [3]-[4], or the proposed by the formulations based on synchronous reference frame, [2], can not be applied to n-phase systems.

This paper presents a definition of the instantaneous imaginary power applied to n- phase systems based on specific mathematical operations.

This paper is organized as follows. In section 3, the fundamental definitions are introduced, using the suitable mathematical operations. In section 4, the current vector is decomposed in two orthogonal components: the instantaneous active power and the instantaneous reactive power. In section 5, the expressions obtained in previous sections are particularized to a three-phase three-wire system and to a three-phase four-wire system. In section 6 simulation results are presented corresponding to a six-phase system whose load consists of a 12 pulses rectifier. In section 7 the most important conclusions are presented.

II. MATHEMATICAL FOUNDATIONS

Based on Fig. 1, the voltage vector is defined as follows:

$$\vec{u} = \mathbf{u} = \begin{bmatrix} u_1 & u_2 & \dots & u_n \end{bmatrix}^T \tag{1}$$

And the current vector as:

$$\vec{i} = \mathbf{i} = \begin{bmatrix} i_1 & i_2 & \dots & i_n \end{bmatrix}^T \tag{2}$$

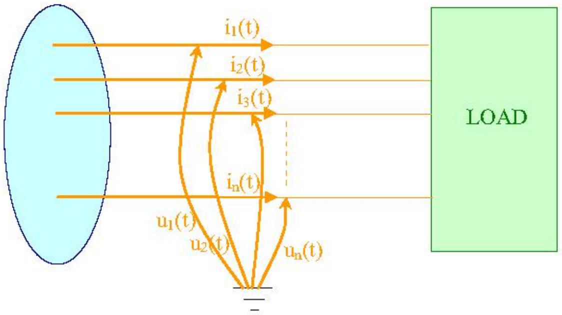

Fig. 1 n wire system

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On the other hand, this formulation defines the instantaneous active power as:

$$p(t)=\vec{u}(t)\cdot\vec{i}(t)=\mathbf{u}^T\mathbf{i} \qquad (3)$$

and the instantaneous imaginary power matrix as the order n hemisymetrical tensor, $\vec{q}(t)$, which is calculated as the exterior product of current and voltage vector as follows:

$$\mathbf{q}(t)=\vec{i}(t)\wedge\vec{u}(t) \qquad (4)$$

The exterior product is calculated as the difference of the next dyadic products:

$$\mathbf{q}(t)=\vec{i}(t)\wedge\vec{u}(t)=\left(\vec{i}\otimes\vec{u}\right)-\left(\vec{u}\otimes\vec{i}\right) \qquad (5)$$

The dyadic product of current vector over voltage vector is defined as:

$$\vec{i}\otimes\vec{u}=\begin{bmatrix} i_1 & i_2 & \dots & i_n \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ \dots \\ u_n \end{bmatrix}=\begin{bmatrix} i_1u_1 & i_1u_2 & \dots & i_1u_n \\ i_2u_1 & i_2u_2 & \dots & i_2u_n \\ \dots & \dots & \dots & \dots \\ i_nu_1 & i_nu_2 & \dots & i_nu_n \end{bmatrix} \qquad (6)$$

And the dyadic product of voltage vector over current vector is defined as:

$$\vec{u}\otimes\vec{i}=\begin{bmatrix} u_1 & u_2 & \dots & u_n \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ \dots \\ i_n \end{bmatrix}=\begin{bmatrix} u_1i_1 & u_1i_2 & \dots & u_1i_n \\ u_2i_1 & u_2i_2 & \dots & u_2i_n \\ \dots & \dots & \dots & \dots \\ u_ni_1 & u_ni_2 & \dots & u_ni_n \end{bmatrix} \qquad (7)$$

