The EHTS and the persistence in the spread reconsidered. A fractional cointegration approach.

Abstract

In this paper, we consider the possibility that a fractionally cointegrated vector autoregressive (FCVAR) model could serve as a novel empirical tool for examining the US term structure of interest rates. This econometric approach allows one to test the existence of a long-run relationship between short- and long-term interest rates and spread persistence together. As one of the main contributions of this paper, we elaborate on new scenarios of the degree of noncontemplative EHTS fulfillment. Monthly time series are constructed with nine different maturities of interest rates for the period of 1993 to 2018. The results obtained contribute new scenarios not previously presented in the literature. We also find that the persistence of spread is the stronger the larger the difference in maturity is between considered interest rates, revealing a long memory process. Importantly, as the main implication of our study, we find that this persistence implies a gradual loss of controlling power over interest rates by FED.

1. Introduction

Since the financial crisis suffered in the USA at the end of the 2000s, the country's economic growth has been increasing steadily. Moreover, while the Fed has established an interest rate of 2.25% (December 2018) to inject more liquidity into the economy and to stimulate prices, the dollar has suffered downward pressure. Nonetheless, a gradual incremental increase in interest rates would involve the announcement of monetary normalization and would therefore denote a positive signal emerging from an extremely unique situation. In this case, rates would increase through an upward effect on short-term rates, which would gradually spread to long-term interest rates. If this rise in interest rates corresponds to inflationary pressure, negative impacts should be mitigated.

According to this framework, in a competitive financial environment, the term structure should move in assembly with predictions of the Expectation Hypothesis of term structure[1] (EHTS hereafter) such that returns respond to international market forces, as the term structure of interest rates has always been viewed as crucial to assessing the impact of monetary policy and its transmission mechanisms. Indeed, when a monetary policy is effective, changes in short-term policy interest rates should impact long-term ones (Holmes *et al.* (2015)). However, the EHTS can be variated due to changes in the economy, i.e., changes in monetary policy or the financial market. Thus, potential unsteadiness in the relationship between short- and long-term interest rates could produce confusing results (Esteve *et al.* (2013)).

The literature of this topic has attempted to demonstrate the EHTS based on different contexts and methodologies, and the data reveal contradictory results for the USA. Thus, the EHTS is accepted as a forecasting tool (Poole et al. (2002)) for its economic implications for monetary policy (Weber and Wolters (2012, 2013). However, there is evidence of cases for which this hypothesis does not hold; for example, when applied to G7 countries, the EHTS is supported for all countries except for the USA (Hardouvelis, 1994). Finally, it is well known that linear cointegration provides less power and fails to detect a long-run relationship between short- and long-term interest rates (Araç and Yalta, 2015).

The EHTS is often tested using cointegration techniques or by imposing stationary conditions on the spread (Li and Davis, 2017). However, the fractionally cointegrated VAR (FCVAR hereafter) allows one to sidestep some limitations of standard cointegration and of the stationarity of the spread, i.e., it is integrated at order zero. The FCVAR model has been applied in reference to financial markets and political economics. Rossi and Santucci de Magistris (2013) used this methodology to study the relationship between spot and futures markets, and Caporin *et al.* (2013) applied the FCVAR model on high and low prices to predict stock prices. Jones *et al.* (2014) examined the fractional relationship between Canadian political support and macroeconomic variables. Dolatabadi *et al.* (2015, 2016) applied the FCVAR model for the analysis of price discovery in commodity spot and futures markets and, more recently, Gil-Alana and Carcel (2018) did the same for exchange rates. This methodology is useful in that it allows us to test cointegration between interest rates of different maturities and spread

[1] In this paper, we use the term EHTS as Expectation Hypothesis of term structure, following the common use in the literature.

stationarity simultaneously, unlike what is possible with the traditional cointegration method; with the traditional method, different studies have executed this exercise separately; they study cointegration relationships between interest rates or the stationarity of the spread (see Vides *et al.* (2018) for a survey).

Thus, regarding the Expectations Hypothesis of Term Structure, this is the first study in which the FCVAR model is applied, providing significant results for practitioners and policy makers. This paper is novel in that it recognizes that premises of standard cointegration testing (*I(1)/I(0)* dichotomy) time series variables integrated at order one and comoved at order zero are too restrictive, i.e., linear combinations of *I(1)* nonstationary processes are *I(0)* stationary. In this sense, the empirical literature has shown that many economic and financial time series hold long-range dependence in the autocorrelation function but do not precisely exhibit a unit root process, i.e., the long memory process[2]. For this reason and according to our research, we reject traditional cointegration assumptions that all interest rates cannot move away from one another for long periods of time and that they are unit roots or *I(1)*; they follow dichotomy *I(0)/I(1)*, such that they follow a fractional process *I(d)*. We also discard the notion that the error term follows a stationary process (*I(0)*) in cases of the cointegration of both variables. In turn, the rigidity of the traditional approach is overcome in favor of allowing the series to be cointegrated, and the error term does not necessarily need to be *I(0)*; for example, we allow the error term to be cointegrated in order *I(d – b),* unlike other techniques which assume that the error term is *I(0)* such as Hansen and Seo's Threshold cointegration or Gregory-Hansen test, the Kejriwal-Perron test, the Hatemi-J test and the Arai-Kurozumi test for structural breaks. To the best of our knowledge, the EHTS based on investors exploiting arbitrage opportunities under empirical approaches usually assumes that the relation between short- and long-term interest rates follows a *I(0)/I(1)* process; however, fractional cointegration may contradict this assumption such that, in the presence of a unitary long-run relationship between interest rates with different maturities, shocks that affect this cointegration relationship can be long-lived and even non-stationary. In turn, this methodology allows one to establish different scenarios of EHTS fulfillment. Therefore, our new approach uses the FCVAR model developed by Johansen and Nielsen (2012) and Nielsen and Popiel (2016), which is an expansion of the traditional cointegrated VAR (CVAR hereafter) proposed by Johansen (1995), enabling us to establish the number of equilibrium relations via cointegrating rank testing to estimate memory parameters, long-run cointegrating relations with adjustment parameters, and short-run lagged dynamics. Finally, our primary results suggest that the EHTS is accepted, but it is accepted to different degrees depending on the maturity of interest rates based on different scenarios when maturity values are greater or less than 5 years. These results highlight the need to relax the rigid assumptions of traditional cointegration (CVAR), as they have led us to noncontemplative scenarios.

The rest of the paper is organized as follows. The following section presents a brief review of the literature, and section 3 describes the methodology used for the work. Section 4 discusses our empirical results and conclusions, from which we identify the economic policy implications given in section 5.

---

[2] Baillie and Bollerslev (1994a), *Baillie (1996),* Dueker and Startz (1998) and Mohanty *et al.* (1998) among others noted that fractional processes may better describe the long memory of macroeconomic variables.

2. A brief literature review

The term structure has long been established as reflecting economic agents' anticipations of future events and as an indicator of monetary policy in academic articles on term structures written over the past century, which serves as testament to the practical importance of this topic and to its intrinsic academic appeal (see Vetzal (1994) for a survey).

The EHTS shows the relationship between short- and long-term interest rates and is the most influential theory explaining term structure relations. The hypothesis states that long-term interest rates are determined by an average of the current and expected short-term interest rate. Thus, this relationship between short- and long-term interest rates implies that their spread contains significant information on future changes in short-term rates and plays an important role in the potential effectiveness of monetary policy, which involves the control of short-term policy rates by central banks and economy affected by monetary impulses through long-term interest rate movements (Bernanke and Blinder, 1992).

As it has been mentioned, the EHTS has economic implications for macroeconomics and finance and for the shape of the yield curve (see Shiller (1990) for a survey). According to the EHTS, an upward sloping yield curve implies that future short-term rates are expected to rise. Conversely, under a downward sloping yield curve, future short-term rates are expected to fall; for example, the slope of the yield curve serves as an important source of information on real economy evolution. As a consequence, Estrella and Hardouvelis (1991) found that a positive curve slope is associated with future increases in real economic activity using macroeconomic variables and providing significant predictive power. One implication of the EHTS as noted by Fama (1984) and Fama and Bliss (1987) relates to the fact that the forward rate is an unbiased predictor of future spot rates. As another implication of this hypothesis, the spread between the long-term interest rate and short-term interest rate –the term spread– is an unbiased predictor of future short-run changes in long-term rates (Mankiw, 1986; Campbell and Shiller, 1991; Campbell 1995).

In focusing in the most important economy in the world, i.e., the USA, empirical research on the EHTS has been long been conducted, and two strands of inquiry provide support or arguments on this issue. Several studies find evidence in support of the EHTS, e.g., Campbell and Shiller (1987) found partial support for the present value of the term structure of interest rates by examining the statistical significance of the EHTS. Additionally, changes in regimes of short- and long-term rates of US treasury bills support the EHTS (Hamilton, 1988), cointegrating US treasury bills among interest rates (Hall *et al.*, 1992). Further, Poole and Rasche (2000) and Poole *et al.* (2002) demonstrated that the market is able to anticipate changes in the FED's target federal fund rate. There is also evidence in support of the EHTS on the relationship between short- and long-term rates for Europe and the USA (Lanne (2003); Brüggemann and Lütkepohl (2005)), thus combining yield factors and macroeconomic variables to relate to the EHTS, serving as evidence in favor of certain regimes (Diebold *et al.*, 2006). Weber and Wolters (2012; 2013) applied the vector error correction (VEC) for the US term structure to offer an economic explanation for deviations from the EHTS. More recently, Holmes *et al.* (2015) examined the term structure of interest rates using a pairwise stationary approach and showed that the EHTS holds over the long-term, i.e., short-run policy changes affect long-term rates.

