Instantaneous Reactive Power Theory: A Reference in the Nonlinear Loads Compensation

Reyes S. Herrera and Patricio Salmerón

***Abstract*—The instantaneous reactive power theory was published 25 years ago, in an IEEE TRANSACTIONS. Since then, it has been the most used in nonlinear load compensation with active power filters. Its application allows constant source power to be obtained after compensation in a simple way. Moreover, some researches have showed up some limitations of the theory, i.e., it goes optimally with source voltage balanced and sinusoidal, but not so good with source voltage unbalanced and/or nonsinusoidal, since the source current obtained is not balanced and sinusoidal. This paper presents a new compensation strategy in phase coordinates, equivalent to the original theory's one. Its simplicity, due to the nonnecessity of coordinate mathematical transformation, makes easier the modifications necessary to obtain alternative compensation objectives. In this way, this paper presents those modifications and derives compensation strategies that obtain alternative compensation objectives: unity power factor or balanced and sinusoidal source current. Finally, compensation strategies are applied to a practical power system, and the results are presented.**

***Index Terms*—Active power filters (APFs), instantaneous reactive power theory, power quality.**

I. INTRODUCTION

THE instantaneous reactive power theory was initially published in English in the Proceedings of the International Power Electronics Conference in 1983 [1]. However, it was in 1984, after its publication in an IEEE TRANSACTIONS, when this theory became well known worldwide [2]. Since then, the instantaneous reactive power theory has been the most used compensation strategy in active power filters (APFs). Indeed, the strategy proposed obtains sinusoidal and balanced currents, constant instantaneous power, and unity power factor in the source side when the voltage applied is balanced and sinusoidal [2]. In any other case, i.e., when the voltage is unbalanced and/or nonsinusoidal, the instantaneous power is constant after compensation in the source side, but the current is not balanced and sinusoidal, and the power factor is not the unity [3], [4].

Thus, from the point of view of research, the publication of the instantaneous reactive power theory caused a great impact in compensation techniques. Therefore, many approaches have been published since then [5]–[14]. In fact, in the 1990s, the interest was specially focused on the study of three-phase four-wire systems at most general conditions: unbalanced and nonsinusoidal source and nonlinear unbalanced load. The first objective was to find control strategies which allow the neutral current elimination with a null average power transferred by the compensator. Thus, besides the original formulation, among others, the modified p–q or cross product formulation [5]–[7] stand out. A comparative evaluation of those theories was carried out when they were applied to obtain active power line conditioners control algorithms for unbalanced systems with nonsinusoidal voltage. At these conditions, each theory AQ1 produced different results, without obtaining the opportunity to establish, in a general way, the advantage of any one theory over the others [8]. Other remarkable formulations are the d–q [9], or its alternative, the id–iq [10] in the rotating frame, the p–q–r formulation [14], and the vectorial formulation [11], [12]. All of them relate the energy transfer in a three-phase system in function to the instantaneous power (instantaneous real power) $p(t)$ and to the instantaneous imaginary (or reactive) power, depending on the formulation. This last quantity establishes the difference between the instantaneous reactive power theory and the rest of other possible theories about the electric power.

All of these works have been published trying to improve the results obtained by the instantaneous reactive power theory in three-phase four-wire systems in any voltage supply conditions. In [13], the results of applying the compensation strategies derived from those relevant theories to a same power system are presented. It shows that none of those theories obtain balanced and sinusoidal source current if the voltage is unbalanced and nonsinusoidal.

In this paper, the instantaneous reactive power theory is presented, and its compensation strategy is applied to a three-phase four-wire power system. In addition, an equivalent formulation developed in phase coordinates is presented. It is not a new theory but a different formulation. Thus, the results obtained by both are the same. However, the simplicity of the new formulation makes easier the derivation of compensation strategy. Moreover, the simplicity of the new formulation in phase coordinates allows alternative compensation strategies to be obtained which produce balanced and sinusoidal source current in any voltage supply conditions. The results obtained when applying the compensation strategies in phase coordinates to a three-phase four-wire system are presented.