So, the instantaneous imaginary power matrix gets the next form:

$$\mathbf{q}=\vec{i}\wedge\vec{u}=\begin{bmatrix} 0 & i_1u_2-u_1i_2 & \dots & i_1u_n-u_1i_n \\ u_1i_2-i_1u_2 & 0 & \dots & i_2u_n-u_2i_n \\ \dots & \dots & \dots & \dots \\ u_1i_n-i_1u_n & u_2i_n-i_2u_n & \dots & 0 \end{bmatrix} \qquad (8)$$

III. CURRENT VECTOR DECOMPOSITION

Firstly, the instantaneous active current, $\vec{i}_p$, is defined proportional to the voltage vector as follows:

$$\vec{i}_p=\frac{p(t)}{\mathbf{u}^T\mathbf{u}}\mathbf{u}=\frac{\left(u_1i_1+u_2i_2+\dots+u_ni_n\right)}{\vec{u}\cdot\vec{u}}\begin{bmatrix} u_1 \\ u_2 \\ \dots \\ u_n \end{bmatrix} \qquad (9)$$

expression equivalent to the next:

$$\vec{i}_p=\frac{\left(\vec{u}\otimes\vec{i}\right)}{\vec{u}\cdot\vec{u}}\vec{u}=\frac{1}{\vec{u}\cdot\vec{u}}\begin{bmatrix} u_1i_1u_1+u_1i_2u_2+\dots+u_1i_nu_n \\ u_2i_1u_1+u_2i_2u_2+\dots+u_2i_nu_n \\ \dots \\ u_ni_1u_1+u_ni_2u_2+\dots+u_ni_nu_n \end{bmatrix} \qquad (10)$$

On the other hand, the current vector can be expressed as:

$$\vec{i}=\frac{\left(\vec{i}\otimes\vec{u}\right)}{\vec{u}\cdot\vec{u}}\vec{u} \qquad (11)$$

Therefore, from (10) and (11), the instantaneous reactive current is defined as follows:

$$\vec{i}_q=\frac{\mathbf{q}(t)\cdot\mathbf{u}}{\mathbf{u}^T\mathbf{u}}=\frac{\left(\vec{i}\wedge\vec{u}\right)\cdot\vec{u}}{\vec{u}\cdot\vec{u}}=\frac{\left[\left(\vec{i}\otimes\vec{u}\right)-\left(\vec{u}\otimes\vec{i}\right)\right]\cdot\vec{u}}{\vec{u}\cdot\vec{u}} \qquad (12)$$

So, the load current vector is decomposed in two orthogonal components: the instantaneous active current proportional to the voltage and which carries the whole instantaneous real power and the instantaneous reactive current orthogonal to the voltage vector and which does not carry instantaneous active power, but instantaneous imaginary power.

The instantaneous real power is a real number and the instantaneous imaginary power is a n order hemisymetrical tensor where n is the wires number of the power system.

Several compensation approaches can be achieved applying this new formulation: instantaneous or average. Into the second approach framework, several objectives can be considered: constant source current, unity power factor and balanced sinusoidal source current, [10]-[11].

IV. APPLICATION TO THREE-PHASE SYSTEMS

Appling the definitions derived from the new formulation presented in this paper to a three-phase three-wire system, fig. 2, the instantaneous imaginary power matrix is expressed as:

$$\mathbf{q}=\begin{bmatrix} 0 & u_2i_1-u_1i_2 & u_3i_1-u_1i_3 \\ u_1i_2-u_2i_1 & 0 & u_3i_2-u_2i_3 \\ u_1i_3-u_3i_1 & u_2i_3-u_3i_2 & 0 \end{bmatrix} \qquad (13)$$

This matrix can be expressed as a vector way as follows:

$$\vec{q} = \begin{bmatrix} u_2 i_3 - u_3 i_2 \\ u_3 i_1 - u_1 i_3 \\ u_1 i_2 - u_2 i_1 \end{bmatrix} \tag{14}$$

It is identical to the instantaneous reactive power vector defined by Peng et all, [4], in the development of the modified p-q formulation in phase coordinates system.