However, others present arguments against the EHTS for the USA such as Hardouvelis (1994), who used a VAR model designed to forecast changes in long-term interest rates using the term spread of G7 countries and who showed that the EHTS is supported in all countries except for the USA. Nevertheless, Thornton (2005) tested the EHTS for federal funds rates to find that the market's expectations are less able to forecast federal funds rates. Conversely, Guidolin and Thornton (2010) concluded that future short-term rates have deep implications for policy makers, suggesting that whether or not the EHTS is true, an inability to predict the future short-term rate would imply that both long- and short-term rates are equal and that this relation is inconsistent. In turn, the conventional theory of the term structure of interest rates is threatened. Finally, Bulkley *et al.* (2011; 2015) identified the failure of the EHTS based on bond yields of US Treasury securities. Despite initial controversy, this literature has shown that it is possible to establish a relationship between short- and long-term rates.

From an empirical point of view, the issue has best been explained through the concept of cointegration provided by Engle and Granger (1987). Several studies have adopted a linear adjustment process, i.e., interest rates maintain a long-term equilibrium relationship such that the interest spread does not tend to increase or decrease over time, reverting to its mean (see Engle and Granger (1987); Campbell and Shiller (1987); and Shea (1992), for instance). Engsted and Targgaard (1994) studied the EHTS for the US term structure and concluded that the EHTS cannot be rejected using a VECM. Hansen and Seo (2002) also used a threshold VECM to demonstrate that their results are roughly consistent with the term structure selected, and Seo (2003) utilized a trivariate threshold VECM and found evidence in support of nonlinear mean-reversion in the term structure of interest rates. Meanwhile, Esteve *et al.* (2013) studied cointegration with multiple structural breaks for Spain, and evidence of the EHTS was not found. It is well known that linear cointegration provides less power and fails to detect a long-run relationship between short- and long-term interest rates (Araç and Yalta, 2015); thus, several authors, such as Clarida *et al.* (2006) and Mili *et al.* (2012), show nonlinearities in the relationship between interest rates. Finally, Lange (2018) shows that the Canadian term spread is mean-stationary and related to macroeconomics. From this premise, our objective is to test the EHTS with a nonlinear cointegrating approach and to demonstrate robustness with different tests.

In focusing on the spread there are some arguments in its treatment in the sense that there are two main ways to check it. On one half, Baillie and Bollerslev (1994a, b), Tkacz (2001) and Cassola and Morana (2008) among others suggest that the spread could follow a fractional order of integration. On the other half, Strohsal and Weber (2014) and Holmes *et al.* (2015), for instance, show that the spread degree of integration could be different from *I(0)* but while supporting the EHTS. Although we have studied cointegration and spread from a fractional perspective separately, our approach allows us to analyze them together.

In other words, the analysis demonstrate a new approach that involves using the fractionally cointegrated vector autoregressive (FCVAR) model developed by Johansen and Nielsen (2012) and Nielsen and Popiel (2016), which is an expansion of the CVAR proposed by Johansen (1995) and which enables us to establish the number of equilibrium relations with a cointegrating rank test to estimate memory parameters, long-run cointegrating relations with adjustment parameters, and short-run lagged dynamics. This econometric approach allows for jointly testing the existence of a long-run relationship between short- and long-term interest rates and spread persistence.

2.1. Did Quantitative Easing affect the term structure of interest rates?

This subsection focuses on events that have recently occurred in the USA and on measures applied by the Fed. At the start of the 2000s, the longest growth period in the history of the United States ended due to a fall in investment caused by the collapse of the dot-com bubble and the September 11 attacks. This situation was reversed with the implementation of painful fiscal adjustments and from the costs of wars in Afghanistan and Iraq (Kraay and Ventura, 2007). Afraid that the economy would slip back into recession, the Fed kept the federal fund rate extremely low, reaching a low of 1% by the middle of 2003. As the momentum and prices of joint expansion began to rise, the target of federal funds increased slowly in a series of movements to 5.25% in mid-2006 (Labonte and Makinen, 2008). Finally, by the end of 2007 the subprime mortgage market collapsed and quickly spread to the rest of the world. The US government responded with a fiscal stimulus package and unprecedented bank bailout; in spite of the NBER declaring a recession for more than a year after the end date (June 2009), the measures applied by the Fed according to the Federal Reserve of Sant Louis correspond to Quantitative Easing (QE, hereafter) programs, which focused primarily on the type and quantity of asset acquisition that would affect financial market conditions, inflation and ultimately economic activity (Williamson, 2017). This QE program was announced on November 25, 2008, and it involved three main phases (or QE) occurring between 2009 and 2014, as follows:

- The QE1 lasted from December 2008 to March 2010, and $175 billion in agency securities and $1.25 trillion in agency mortgage-backed securities (MBS) were purchased.

- The QE2 spanned November 2010 to June 2011, during which time $600 billion in long-maturity Treasury securities were acquired. From September 2011 to December 2012, this acquisition was applied to the so-called *Operation Twist*. The measure involved a swap of more than $600 billion consisting of the acquisition of Treasury securities with maturities of six to thirty years and the sales of Treasury securities with maturities of three years or less.

- Finally, the QE3 covered the period of September 2012 to October 2014 and was based on the purchase of MBS and long-term Treasury securities initially set at $40 billion per month for MBS and $45 billion per month for values of long-term Treasury securities.

Until September 2017 the Fed began to very gradually reduce the balance sheet to a more typical value. However, Wright (2011) has shown that these announcement effects were short lived, lasting only a few months.

Simultaneously the Fed faced the subprime mortgage crisis by reducing the federal funds target from 5.25% to a range of 0% to 0.25% in December 2008, which economists call the *zero lower bound*. From this monetary policy measure, economic expansion and the unemployment rate were valued at close to the Fed's estimate of full employment when it began raising rates in December 2015. The Fed has since continued to raise rates more slowly than it initially intended over a series of steps to incrementally tighten

monetary policy. The Fed raised rates once in 2016 and three times in 2017 by 0.25 percentage points each time. Fed has raised the federal funds rate three times this year to a range between 2% and 2.25%.
Table 1 summarizes the abovementioned events to provide clearer review of the discussed measures.

Table 1: Summary of Fed measures

Year	Event	Measure
Early 2000s	Dot-com bubble and September 11th attacks	A painful fiscal adjustment due to the cost of the Afghanistan and Iraq wars. Fed causes federal fund rates to reach a low of 1% by mid-2003
Mid-2006	Economic expansion	Fed increases federal funds to 5.25%
Mid-2007	Subprime mortgage crisis	Fiscal stimulus and bank bailout
Nov. 2008		Announcement of Quantitative Easing program
Dec. 2008		Fed establishes federal fund target to a range of 0 – 0.25% (zero lower bound)
Dec. 2008	Quantitative Easing 1	Purchase of $175 billion in agency securities and of $1.25 trillion in agency MBS
Nov. 2010	Quantitative Easing 2	Acquisition of $600 billion in long-maturity Treasury securities
Sept. 2012	Quantitative Easing 3	purchase of $40 billion per month for MBS.
Dec. 2015	Economic expansion and unemployment rate close to the Fed's estimate	Fed raises rates once in 2016 and three times in 2017 by 0.25 percentage points each time. At present, federal fund rates range from 2 – 2.25%.

3. Methodology

Our econometric strategy involves obtaining and analyzing the model estimation at a monthly frequency, and we then perform statistical tests of cointegration, exclusion and weak exogeneity based on the fundamental equation for the EHTS in an econometric context.