This paper is organized as follows. In Section II, the instantaneous reactive power theory is presented. In Section III, the formulation developed in phase coordinates is presented, as well. The results obtained when applying the compensation strategy in phase coordinates to a practical nonlinear three-phase system are presented. In Section IV, alternative strategies corresponding to different compensation objectives are derived. They are applied to the practical system, and the results are presented. Finally, in Section V, some conclusions are established.

Manuscript received May 29, 2008; revised January 27, 2009.

The authors are with the Department of Electrical Engineering, Escuela Politécnica Superior, University of Huelva, Campus of La Rábida, 21819 Palos de la Frontera, Spain (e-mail: reyes.sanchez@die.uhu.es; patricio@uhu.es).

Digital Object Identifier 10.1109/TIE.2009.2014749

II. Instantaneous Reactive Power Theory

The voltage vector in phase coordinates corresponding to a three-phase system is expressed as follows:

$$\vec{u} = [u_1 \quad u_2 \quad u_3]^t. \tag{1}$$

Therefore, the current vector

$$\vec{i} = [i_1 \quad i_2 \quad i_3]^t. \tag{2}$$

The instantaneous reactive power theory, also named p–q formulation [1], [2], is based on the Clarke coordinates transformation, which, applied to the voltage and current vectors in phase coordinates, gives those vectors in $0\alpha\beta$ coordinates

$$\begin{bmatrix} u_0 \\ u_\alpha \\ u_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}. \tag{4}$$

In the new coordinates system, three power terms are defined: zero-sequence instantaneous real power p_0, instantaneous real power $p_{\alpha\beta}$, and instantaneous imaginary power $q_{\alpha\beta}$

$$p_0(t) = u_0 i_0 \tag{5}$$

$$p_{\alpha\beta}(t) = [u_\alpha \quad u_\beta] \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = u_\alpha i_\alpha + u_\beta i_\beta \tag{6}$$

$$\begin{aligned} q_{\alpha\beta} &= \|\vec{q}_{\alpha\beta}(t)\| = \left\| [u_\alpha \quad u_\beta]^t \wedge [i_\alpha \quad i_\beta]^t \right\| \\ &= (-u_\beta i_\alpha + u_\alpha i_\beta) \end{aligned} \tag{7}$$

where the instantaneous imaginary power has been defined as the norm of the instantaneous imaginary power vector $\vec{q}_{\alpha\beta}(t)$. This has been defined as the cross product of voltage and current vector in $\alpha\beta$ coordinates.

Equations (5)–(7) define the three power variables, which may be expressed in matrix form as follows:

$$\begin{bmatrix} p_0 \\ p_{\alpha\beta} \\ q_{\alpha\beta} \end{bmatrix} = \begin{bmatrix} u_0 & 0 & 0 \\ 0 & u_\alpha & u_\beta \\ 0 & -u_\beta & u_\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix}. \tag{8}$$

From this equation, current may be expressed according to the power quantities as

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{u_0 u_{\alpha\beta}^2} \begin{bmatrix} u_{\alpha\beta}^2 & 0 & 0 \\ 0 & u_0 u_\alpha & -u_0 u_\beta \\ 0 & u_0 u_\beta & u_0 u_\alpha \end{bmatrix} \begin{bmatrix} p_0 \\ p_{\alpha\beta} \\ q_{\alpha\beta} \end{bmatrix} \tag{9}$$

where $u_{\alpha\beta}^2 = u_\alpha^2 + u_\beta^2$.

From now on, it is possible to talk about compensation. The compensation current in matrix form derived from the instantaneous reactive power theory is

$$\begin{bmatrix} i_{C0} \\ i_{C\alpha} \\ i_{C\beta} \end{bmatrix} = \frac{1}{u_0 u_{\alpha\beta}^2} \begin{bmatrix} u_{\alpha\beta}^2 & 0 & 0 \\ 0 & u_0 u_\alpha & -u_0 u_\beta \\ 0 & u_0 u_\beta & u_0 u_\alpha \end{bmatrix} \begin{bmatrix} p_{C0} \\ p_{C\alpha\beta} \\ q_{C\alpha\beta} \end{bmatrix} \tag{10}$$

where the subindex "C" means compensation component. It is, p_{C0} means the compensation zero-sequence instantaneous power, $p_{C\alpha\beta}$ the compensation instantaneous power without zero-sequence, and $q_{C\alpha\beta}$ the compensation instantaneous imaginary power. The values assigned to p_{C0}, $p_{C\alpha\beta}$, and $q_{C\alpha\beta}$ are established applying the constant power compensation developed along the present section.