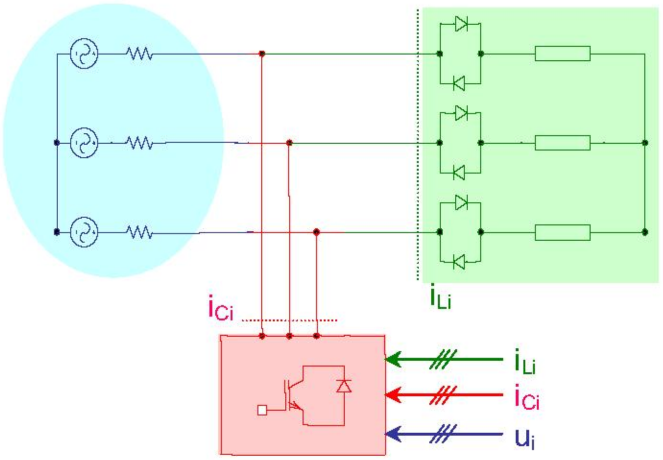

Fig. 2 Tree-phase three-wire power system

Therefore, the current vector is expressed in the next way:

$$\vec{i}(t) = \vec{i}_p(t) + \vec{i}_q(t) =$$
$$= \frac{p(t)}{\vec{u}\cdot\vec{u}}\vec{u} + \frac{1}{\vec{u}\cdot\vec{u}} \begin{bmatrix} (u_3 i_1 - u_1 i_3)u_3 - (u_1 i_2 - u_2 i_1)u_2 \\ (u_1 i_2 - u_2 i_1)u_1 - (u_2 i_3 - u_3 i_2)u_3 \\ (u_2 i_3 - u_3 i_2)u_2 - (u_3 i_1 - u_1 i_3)u_1 \end{bmatrix} \tag{15}$$

The instantaneous reactive current must be expressed as:

$$\vec{i}_q(t) = \frac{\vec{q}(t) \wedge \vec{u}(t)}{\vec{u}\cdot\vec{u}} \tag{16}$$

It is identical to the instantaneous reactive current expression presented by the modified p-q formulation, [4], in phase coordinates system.

V. ORIGINAL P-Q FORMULATION IN THE NEW APPROACH FRAMEWORK

In the previous sections the modified p-q formulation is obtain from the tensorial approach. In the same way, the original p-q formulation equations are obtained in the present section. In fact, voltage and current are expressed in the 0αβ coordinates system. So, the voltage vector is:

$$\vec{u} = \begin{bmatrix} u_0 & u_\alpha & u_\beta \end{bmatrix}^T \tag{17}$$

where u_0 is the zero-sequence component, u_α the α component and u_β the β component.

On the other hand, the current vector is expressed as:

$$\vec{i} = \begin{bmatrix} i_0 & i_\alpha & i_\beta \end{bmatrix}^T \tag{18}$$

where i_0 is the zero-sequence component, i_α the α component and i_β the β component.

According to the p-q formulation, the zero-sequence component is submitted to an independent treatment. So, two other vectors can be defined: the αβ voltage vector or voltage vector without zero-sequence component, $\vec{u}_{\alpha\beta}$, and the αβ current vector or current vector without zero-sequence component, $\vec{i}_{\alpha\beta}$:

$$\vec{u}_{\alpha\beta} = \begin{bmatrix} 0 & u_\alpha & u_\beta \end{bmatrix}^T \tag{19}$$

$$\vec{i}_{\alpha\beta} = \begin{bmatrix} 0 & i_\alpha & i_\beta \end{bmatrix}^T \tag{20}$$

Therefore, the zero-sequence vectors can be defined. In fact, the voltage zero-sequence vector is expressed as:

$$\vec{u}_0 = \begin{bmatrix} u_0 & 0 & 0 \end{bmatrix}^T \tag{21}$$

And the current zero-sequence vector as:

$$\vec{i}_0 = \begin{bmatrix} i_0 & 0 & 0 \end{bmatrix}^T \tag{22}$$

As can be seen, the zero-sequence current vector and the αβ current vector are orthogonal and so on the respective voltage vectors.