The EHTS model

The fundamental equation of the EHTS of an $n > 1$ period bond R_t (i.e., long-term interest rate) is equal to the average of the current and expected r_t (i.e., short-term interest rates) set of $n \leq 1$ periods plus a constant term. The relationship can be expressed in the following form:

$$R_t = \frac{1}{n}\sum_{k=0}^{n-1} E_t[r_{t+k}] + \Phi_t^*, \qquad (1)$$

where Φ_t^* is a potential stationary term, and E_t is the expectations operator at time t for the evolution of short-term interest rates driving the longer-term interest rate. To test the EHTS in the context of cointegration theory, the common linear mode used is:

$$R_t = c + \beta r_t + \varepsilon_t \qquad (2)$$

In agreement with Campbell and Shiller (1987), R_t and r_t should be non-stationary and related through a cointegration relationship with parameters (1,-1). This implies that β_R and β_r are cointegrated constants and that their combination involves a stationary process while the spread of the interest rate reverts to the mean. When the spread is stationary, the long- and short-term rates are driven by a common stochastic trend and do not allow for arbitrage opportunities because market forces adjust to correct any temporary disequilibrium. As the EHTS suggests, the interest rate spread is an optimal forecast[3] of future changes in long-term interest rates. Thus, the market's expectations of the short-rate developments in the bond yield are reflected in the slope of the term structure with a one-to-one relation, $\beta = 1$.[4]

According to the existing literature, cointegration implies a VECM such as:

$$\begin{pmatrix} \Delta R_t \\ \Delta r_t \end{pmatrix} = \begin{pmatrix} \alpha_R \\ \alpha_r \end{pmatrix} (R_{t-1} - \beta r_{t-1} - c) + \sum_{i=1}^{n} \Gamma_i \begin{pmatrix} \Delta R_{t-i} \\ \Delta r_{t-i} \end{pmatrix} + \begin{pmatrix} w_{1t} \\ w_{2t} \end{pmatrix} \quad (3)$$

With adjustment parameters α, cointegration coefficient β, restricted constant (c), lag length (n) and errors w. Γ_i are 2 x 2 parameter matrices in short-run dynamics. Adjustment coefficients α_R and α_r capture the speed of R_t and r_t adjustment towards equilibrium. Furthermore, according to the EHTS, the absolute values of estimates of α_R are much smaller than α_r, and we suggest that the correction included in the equation for short-term interest rates overshoots the long-run equilibrium, i.e., the spread defined by the difference between long- and short-term interest rates.

For this work the FCVAR model allows us to study the common long-run equilibrium relationship between long- and short-term interest rates. The model is a generalization of Johansen's (1995) cointegrated vector autoregressive (CVAR) model to allow for fractional processes of order d that cointegrate to order $d – b$. As we conduct our analysis using a bivariate fractional cointegration approach, we recognize that the standard unit root and cointegration test may be too restrictive (*I(1)/I(0)* dichotomy). The CVAR and FCVAR differ in part because for the CVAR, the error correction term (the spread when the EHTS is supported) is *I(0),* while for the FCVAR, this assessment is not restricted (the integration order may be different from zero, reflecting a long-memory process); thus, the assumption that the spread term of *I(0)* could reflect an *I*(d) process is rejected. A more generalized *I*(d)-type specification has been adopted based on the possibility of fractional orders of integration. Cointegration without these values is unrestricted.

Accordingly, the FCVAR model allows us to identify several degrees or scenarios of EHTS fulfillment. Once the test shows that there is cointegration, the degree of integration in the spread allows us to identify up to three different scenarios. This idea is illustrated more fully in the following subsection and is illustrated in table 3, in which new possibilities allowed through the application of the FCVAR as a generalization of traditional approaches are broken down.

Table 2. New possibilities arising when applying the FCVAR

[3] Baillie and Bollerslev (1994a) discovered that cointegrating relationships may not be precisely valued at *I*(0), implying that a fractionally cointegrated relationship may yield noticeable gains in forecast accuracy only within the context of a longer-term forecast.

[4] When $\beta = 1$ we assume that the difference between short- and long-term interest rates is the term spread.

		Cointegration/Long-run relationship	
		Yes	*No*
Spread	*Stationary*	**Several degrees of the EHTS (see table 3)**	Controversy
	Nonstationary		No EHTS

Fractional cointegration model – FCVAR methodology

This model is presented in Johansen (2008a, 2008b) and is further developed in Johansen and Nielsen (2012) and Nielsen and Popiel (2016). It offers the advantage of being applicable to stationary and non-stationary time series. Our objective is to study the EHTS under fractional cointegration conditions.

In introducing the FCVAR model, we begin with the well-known non-fractional CVAR model with $Y_t = 1, \dots, T$ as a p-dimensional *I(1)* time series. Thus, the CVAR model is:

$$\Delta Y_t = \alpha\beta' Y_{t-1} + \sum_{i=1}^{k} \Gamma_i \, \Delta Y_{t-i} + \varepsilon t = \alpha\beta' L Y_t + \sum_{i=1}^{k} \Gamma_i \, \Delta L^i Y_t + \varepsilon t \qquad (4)$$

The fractional difference operator is Δ, and the fractional lag operator is $\Delta = (1 - \mathrm{L})$. We replace lag operators with fractional counterparts Δ^b and $\Delta^b = (1 - \mathrm{L}_b)$ and apply $Y_t = \Delta^{d-b} X_t$ such that:

$$\Delta^d X_t = \alpha\beta' L_b \Delta^{d-b} X_t + \sum_{i=1}^{k} \Gamma_i \, \Delta^d L_b^i X_t + \varepsilon t. \qquad (5)$$

As always, εt is p-dimensional independent and identically distributed with a mean of zero and covariance matrix Ω. Parameters α and β are $p \, x \, r$ matrices where $0 \leq r \leq p, d \geq b > 0$. In matrix β, the columns denote cointegrating relationships, and $\beta' X_t$ assumes the existence of a common stochastic trend integrated at order *d*. Short-term parts from the long-run equilibrium are integrated at order *d - b*, but when *d - b* < *1/2* asymptotically, a zero-mean stationary process occurs. The coefficients of α correspond the rate of adjustment to equilibrium. Therefore, $\alpha\beta'$ is the adjustment long-run, ρ' is the restricted constant term and Γ_i represents the short-run behavior of the variables. We in turn develop the final model:

$$\Delta^d X_t = L_d \alpha(\beta' X_t + \rho') + \sum_{i=1}^{k} \Gamma_i \, \Delta^d L_d^i X_t + \varepsilon t. \qquad (6)$$

When the VAR model is applied with *d* = *b* = 1 (CVAR), X_t is integrated at order *d*, and *b* denotes the strength of cointegrating relationships (as the value of *b* increases persistence declines in cointegrating relationships). The error correction term is integrated from order (*d* – *b*), which is *I(0)* in the case of standard cointegration (*d*=*b*=1). However, in fractional cointegration these axioms are relaxed because *(d – b)* = *0*, i.e., the error correction term exhibits short-run stationary behavior, or because *(d – b)* > *0*; i.e., there is a long memory process, and the error correction term will revert to its mean over the long run. As the cointegrating vector is (1, -*β*), we can interpret the difference (*d* – *b*) as the order of integration for the cointegrating error or as the degree of persistence (H_1^{d-b}). According to Table 1 given in Tkacz (2001), when (*d* – *b*) = 0, the cointegrating error follows a stationary process, and the shock duration is short-lived. When 0 < (*d* – *b*) <

0.5, a stationary process occurs, and the shock duration is long-lived. Finally, when 0.5 < (d – b) < 1, the cointegrating error involves a non-stationary process while mean-reverting, and the shock durations are long. Thus, one of the main contributions of this paper lies in its elaboration of new conditions of the degree of EHTS fulfillment while establishing β as a "different condition of relationship strength" and while focusing on the persistence of the error term. These conditions are synthesized in table 3.

Table 3. EHTS scenarios

Order of integration of the error correction term (ECT)	Value of *β (assuming cointegration)*	
	β = 1	**0 < β < 1**
I(d – b) = I(0)	Theoretical EHTS and the shock duration is **short-lived**.	Weak EHTS and the shock duration is **short-lived**.
I(0) < I(d – b) < I(0.5)	EHTS and the shock duration is **long-lived**.	Weak EHTS and the shock duration is **long-lived**.
I(0.5) < I(d – b) < I(1)	EHTS and the ECT follows a **non-stationary** process, although mean-reverting and shock durations are **long-lived**.	Weak EHTS and the ECT follows a **non-stationary** process, although mean-reverting and shock durations are **long-lived**.

Notes: The shaded area corresponds to the traditional *EHTS*.

Johansen and Nielsen (2012) show that maximum likelihood estimators $(d, \alpha, \Gamma_i, \dots, \Gamma_k)$ are asymptotically normal and that the maximum likelihood estimator of (β, ρ) is asymptotically mixed normal.

To determine the number of stationary cointegrating relations following the hypotheses of the rank test based on a series of LR tests, in the FCVAR model, we test hypothesis $H_0\colon rank(\Pi) = r$ against alternative hypothesis $H_1\colon rank(\Pi) = p$. $L(d, b, r)$ is the profile likelihood function of rank r, where (α, β, Γ) is reduced by rank regression (see Johansen and Nielsen (2012)). For the model with a constant, we test $H_0\colon rank(\Pi, \mu) = r$ against the alternative $H_1\colon rank(\Pi, \mu) = p$, and the profile likelihood function given rank r is $L(d, r)$, where parameters $(\alpha, \beta, \rho, \Gamma)$ are focused out.

To maximize the profile likelihood distribution under both hypotheses, the LR test statistics are now $LR_t(q)$. The asymptotic distribution of $LR_t(q)$ depends on parameter b and on $q = n - r$. MacKinnon and Nielsen (2014) based their numerical distribution functions on the asymptotic critical values of an LR rank test. In cases of "weak cointegration", i.e., 0 < b < 1/2, $LR_t(q)$ has a standard asymptotic distribution, LRt(q) $\overset{D}{\rightarrow} \chi^2(q^2)$.