Besides:

$$\vec{u}\cdot\vec{u}_{\alpha\beta} = \vec{u}_{\alpha\beta}\cdot\vec{u}_{\alpha\beta} \tag{23}$$

and:

$$\vec{u}\cdot\vec{u}_0 = \vec{u}_0\cdot\vec{u}_0 \tag{24}$$

The αβ current vector can be decomposed in two orthogonal components by means of the tensorial develop. It is:

$$\vec{i}_{\alpha\beta} = \frac{\vec{i}_{\alpha\beta}\otimes\vec{u}_{\alpha\beta}}{\vec{u}_{\alpha\beta}\cdot\vec{u}_{\alpha\beta}}\vec{u}_{\alpha\beta} =$$
$$= \frac{\vec{u}_{\alpha\beta}\otimes\vec{i}_{\alpha\beta}}{\vec{u}_{\alpha\beta}\cdot\vec{u}_{\alpha\beta}}\vec{u}_{\alpha\beta} + \frac{\vec{i}_{\alpha\beta}\wedge\vec{u}_{\alpha\beta}}{\vec{u}_{\alpha\beta}\cdot\vec{u}_{\alpha\beta}}\vec{u}_{\alpha\beta} = \vec{i}_{\alpha\beta p} + \vec{i}_{\alpha\beta q} \tag{25}$$

where $\vec{i}_{\alpha\beta p}$ defines the projection of the αβ current vector over the αβ voltage vector and is called αβ instantaneous active current and the $\vec{i}_{\alpha\beta q}$ represents the projection of the αβ current vector over and orthogonal voltage vector and is called instantaneous reactive current.

In fact, developing the αβ instantaneous active current presented in the equation (25), it is obtained the next:

$$\vec{i}_{\alpha\beta p} = \frac{p_{\alpha\beta}(t)}{u_{\alpha\beta}^2}\vec{u}_{\alpha\beta} \tag{26}$$

where $p_{\alpha\beta}(t)$ is the defined by Akagi et all, [1]:

$$p_{\alpha\beta}(t) = u_\alpha i_\alpha + u_\beta i_\beta \tag{27}$$

The current vector presented in equation (26) involves two components. The first one is the instantaneous active current in the axe α, $\vec{i}_{\alpha p}$, [1]:

$$\vec{i}_{\alpha p} = \frac{p_{\alpha\beta}(t)}{u_{\alpha\beta}^2}u_\alpha \tag{28}$$

The second one is the instantaneous active current in the axe β, $\vec{i}_{\beta p}$, [1]:

$$\vec{i}_{\beta p} = \frac{p_{\alpha\beta}(t)}{u_{\alpha\beta}^2}u_\beta \tag{29}$$

On the other hand, developing the $\vec{i}_{\alpha\beta q}$ expression in equation (25), it is obtained the next:

$$\begin{aligned}\vec{i}_{\alpha\beta q} &= \frac{\vec{i}_{\alpha\beta} \wedge \vec{u}_{\alpha\beta}}{\vec{u}_{\alpha\beta} \cdot \vec{u}_{\alpha\beta}}\vec{u}_{\alpha\beta} = \\ &= \frac{1}{u_\alpha^2 + u_\beta^2}\begin{bmatrix} 0 \\ i_\alpha u_\beta^2 - i_\beta u_\alpha u_\beta \\ i_\beta u_\alpha^2 - i_\alpha u_\alpha u_\beta \end{bmatrix} = \\ &= \frac{u_\alpha i_\beta - u_\beta i_\alpha}{u_{\alpha\beta}^2}\begin{bmatrix} 0 \\ -u_\beta \\ u_\alpha \end{bmatrix}\end{aligned} \tag{30}$$

The current expression shown in equation (30) involves two components. In fact, the α component is the corresponding to the original p-q formulation instantaneous reactive current in axe α, $\vec{i}_{\alpha q}$, [1]:

$$\vec{i}_{\alpha q} = -\frac{q_{\alpha\beta}(t)}{u_{\alpha\beta}^2}u_\beta \tag{31}$$