3. Empirical analysis

The purpose of the present study is to test the existence of EHTS values with 3 to 240 months of maturity. The first step here involves testing the existence of a common trend, i.e., a long-run relationship between short- and long-term rates. For this cointegration analysis, we determine if fractional cointegration is more appropriate than standard cointegration. As a second step, we study the potential relation one to one, i.e., the cointegrating vector (1,-1). The next step involves analyzing the adjustment coefficients, and finally we examine the fractional cointegration degree (persistence).

Table 4. Empirical Research Method

	Procedure	**Hypotheses**
Step 1	Fractional Cointegration?	H_1^d: Is the fractional cointegration more appropriate than traditional cointegration?

Step 2	Estimation of β	$\boldsymbol{H_1^{\beta}}$: The cointegrating vector is (1, -1)
Step 3	Estimation of adjustment coefficients (α_R, α_r) (FVECM)	$\boldsymbol{H_1^{\beta}} \cap \boldsymbol{H_1^{\alpha_i}}$: Variables are weakly exogenous under the restrictions of the cointegrating vector (1, -1)
Step 4	Degree of spread persistence, i.e., order of integration ($d - b$)	$\boldsymbol{H_1^{d-b}}$: Is the spread a long memory process?

Data description

For our empirical analysis, we employ a monthly sample of Treasury Constant interest rates of 9 different maturities for the period of October 1993 to December 2018 (amounting 303 observations for each interest rate series). The data correspond to 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year and 20-year constant maturity rates. The data are gathered from Federal Reserve Economic Data (FRED) assembled by the Economic Research Division of the Federal Reserve Bank of St. Louis. As 1-month Treasury Constant maturity rate data are only available from January 2001, we use these maturities to determine the availability of consistent interest rate data for the period studied. Interest rates are measured as percentages and are shown in figure 1. We consider 3-month, 6-month and 1-year periods as short-run periods. A period of 1 year is defined as short-term to render our estimation more robust.[5] On the other hand, we define the rest of the maturity rates as long-run. Table 5 shows descriptive statistics associated with each interest rate for different maturities. In terms of volatility, the variables exhibit similar behavior and figure 1 presents a graphical analysis of time series dynamics traced for all maturities.

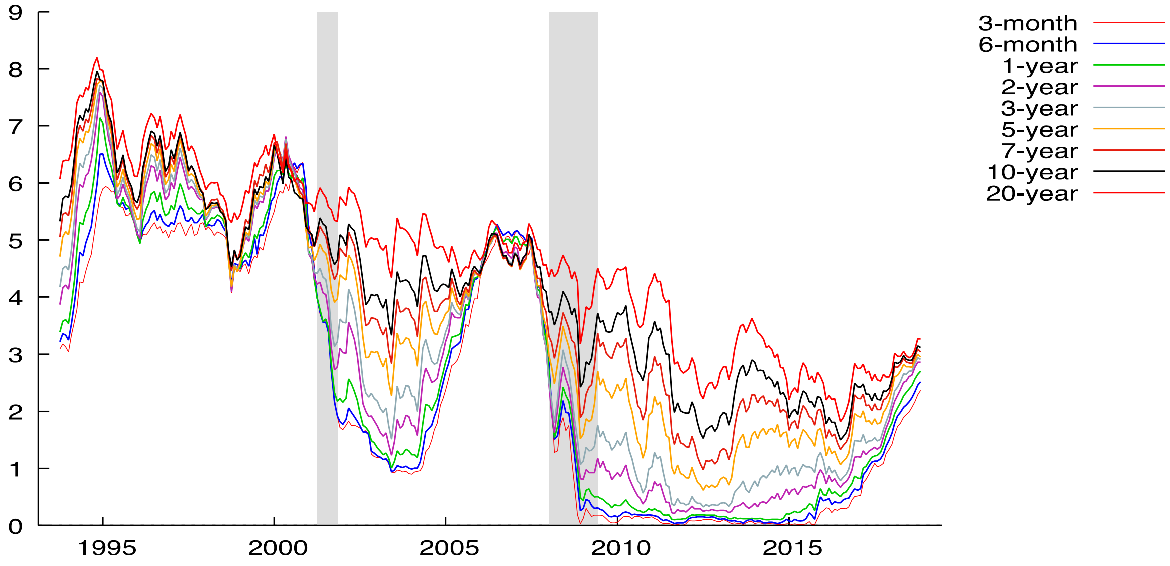
Table 5. Descriptive statistics for the data

	3-month	6-month	1-year	2-year	3-year	5-year	7-year	10-year	20-year
Mean	2.459	2.597	2.729	3.005	3.217	3.607	3.922	4.168	4.696
Median	1.760	1.970	2.220	2.610	2.870	3.360	3.790	4.150	4.810
Min	0.010	0.040	0.100	0.210	0.330	0.620	0.980	1.500	1.820
Max	6.360	6.510	7.140	7.590	7.710	7.780	7.830	7.960	8.200
S.D.	2.180	2.221	2.217	2.193	2.109	1.918	1.783	1.566	1.566

Notes: For 10/1993 to 12/2018

Figure 1. Time series traced for employed variables

[5] We also estimate pairs of short-run maturities, i.e., 3-month – 6-month, 3-month – 1-year and 6-month – 1-year.



4.1 Univariate analysis

As a preliminary step we estimate the order of the fractional integration of the interest rates. There are several means of estimating the fractional differencing parameter in semiparametric contexts. Although the semiparametric log-periodogram regression method proposed by Geweke and Porter-Hudak (1983) is the most widely used, the method was modified and further developed by Robinson (1995) and has been analyzed by Velasco (1999) and Phillips and Shimotsu (2002) among others. To develop a fractionally cointegrated model, we first discuss long memory univariate results; then, we proceed to the estimation of fractional parameter *d* for each univariate series with results presented in table 6. The three columns show semiparametric log-periodogram regression estimates drawn from Geweke and Porter-Hudak (1983) computed with bandwidths of [6] $m = T^n$, which is equivalent to Fourier frequencies, where m is the integer, *T* is the number of observations and *n* is the bandwidth size. The estimates are consistent with joint estimates presented below. As is shown in table 6, the values for *d* decrease as maturities increase, becoming mean-reverting values.

Table 6. Univariate analysis. GPH estimates

	3-month	6-month	1-year	2-year	3-year	5-year	7-year	10-year	20-year
$m = [T^{0.4}] = 9$	1.216 (0.298)	1.266 (0.316)	1.313 (0.306)	1.335 (0.319)	1.243 (0.247)	1.018 (0.187)	0.871 (0.233)	0.678 (0.389)	0.658 (0.213)
$m = [T^{0.5}] = 17$	1.491 (0.183)	1.480 (0.189)	1.377 (0.163)	1.247 (0.168)	1.191 (0.162)	1.069 (0.163)	0.936 (0.157)	0.775 (0.208)	0.776 (0.157)
$m = [T^{0.6}] = 30$	1.467 (0.107)	1.388 (0.107)	1.309 (0.099)	1.151 (0.101)	1.069 (0.098)	0.968 (0.098)	0.889 (0.093)	0.794 (0.118)	0.838 (0.106)

Notes: GPH denotes the Geweke and Porter-Hudak semiparametric log-periodogram regression estimator. Standard errors are given in parenthesis beneath estimates of *d*. The sample size is 303.

4.2 Cointegration analysis

[6] To test the presence of unit roots, estimates were obtained from first-differenced data, as the original series may exceed 0.5, and the test requires that results are limited to an interval of $-0.5 < d < 0.5$ while adding 1 to obtain the proper estimates of *d*. According to the literature, the bandwidth size ranges from 0.25 to 0.8. For our study, the three bandwidths selected are valued at 0.4, 0.5 and 0.6.

In this subsection, we present the procedure used to derive the results shown below. First, to select the FCVAR lag length, we use AIC criteria so that the lag length selected is different for each pair of variables studied. As can be observed when the short-term interest rate referenced is for a 3-month period, the optimal lag length is six in all cases (table A1a of the appendix). When 6 months is the reference short-term period, the result is diverse and for 1 year the result ranges between four and five (tables A1b and A1c of the appendix).

We then determine that there is a long-run relationship between each pair of maturities selected, and we test the cointegration rank before testing the hypothesis for the fractional parameter. We in turn find that the number of cointegrating vectors is one in almost most of cases (table A2 of the appendix). Once a rank cointegration test is conducted, we test hypothesis H_1^d, which tests whether fractional cointegration is more appropriate than traditional cointegration (the CVAR model), i.e., the null hypothesis is $d = 1$, and its rejection implies that the FCVAR model is more suitable than traditional cointegration. Accordingly, by assuming *I(1)* cointegration or an *I(0)* VAR model, we may be misspecifying the model estimates, parameters, test restrictions and implied dynamics, such as the term spread. In contrast, whether there is a stochastic trend of an order lower than unity, economic shocks have temporary mean-reverting effects on the relevant variables. This allows for more flexibility when theoretical and macrodynamic principles are applied, and the stochastic long-run term structure trend can in turn be established by shocks with transitory effects on interest rates (Abbritti *et al.*, 2018). In this sense, Table 7 shows the results of an LR test and shows that the CVAR is rejected in favor of the FCVAR, i.e., fractional cointegration is appropriate for this study.

Table 7. H_1^d: LR test, CVAR vs. FCVAR

	Maturities							
3-month vs.	6-month	1-year	2-year	3-year	5-year	7-year	10-year	20-year
H_1^d	10.220 (0.001)	12.808 (0.000)	17.197 (0.000)	28.038 (0.000)	23.276 (0.000)	22.781 (0.000)	20.125 (0.000)	20.387 (0.000)
6-month vs.		1-year	2-year	3-year	5-year	7-year	10-year	20-year
H_1^d		25.650 (0.000)	29.549 (0.000)	25.468 (0.000)	20.811 (0.000)	15.407 (0.000)	19.705 (0.000)	16.308 (0.000)
1-year vs.			2-year	3-year	5-year	7-year	10-year	20-year
H_1^d			16.667 (0.000)	15.051 (0.000)	16.418 (0.000)	16.202 (0.000)	16.536 (0.000)	18.463 (0.000)

Notes: Following Jones *et al.* (2014), the significance level is set to 10% for exclusion. The sample size is 303. LR statistics and *P*-values are in parenthesis below LR test values.

We next estimate the long-run relationship between long- and short-term rates (see equation 2). The estimated values are shown in table 8. It can be observed that parameter β is close to 1.[7]

Table 8. Cointegrating vector (1, -β)

Maturities

[7] For every estimation, we check residuals for serial correlations using a multivariate Ljung-Box Q-test $Q_{\hat{\varepsilon}}$ with h = 12 lags because our data are monthly. The results show no evidence of serial correlations in the residuals for every estimation, and the Ljung-Box Q-test shows no signs of misspecification, indicating that the model is well specified (see table A3 of the appendix).

3-month vs.	6-month	1-year	2-year	3-year	5-year	7-year	10-year	20-year
	[1,-1.029]	[1,-1.048]	[1,-1.085]	[1,-1.089]	[1,-1.130]	[1,-1.122]	[1,-1.142]	[1,-1.309]
6-month vs.		1-year	2-year	3-year	5-year	7-year	10-year	20-year
		[1,-1.025]	[1,-0.981]	[1,-0.998]	[1,-1.143]	[1,-1.144]	[1,-1.156]	[1,-1.263]
1-year vs.			2-year	3-year	5-year	7-year	10-year	20-year
			[1,-1.094]	[1,-1.121]	[1,-1.162]	[1,-1.144]	[1,-1.151]	[1,-1.266]

Recalling that the EHTS implies that series are cointegrated while the cointegrating vector between each variable is constrained in (1,-1), H_1^{β}, we must test the existence of this vector. From the LR test illustrated in table 9, we do not reject this parameter restriction when a given maturity exceeds 5 years when using any short-term rate. However, when we select a maturity of 2 or 3 years, the parameter restriction is rejected.[8] The traditional prism of the EH establishes that there must be a linear combination between short- and long-term interest rates constrained by a (1,-1) vector. Nonetheless, from tables 2 and 3 presenting data on different degrees of maturity, it may be observed that for a given combination of interest rates, the EHTS illustrates a different vision from a traditional perspective. Then, the EH is referred to as "weak", as Esteve *et al.* (2013) shows. In our study, a "weak-EH" refers to combinations of interest rate pairs with values of $0 < \beta < 1$ based on new scenarios in the conception of this hypothesis. Our results may be interpreted in two ways. On one hand, maturities of over 5 years meet the abovementioned requirements such that the EHTS is strongly supported due to this level of interest rate maturity. On the other hand, for maturities of less than 5 years, i.e., 1, 2 and 3 years, following Esteve *et al.* (2013), such results support a weak version of the EHTS for interest rates. For this reason, with the FCVAR as shown in table 3, we establish different scenarios.

Table 9. H_1^{β}: LR test statistics of the hypothesis $\beta = 1$ (cointegrating vector is (1,-1))

	Maturities							
3-month vs.	6-month	1-year	2-year	3-year	5-year	7-year	10-year	20-year
LR test	8.615	8.506	7.300	3.881	1.319	0.746	0.502	1.123
P-value	(0.003)	(0.004)	(0.007)	(0.049)	(0.251)	(0.388)	(0.478)	(0.289)
6-month vs.		1-year	2-year	3-year	5-year	7-year	10-year	20-year
LR test		5.245	20.036	9.572	1.193	0.728	0.518	0.710
P-value		(0.022)	(0.000)	(0.002)	(0.275)	(0.393)	(0.472)	(0.399)
1-year vs.			2-year	3-year	5-year	7-year	10-year	20-year
LR test			5.185	2.721	1.357	0.769	0.498	0.762
P-value			(0.023)	(0.099)	(0.244)	(0.380)	(0.480)	(0.383)

Notes: Following Jones *et al.* (2014), the significance level is set to 10% for exclusion. The sample size is 303.

As our next step, according to the existing cointegration literature, we estimate an FVECM (see equation 9) testing the significance of adjustment coefficients of the joint hypothesis, $H_1^{\beta} \cap H_1^{\alpha_i}$, as shown in table 10[9], using an LR test, and we find that only those coefficients associated with short-term rates (α_r) are significant, implying that the spread offers predictive power on the behavior of future short-term rates consistent with the

[8] We also reject combinations of short-term rates with an LR test.

[9] We focus on maturities where the EHTS is supported according to results given in table 9.

EHTS. Finally, as expected, adjustment coefficients of the short-term rate are positive, which serves as extra evidence in support of the EHTS; conversely, adjustment coefficients of long-term rates are much lower in magnitude than those short-term rates, but the adjustment coefficients are insignificantly different from zero (this finding is based the results of Hansen and Seo (2002)).

Table 10. $H_1^{\beta} \cap H_1^{\alpha_i}$: Adjustment coefficients under cointegration vector (1,-1)

			Maturities					
3-month vs.	6-month	1-year	2-year	3-year	5-year	7-year	10-year	20-year
$H_1^{\beta} \cap H_1^{\alpha_R}$	---	---	---	---	0.006 (0.937)	0.032 (0.858)	0.267 (0.605)	0.709 (0.400)
$H_1^{\beta} \cap H_1^{\alpha_r}$	---	---	---	---	15.440 (0.001)	11.904 (0.001)	12.735 (0.000)	11.740 (0.001)
α_R	---	---	---	---	0.003	-0.005	-0.014	-0.250
α_r	---	---	---	---	0.124	0.076	0.072	0.083
6-month vs.		1-year	2-year	3-year	5-year	7-year	10-year	20-year
$H_1^{\beta} \cap H_1^{\alpha_R}$		---	---	---	0.097 (0.755)	0.020 (0.887)	0.132 (0.716)	0.738 (0.390)
$H_1^{\beta} \cap H_1^{\alpha_r}$		---	---	---	10.798 (0.001)	13.407 (0.000)	9.427 (0.002)	12.172 (0.000)
α_R		---	---	---	0.014	-0.015	-0.009	-0.118
α_r		---	---	---	0.101	0.277	0.056	0.401
1-year vs.			2-year	3-year	5-year	7-year	10-year	20-year
$H_1^{\beta} \cap H_1^{\alpha_R}$			---	---	0.018 (0.893)	0.065 (0.799)	0.306 (0.580)	0.777 (0.378)
$H_1^{\beta} \cap H_1^{\alpha_r}$			---	---	7.246 (0.006)	7.748 (0.005)	7.698 (0.006)	8.115 (0.004)
α_R			---	---	0.018	-0.024	-0.051	-0.109
α_r			---	---	0.279	0.202	0.201	0.313

Notes: Following Jones *et al.* (2014), the significance level is set to 10% for exclusion. The sample size is 303. *P*-values are in parenthesis below LR test values.

Finally, as the cointegrating vector is (1, -1), we can interpret the difference *(d – b)* as the order of integration of the spread or as the degree of persistence (H_1^{d-b}). As stated in the methodology section, when *(d – b) = 0,* the spread follows a stationary process, and the shock duration is short-lived. When *0 < (d – b) < 0.5*, there is a stationary process, and the shock duration is long-lived. Finally, when *0.5 < (d – b) < 1*, the spread follows a non-stationary but mean-reverting process, and the shock duration is long-lived. As we show in table 11, there are two sources of evidence of this difference. On one hand, the order of integration of the spread is noticeably above zero, reflecting a long-memory process. On the other hand, most maturities follow a stationary process; thus the duration of the shock is long-lived. Meanwhile, when the maturity of interest rates is 20 years, the process switches to a non-stationary but mean-reverting process, and the effect of the shock declines at a slower rate than the previous maturities. This result is in line with the results of Weber and Wolters (2012) and Holmes *et al.* (2015).