The β component is the corresponding to the original p-q formulation instantaneous reactive current in axe β, $\vec{i}_{\beta q}$, [1]:

$$\vec{i}_{\beta q} = \frac{q_{\alpha\beta}(t)}{u_{\alpha\beta}^2}u_\alpha \tag{32}$$

VI. SIMULATION RESULTS

In this section, the definitions are applied to a six-phase system constituted by a balanced and sinusoidal source supplying a 12 pulses diode rectifier.

From this power system, the current two orthogonal components are calculated according to the definitions presented in section 4.

Fig. 2 shows the voltage supplied by the source. A balanced a sinusoidal voltage system.

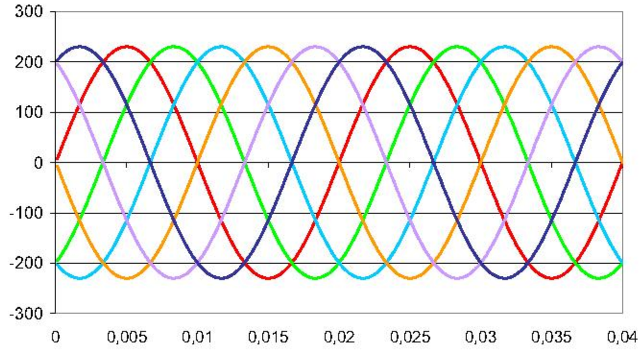

Fig. 2 Six-phase voltage waveform applied to the power system

Fig. 3 shows the six-phase waveform corresponding to the current required by the load. Fig. 4 details the load current phase 1. It is a strongly non linear load.

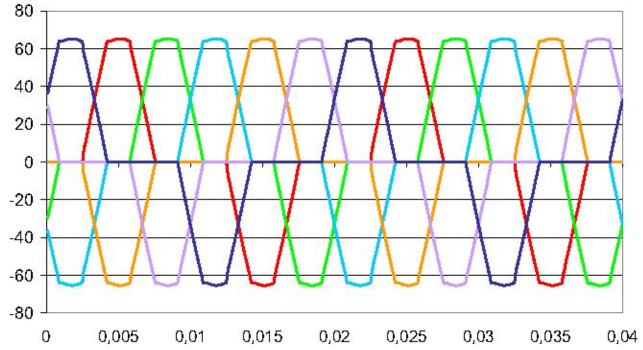

Fig. 3 Six-phase current consumed by the load

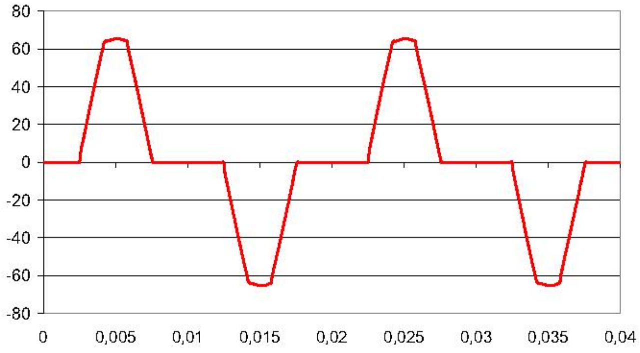

Fig. 4 Load current phase 1

Fig. 5 presents the phase 1 of the instantaneous active current. The corresponding six-phase waveform is balanced.