Table 11. H_1^{d-b}: Spread degree of persistence

Maturities

3-month vs	6-month	1-year	2-year	3-year	5-year	7-year	10-year	20-year
d					1.057	0.989	1.037	1.116
b					0.808	0.837	0.821	0.795
$\boldsymbol{H_1^{d-b}}$	---	---	---	---	*0.239*	*0.151*	*0.216*	*0.321*
6-month vs		1-year	2-year	3-year	5-year	7-year	10-year	20-year
d					1.058	1.132	1.022	1.231
b					0.813	0.640	0.824	0.577
$\boldsymbol{H_1^{d-b}}$		---	---	---	*0.250*	*0.492*	*0.198*	*0.654*
1-year vs.			2-year	3-year	5-year	7-year	10-year	20-year
d					1.128	1.091	1.108	1.197
b					0.646	0.655	0.633	0.584
$\boldsymbol{H_1^{d-b}}$			---	---	*0.482*	*0.436*	*0.475*	*0.613*

Note: Fractional order of integration of the explanatory variables and the errors cointegrating are denoted by d and b.

According to section 2.1, we attempt to answer the following question: Did the QE affect the term structure of interest rates? For this reason, we check if the QE program had any impact on the long-run relationship between each pair of maturities by applying our methodology, i.e., the FCVAR model. We apply November 2008, the date marking as the start of the QE program (as Holmes *et al.* (2015) did), as the breakpoint of our sample. We then apply two regimes: the first regime covers October 1993 to November 2008 and the second covers December 2018 to December 2018. We next apply the FCVAR and obtain the following results[10] (see tables 12a and 12b). On one hand, for the first regime, the results show steady behavior, where most interest rate maturity pairs analyzed are cointegrated in a (1, -1) vector, and the spread follows a stationary process. On the other hand, according to the Regime II estimations, this regime covers the aftermath of the global financial crisis and government efforts to allay the impact of this quarrelsome period. Consequently, the results obtained are very similar to those of the original sample, in which the majority of interest rate pairs apply to another scenario, thus establishing places where the cointegrating vector is not (1, -1), although the spread follows a stationary process. We also find that two pairs of maturities follow a nonstationary but mean-reverting process.

Table 12a. Summary of results Regime I

Order of integration of the error correction term (ECT)	**Value of *β* *(assuming Cointegration)***	
	***β* = 1**	**0 < *β* < 1**
I(*d* – *b*) = I(0)	[3M – 1Y]; [3M – 2Y]; [3M – 3Y] [3M – 5Y]; [3M – 7Y]; [3M – 10Y];[3M – 20Y] [6M – 1Y]; [6M – 3Y]; [6M – 5Y]; [6M – 7Y]; [6M – 10Y]; [6M – 20Y] [1Y – 2Y]; [1Y – 3Y]; [1Y – 20Y]	[3M – 6M]; [6M – 2Y];
I(0) < I(*d* – *b*) < I(0.5)	[1Y – 5Y]; [1Y – 7Y]; [1Y – 10Y]	
I(0.5) < I(*d* – *b*) < I(1)		

Notes: The shaded area corresponds to the traditional *EHTS*. The sample covers October 1993 to November 2008.

Table 12b. Summary of results Regime II

Value of *β* *(assuming Cointegration)*

[10] For reasons of space the results are available upon request.

Order of integration of the error correction term (ECT)	$\beta = 1$	$0 < \beta < 1$
I($d - b$) = I(0)	[1Y – 3Y]	[3M – 6M]; [3M – 1Y]; [3M – 2Y]; [3M – 3Y] [6M – 1Y]; [6M – 2Y]; [6M – 3Y] [1Y – 2Y];
I(0) < I($d - b$) < I(0.5)	[3M – 5Y]; [3M – 7Y]; [3M – 10Y];[3M – 20Y] [6M – 5Y]; [6M – 7Y]; [6M – 10Y] [1Y – 5Y]; [1Y – 7Y]; [1Y – 10Y]	
I(0.5) < I($d - b$) < I(1)	[6M – 20Y] [1Y – 20Y]	

Notes: The shaded area corresponds to the traditional *EHTS*. The sample covers December 2008 to December 2018.

Tables 13 and 14 provide a summary of results, in which whether hypotheses are accepted or rejected is indicated with a tick or cross symbol, respectively, in the results column. In this table, we also present data that are valuable to highlight. From table 14, we observe that each pair of interest rates (short- vs. long-term interest rates) is disposed under a different scenario. As stated above, the results obtained for the second regime are similar to these. At first glance, the results reveal that, while none of the interest rate pairs checked occupy the theoretical EHTS zone, they spread to a weak EHTS or to situations in which the spread follows a long-memory process. If we were to execute this exercise with a traditional cointegration[11] approach, the results would be different because they would occupy the theoretical EHTS zone. Thus, the FCVAR model allows us to avoid rigidity in the stationarity of the spread, showing that we adhere to scenarios not appreciated in traditional cointegration.

Table 13. Summary of results I

	Hypotheses	Results	Observations
Step 1	$\boldsymbol{H_1^d}$: Is fractional cointegration more appropriate than traditional cointegration?	✔	
Step 2	$\boldsymbol{H_1^\beta}$: Cointegrating vector is (1, -1)	✔	Only for 5-year or more interest rates
Step 3	$\boldsymbol{H_1^\beta} \cap \boldsymbol{H_1^\alpha}$: Variables are weakly exogenous under restrictions of the cointegrating vector (1, -1)	✔	Only for short-term interest rates (α_r)
Step 4	$\boldsymbol{H_1^{d-b}}$: Does the spread involve a long memory process?	✔	- Stationary (5, 7, 10-year) - Non-stationary but mean-reverting (20-year)

Table 14. Summary of results II

	Value of β *(assuming Cointegration)*

[11] A summary of estimations of the CVAR is included in the Appendix as Table A4.

Order of integration for the error correction term (ECT)	$\beta = 1$	$0 < \beta < 1$
I(*d* – *b*) = I(0)		[3M – 6M]; [3M – 1Y]; [3M – 2Y]; [3M – 3Y] [6M – 1Y]; [6M – 2Y]; [6M – 3Y] [1Y – 2Y]; [1Y – 3Y]
I(0) < I(*d* – *b*) < I(0.5)	[3M – 5Y]; [3M – 7Y]; [3M – 10Y];[3M – 20Y] [6M – 5Y]; [6M – 7Y]; [6M – 10Y] [1Y – 5Y]; [1Y – 7Y]; [1Y – 10Y]	
I(0.5) < I(*d* – *b*) < I(1)	[6M – 20Y] [1Y – 20Y]	

Notes: The shaded area corresponds to the traditional *EHTS.* The sample size is 303 and the sample covers October 1993 to December 2018.

4. Conclusions

With the EHTS, long-term rates can explain changes in future short-term rates. Understanding the term structure of interest rates has always been viewed as crucial to determining the impact of monetary policies and their transmission mechanisms; it also plays an important role in macroeconomic predictions and in portfolio analysis (Li and Davis, 2017). Indeed, when monetary policy is effective, changes in short-term policy interest rates should impact long-term ones. Based on a fractionally cointegrated VAR model, our analysis considers both cointegration between short- and long-term interest rates and the long memory of their linear combination, i.e., the spread. We describe the spread as the difference between long- and short-term rates. The proposed methodology affords us the opportunity to reject the apriorism of incompatibility, whereby interest rates are cointegrated and the term spread is rendered non-stationary. We also manage to alter the EHTS by extending opportunities raised in the literature.

We use US monthly interest rates for nine different maturities running from October 1993 to December 2018 Our results provide evidence in accordance with the EHTS for 5- to 20-year interest rates; the spread between the short rate and long end of the term structure was found to be an optimal predictor of future short rates from a maturity of a 5-year horizon. Importantly, we find evidence that the spread presents a long memory process, in contrast to the usual assumption of *I(0)*.

Due to the global financial crisis of 2008, the Fed initiated a series of incentives to face such a crisis, i.e., the QE program, establishing this point as a breakpoint and analyzing the resulting regimes. In this regard, we find that the behaviors of short- and long-term rates differ depending on the observed regime, and we highlight the change in pattern occurring from the first regime, where relations appear stable; from the crisis, we observe how this stability disappears a priori, giving rise to a situation very similar to that observed for the entire sample. In sum, these results show that our results may be motivated by the application of the QE program.

According to these empirical results, we reveal persistence in the spread, and we consequently outline some important implications for monetary policy. On one hand, Baillie and Bollerslev (1994a) noted that a long memory spread offers adequate forecasting power at longer horizons. On the other hand, and even more importantly, in line with Cassola and Morana (2008), Hassler and Nautz (2008) and Cömert (2012), we show that the persistence of the spread implies a gradual loss of control power over interest rates of the Fed, particularly when the maturity is 20 years. As persistence in term spread increases, the gap between maturities also increases. This persistence might limit the amount of information contained in short-term rates for future monetary policy, which may affect the Fed's control of long-term interest rates and of the yield curve. To address this issue, the Fed should increase the frequency of money market interventions.

We suggest that the growing difference between short- and long-term interest rates creates a vulnerable link as the term spread increases. In line with Blinder *et al.* (2008), Ben Bernanke's 2012 Jackson Hole speech and Li and David (2017), this connection is essential for further guidance. Furthermore, since the Fed only has power over shorter-end interest rates, its manipulation may influence other short-term interest rates and thus may be necessary for the application of measures affecting longer-term rates when the monetary policy transmission mechanism predicted by the EHTS is not met. Policies oriented over time, such as the QE program, would thus be necessary to maintain this transmission mechanism or the substitutability of interest rates.

References

Abbritti, M., Carcel, H., Gil-Alana, L. A., & Moreno, A. (2018). Term Premium and Quantitative Easing in a Fractionally Cointegrated Yield Curve. *Available at SSRN 3085154*.

Araç, A., & Yalta, A. Y. (2015). Testing the expectations hypothesis for the Eurozone: A nonlinear cointegration analysis. *Finance Research Letters, 15*, 41-48.

Baillie, R. T. (1996). Long memory processes and fractional integration in econometrics. *Journal of econometrics, 73*(1), 5–59.

Baillie, R. T., & Bollerslev, T. (1994a). Cointegration, fractional cointegration, and exchange rate dynamics. *The journal of Finance, 49(2)*, 737-745.

Baillie, R. T., & Bollerslev, T. (1994b). The long memory of the forward premium. *Journal of International Money and Finance, 13(5)*, 565-571.

Bernanke, B. S., & Blinder, A. S. (1992). The federal funds rate and the channels of monetary transmission. *The American Economic Review*, 901-921.

Blinder, A. S., Ehrmann, M., Fratzscher, M., De Haan, J., & Jansen, D. J. (2008). Central bank communication and monetary policy: A survey of theory and evidence. *Journal of Economic Literature*, *46*(4), 910-45.

Brüggemann, R., & Lütkepohl, H. (2005). Uncovered interest rate parity and the expectations hypothesis of the term structure. Empirical results for the US and Europe. *Applied Economics Quarterly, 51*, 143-154.

Bulkley, G., Harris, R. D., & Nawosah, V. (2011). Revisiting the expectations hypothesis of the term structure of interest rates. *Journal of Banking & Finance*, *35*(5), 1202-1212.

Bulkley, G., Harris, R. D., & Nawosah, V. (2015). Can behavioral biases explain the rejections of the expectation hypothesis of the term structure of interest rates?. *Journal of Banking & Finance*, *58*, 179-193.

Campbell, J. Y. (1995). Some lessons from the yield curve. *Journal of Economic Perspectives, 9*, 124-52.

Campbell, J. Y., & Shiller, R. J. (1987). Cointegration and tests of present value models. *Journal of Political Economy, 95*, 1062-1088.

Campbell, J. Y., & Shiller, R. J. (1991). Yield spreads and interest rate movements: a bird's eye view. *Review of Economic Studies, 58*, 495-514.

Caporin, M., Ranaldo, A., and De Magistris, P. S. (2013). On the predictability of stock prices: A case for high and low prices. Journal of Banking & Finance, 37(12):5132– 5146.

Cassola, N., & Morana, C. (2008). Modeling short-term interest rate spreads in the euro money market. *International Journal of Central Bank.*

Clarida, R. H., Sarno, L., Taylor, M. P., & Valente, G. (2006). The role of asymmetries and regime shifts in the term structure of interest rates. T*he Journal of Business*, *79*(3), 1193-1224.

Cömert, H. (2012). Decoupling between the federal funds rate and long-term interest rates: Decreasing effectiveness of monetary policy in the U.S. *No. wp295), Political Economy Reserch Center.*

Diebold, F. X., Rudebusch, G. D., & Aruoba, S. B. (2006). The macroeconomy and the yield curve: a dynamic latent factor approach. *Journal of econometrics*, *131*(1-2), 309-338.

Dolatabadi, S., Nielsen, M. Ø., and Xu, K. (2015). A fractionally cointegrated VAR analysis of price discovery in commodity futures markets. Journal of Futures Markets, 35(4):339–356.

Dolatabadi, S., Nielsen, M. Ø., and Xu, K. (2016). A fractionally cointegrated VAR model with deterministic trends and application to commodity futures markets. Jour- nal of Empirical Finance, 38:623–639.

Dueker, M. & Startz, R. (1998). Maximum-likelihood estimation of fractional cointegration with an application to us and Canadian bond rates. *Review of Economics and Statistics, 80*(3), 420–426.

Engle, R. F., & Granger, C. W. (1987). Cointegration and error correction: Representation, estimation and testing. *Econometrica, 55*, 251-276.

Engsted, T., & Tanggaard, C. (1994). Cointegration and the US term structure. *Journal of Banking and Finance, 18 (1)*, 167-181.

Esteve, V., Navarro-Ibáñez, M., & Prats, M. A. (2013). The Spanish term structure of interest rates revisited: Cointegration with multiple structural breaks, 1974–2010. *International Review of Economics & Finance*, *25*, 24-34.

Estrella, A., & Hardouvelis, G. A. (1991). The term structure as a predictor of real economic activity. *Journal of Finance, 46*, 555-576.

Fama, E. (1984). The information in the term structure. *Journal of Financial Economics, 13 (4)*, 509-28.

Fama, E., & Bliss, R. (1987). The information in long-maturity forward rates. *American Economic Review, 77*, 680-692.

Geweke, J., & Porter-Hudak, S. (1983). The estimation and application of long memory time series models. *Journal of time series analysis, 4(4)*, 221-238.

Gil-Alana, L. A., & Carcel, H. (2018). A fractional cointegration var analysis of exchange rate dynamics. *The North American Journal of Economics and Finance*.

Guidolin, M., & Thornton, D. L. (2010). Predictions of short-term rates and the expectations hypothesis. *Federal Reserve Bank of St. Louis Working Paper Series 2010-013A*.

Hall, A. D., Anderson, H. M., & Granger, C. W. (1992). A cointegration analysis of treasury bill yields. *The Review of Economics and Statistics*, 116-126.

Hamilton, J. D. (1988). Rational-expectations econometric analysis of changes in regime: An investigation of the term structure of interest rates. *Journal of Economic Dynamics and Control, 12*, 385-423.

Hansen, B. E., & Seo, B. (2002). Testing for two-regime threshold cointegration in vector error-correction models. *Journal of Econometrics, 110*, 293-318.

Hardouvelis, G. A. (1994). The term structure spread and future changes in long and short rates in the G7 countries: Is there a puzzle? *Journal of Monetary Economics, 33*, 255-83.

Hassler, U., & Nautz, D. (2008). On the persistence of the Eonia spread. *Economics Letters, 101(3)*, 184-187.

Holmes, M. J., Otero, J., & Panagiotidis, T. (2015). The expectations hypothesis and decoupling of short-and long-term US interest rates: A pairwise approach. *The North American Journal of Economics and Finance*, *34*, 301-313.

Johansen, S. (1995). *Likelihhood based inference on cointegration in the vector autoregressive model.* Oxford, UK: Oxford University Press.

Johansen, S. (2008a). A representation theory for a class of vector autoregressive models for fractional processes. *Econometric Theory,24*, 651-676.

Johansen, S. (2008b). Representation of cointegrated autoregressive processes with application to fractional processes. *Econometric Reviews, 28*, 121-145.

Johansen, S., & Nielsen, M. Ø. (2012). Likelihood inference for a fractionally cointegrated vector autoregressive model. *Econometrica, 80(6)*, 2667-2732.

Jones, M. E., Nielsen, M. Ø., & Popiel, M. K. (2014). A fractionally cointegrated VAR analysis of economic voting and political support. *Canadian Journal of Economics/Revue canadienne d'économique*, *47*(4), 1078-1130.

Kraay, A., & Ventura, J. (2005). *The dot-com bubble, the Bush deficits, and the US current account*. The World Bank.

Labonte, M., & Makinen, G. E. (2008). Monetary policy and the Federal Reserve: current policy and conditions. Congressional Research Service, Library of Congress.

Lange, R. H. (2018). The term structure of liquidity premia and the macroeconomy in Canada: A dynamic latent-factor approach. *International Review of Economics & Finance, 57*, 164 – 182.

Lanne, M. (2003). Testing the expectations hypothesis of the term structure of interest rates in the presence of a potentia regime shift. *Manchester School, 71*, 54-67.

Li, J., & Davis, G. (2017). Rethinking cointegration and the expectation hypothesis of the term structure. *Journal of Empirical Finance*, *44*, 177-189.

MacKinnon, J. G., & Nielsen, M. Ø. (2014). Numerical distribution functions of fractional unit root and cointegration tests. *Journal of Applied Econometrics, 29*, 161-171.

Mankiw, N. G. (1986). The term structure of interest rates revisited. *Brookings Papers on Economic Activity 1*, 61-96.

Mili, M., Sahut, J. M., & Teulon, F. (2012). New evidence of the expectation hypothesis of interest rates: a flexible nonlinear approach. *Applied Financial Economics*, *22*(2), 165-176.

Mohanty, S., Peterson, E. W. F. & Smith, D. B. (1998). Fractional cointegration and the false rejection of the law of one price in international commodity markets. *Journal of Agricultural and Applied Economics, 30*(02), 267–276.

Nielsen, M. Ø., & Popiel, M. K. (2016). A Matlab program and user's guide for the fractionally cointegrated VAR model. *QED working paper*, 1330.