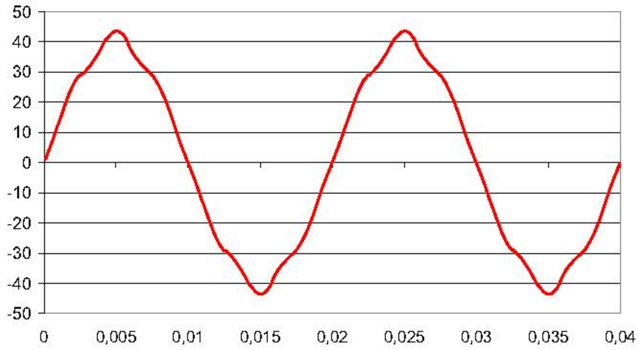

Fig. 5 Six-phase instantaneous active current

It carries the whole instantaneous real power as can be seen in Fig. 6 where the dot product $\vec{u}^T \cdot \vec{i}$ versus the dyadic product $\vec{u} \otimes \vec{i}$ is shown. These two graphs are superimposed. So, effectively, the instantaneous active current carries the whole instantaneous real power.

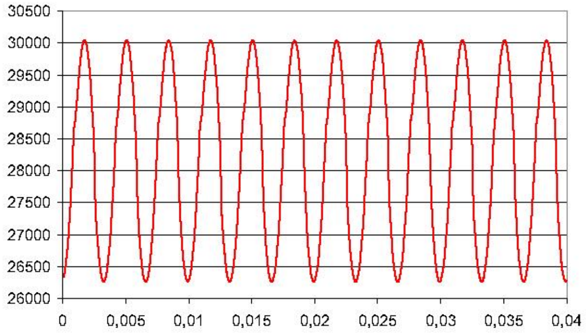

Fig. 6 Instantaneous active power

Fig. 7 presents the phase 1 of the instantaneous reactive current. The corresponding six-phase waveform is balanced.

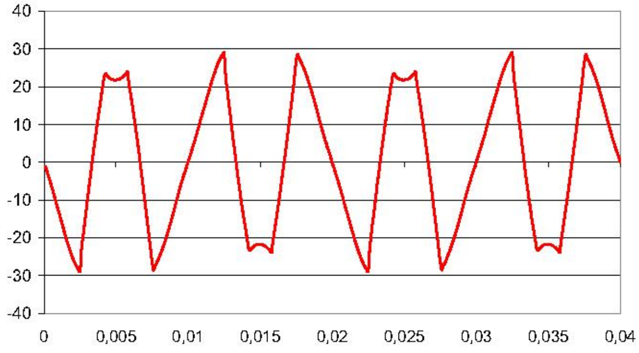

Fig. 7 Instantaneous reactive current phase 1

The instantaneous reactive current carries the whole instantaneous imaginary power. To corroborate it, Fig. 8 presents two graphs. One of them has been calculated as follows:

$$q = \sqrt{s^2(t) - p^2(t)} \qquad (33)$$

where 's' is the instantaneous apparent power calculated according to the next expression:

$$s(t) = \left\|\vec{u}(t)\right\| \left\|\vec{i}(t)\right\| \qquad (34)$$

The another graph has been calculated as the norm of the n order hemisymetrical tensor defined in (8).

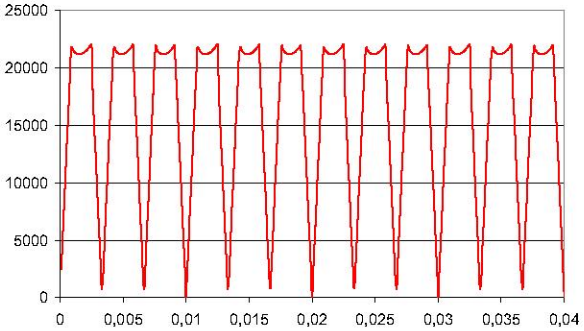

Fig. 8 Instantaneous imaginary power

Fig. 8 two graphs are superimposed, too. So, the instantaneous reactive current carries the whole instantaneous imaginary power.

VII. CONCLUSIONS

In this paper, a instantaneous reactive power theory formulation has been presented. This new formulation proposes a way of calculating the instantaneous imaginary power which can be applied not only to three-phase power systems, but to n-phase power systems. Besides, the equations representative of the p-q formulation are obtained from this new one. The paper presents simulation results, too, where all the calculations have been confirmed.

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