Phillips, P. C., & Shimotsu, K. (2002). Pooled log periodogram regression. *Journal of Time Series Analysis*, *23*(1), 57-93.

Poole, W., & Rasche, R. H. (2000). Perfecting the market's knowledge of monetary policy. *Journal of Financial Services Research. 18(2-3)*, 255-298.

Poole, W., Rasche, R. H., & Thornton, D. L. (2002). Market anticipations of monetary policy actions. *Review-Federal Reserve Bank of Saint Louis*, *84*(4), 65-94.

Robinson, P. M. (1995). Log-periodogram regression of time series with long range dependence. *The annals of Statistics*, 1048-1072.

Rossi, E., & De Magistris, P. S. (2013). Long memory and tail dependence in trading volume and volatility. *Journal of Empirical Finance*, *22*, 94-112.

Seo, B. (2003). Nonlinear mean reversion in the term structure of interest rates. *Journal of Economic Dynamics and Control, 27*, 2243-2265.

Shea, G. S. (1992). Benchmarking the expectations hypoyhesis of the interest-rate term structure: An analysis of cointegration vectors. *Journal of Business & Economic Statistics, 10*, 347-366.

Shiller, R. J. (1990). The term structure of interest rates. *Handbook of monetary economics, 1*, 627-722.

Strohsal, T., & Weber, E. (2014). Mean-variance cointegration and the expectations hypothesis. *Quantitative Finance, 14(11)*, 1983-1997.

Thornton, D. L. (2005). Tests of the expectations hypothesis: Resolving the anomalies when the short-term rate is the federal funds rate. *Journal of Banking & Finance, 29(10)*, 2541-2556.

Tkacz, G. (2001). Estimating the fractional order of integration of interest rates using a wavelet OLS estimator. *Studies in Nonlinear Dynamics and Econometrics, 5(1)*, 19-32.

Velasco, C. (1999). Non-stationary log-periodogram regression. *Journal of Econometrics*, *91*(2), 325-371.

Vetzal, K. R. (1994). A survey of stochastic continuous time models of the term structure of interest rates. *Insurance: Mathematics and Economics, 14(2)*, 139-161.

Vides, J. C., Iglesias, J., & Golpe, A. A. (2018). The Term Structure Under Non-linearity Assumptions: New Methods in Time Series. In *New Methods in Fixed Income Modeling* (pp. 117-136). Springer, Cham.

Weber, E., & Wolters, J. (2012). The US term structure and central bank policy. *Applied Economic Letters, 19*, 41-45.

Weber, E., & Wolters, J. (2013). Risk and policy shocks on the us term structure. *Scottish Journal of Political Economy, 60*, 101-119.

Williamson, S. D. (2017). Quantitative easing: how well does this tool work?. *The Regional Economist*, *25*(3).

Wright, J. H. (2012). What does monetary policy do to long-term interest rates at the zero lower bound?. *The Economic Journal*, *122*(564), F447-F466.

Appendix

Table A1a. Lag length selection when 3-month is reference of short-term

Lags	6-month	1-year	2-year	3-year	5-year	7-year	10-year	20-year
0	-1060.86	-804.88	-575.44	-478.43	-392.76	-370.20	-372.69	-402.30
1	-1063.84	-799.23	-575.68	-481.68	-395.14	-372.49	-373.13	-403.58
2	-1068.39	-812.54	-585.67	-489.85	-396.41	-372.20	-371.58	-401.48
3	-1077.42	-823.24	-588.59	-490.21	-397.68	-375.51	-379.19	-409.37
4	-1070.59	-818.39	-585.15	-488.32	-396.18	-373.35	-378.05	-409.51
5	-1070.95	-820.87	-587.62	-491.14	-401.54	-378.50	-382.91	-412.81
6	**-1090.25**	**-830.45**	**-596.63**	**-500.08**	**-410.40**	**-386.98**	**-393.02**	**-420.13**

Notes: Bold indicates lag order selected

Table A1b. Lag length selection when 6-month is reference of short-term

Lags	1-year	2-year	3-year	5-year	7-year	10-year	20-year
0	-1140.82	-740.26	-604.16	-487.62	-449.86	-440.04	-455.31
1	-1139.94	-743.54	-606.96	-487.99	-450.04	-438.71	-455.01
2	-1156.86	-754.32	**-615.36**	-488.79	-447.51	-435.74	-452.09
3	-1158.26	-751.87	-611.76	-481.50	-445.15	-436.79	-452.14
4	-1158.30	**-756.91**	-608.64	-480.72	-444.23	-438.85	-454.56
5	**-1167.29**	-748.48	-611.52	-492.80	**-455.94**	-447.08	**-462.09**
6	-1165.55	-749.57	-612.59	**-493.00**	-455.95	**-447.33**	-460.16

Notes: Bold indicates lag order selected

Table A1c. Lag length selection when 1-year is reference of short-term

Lags	2-year	3-year	5-year	7-year	10-year	20-year
0	-950.22	-736.94	-553.15	-491.79	-465.73	-463.49
1	-948.32	-734.64	-549.23	-487.93	-460.95	-459.51
2	-951.53	-738.20	-548.39	-486.72	-459.99	-457.47
3	-949.54	-734.80	-547.46	-489.07	-467.25	-465.37
4	-953.47	-739.57	-551.39	-492.83	-470.00	-467.57
5	**-953.82**	**-742.17**	**-561.41**	**-503.20**	**-482.01**	**-479.59**
6	-952.56	-740.18	-559.52	-500.47	-480.72	-477.21

Notes: Bold indicates lag order selected

Table A2. Rank test

Rank test when 3-month is reference of short-term

rank	6-month	1-year	2-year	3-year	5-year	7-year	10-year	20-year
0	27.562 (0.003)	10.987 (0.027)	17.532 (0.011)	24.088 (0.010)	19.407 (0.001)	17.384 (0.013)	17.551 (0.089)	16.119 (0.140)
1	4.544 (0.288)	1.981 (0.691)	1.052 (0.860)	0.632 (0.940)	2.017 (0.627)	1.493 (0.747)	2.665 (0.515)	2.348 (0.566)

Rank test when 6-month is reference of short-term

rank		1-year	2-year	3-year	5-year	7-year	10-year	20-year
0		34.212 (0.000)	16.412 (0.003)	17.255 (0.002)	9.882 (0.042)	19.056 (0.046)	8.663 (0.267)	16.484 (0.105)
1		2.658 (0.479)	1.784 (0.182)	0.372 (0.542)	1.575 (0.723)	1.107 (0.652)	2.169 (0.613)	1.502 (0.522)

Rank test when 1-year is reference of short-term

rank			2-year	3-year	5-year	7-year	10-year	20-year
0			17.975 (0.054)	15.612 (0.124)	15.615 (0.122)	15.609 (0.126)	14.825 (0.154)	13.930 (0.205)
1			0.627 (0.775)	0.488 (0.842)	0.929 (0.703)	0.812 (0.742)	1.131 (0.644)	0.891 (0.684)

Notes: The top of the table shows the LR statistics and *P*-values are in parenthesis. We follow the rank test procedure for small samples (the sample size is 303) suggested by Jones *et al.* (2014).

Table A3. Ljung-Box Q-test

	Maturities							
3-month vs	6-month	1-year	2-year	3-year	5-year	7-year	10-year	20-year
$Q_{\hat{\varepsilon}}$	59.483 (0.124)	43.407 (0.661)	26.770 (0.994)	22.143 (0.999)	19.726 (1.000)	20.548 (1.000)	22.471 (0.999)	20.366 (1.000)
6-month vs		1-year	2-year	3-year	5-year	7-year	10-year	20-year
$Q_{\hat{\varepsilon}}$		60.490 (0.107)	73.098 (0.011)	67.618 (0.032)	30.501 (0.977)	38.576 (0.832)	33.013 (0.951)	40.404 (0.774)
1-year vs.			2-year	3-year	5-year	7-year	10-year	20-year
$Q_{\hat{\varepsilon}}$			49.023 (0.432)	44.599 (0.613)	42.825 (0.684)	42.219 (0.708)	41.013 (0.752)	39.010 (0.819)

Notes: Following Jones et al. (2014), the significance level is set to 10% for exclusion. The sample size is 303. P-values are in parenthesis below LR test values.

Table A4. CVAR results

Order of integration of the error correction term (ECT)	**Value of *β***	
	***β* = 1**	**0 < *β* < 1**
I(*d* – *b*) = I(0)	[3M – 2Y]; [3M – 3Y]; [3M – 5Y];[3M – 7Y]; [3M – 10Y] [6M – 1Y]; [6M – 2Y]; [6M – 3Y]; [6M – 5Y]; [6M – 7Y]; [6M – 10Y] [1Y – 2Y]; [1Y – 3Y] [1Y – 5Y]; [1Y – 7Y]; [1Y – 10Y]	[3M – 6M]; [3M – 1Y];
I(0) < I(*d* – *b*) < I(0.5)		
I(0.5) < I(*d* – *b*) < I(1)		

Notes: The shaded area corresponds to the traditional *EHTS.*