Moreover, in addition to the constant power compensation imposed by the original p–q authors, they add the constraint of eliminating the neutral current. Therefore, the current zero-sequence component must be

$$i_{C0} = i_{L0} = \frac{p_{L0}}{u_0} \tag{11}$$

where the subindex "L" means incoming to the load.

On the other hand, the p–q formulation evolves the total compensation of the instantaneous imaginary power. Therefore,

$$q_{C\alpha\beta} = q_{L\alpha\beta}. \tag{12}$$

With respect to instantaneous power, the p–q formulation considers the constraint of eliminating the active power supplied by the compensator besides the achievement of constant source power. In this way, the instantaneous power required by the load is

$$p_L(t) = p_S(t) + p_C(t) \tag{13}$$

where subindex "S" means "source component." The load instantaneous power can be expressed as follows, too:

$$p_L(t) = \tilde{p}_{L\alpha\beta}(t) + P_{L\alpha\beta} + \tilde{p}_{L0}(t) + P_{L0} \tag{14}$$

where the uppercase P means the instantaneous real power average value, and the symbol "$\sim$" over the letter, the oscillating component [4].

The compensation instantaneous power can be divided into its zero-sequence component and its $\alpha\beta$ component and according to (11)

$$p_C(t) = p_{C0}(t) + p_{C\alpha\beta}(t) = p_{L0}(t) + p_{C\alpha\beta}(t). \tag{15}$$

Considering (13)–(15)

$$\begin{aligned} p_S(t) + p_C(t) &= p_S(t) + p_{C\alpha\beta}(t) + p_{C0}(t) \\ &= p_L(t) = \tilde{p}_{L\alpha\beta}(t) + P_{L\alpha\beta} + \tilde{p}_{L0}(t) + P_{L0}. \end{aligned} \tag{16}$$

Taking into account that the source must supply the constant component of the instantaneous power incoming to the load

$$p_{S0}(t) = P_{L\alpha\beta} + P_{L0}. \tag{17}$$

Moreover, introducing (17) in (16), it is

$$\begin{aligned} P_{L\alpha\beta} + P_{L0} + p_{C\alpha\beta}(t) + p_{C0}(t) \\ = \tilde{p}_{L\alpha\beta}(t) + P_{L\alpha\beta} + \tilde{p}_{L0}(t) + P_{L0}. \end{aligned} \tag{18}$$

Operating and according to (11), (18) is

$$P_{L\alpha\beta} + P_{L0} + p_{C\alpha\beta}(t) = \tilde{p}_{L\alpha\beta}(t) + P_{L\alpha\beta}. \tag{19}$$

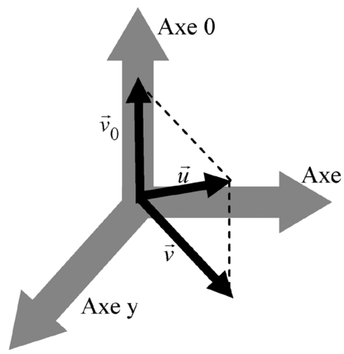


Fig. 1. Voltage vectors in a three-phase system.

Finally, the $\alpha\beta$-component of the instantaneous real power transferred by the compensator is

$$p_{C\alpha\beta} = \tilde{p}_{L\alpha\beta} - P_{L0}. \tag{20}$$

Therefore, the complete compensation strategy in matrix form is as follows:

$$\begin{bmatrix} i_{C0} \\ i_{C\alpha} \\ i_{C\beta} \end{bmatrix} = \frac{1}{u_0 u_{\alpha\beta}^2} \begin{bmatrix} u_{\alpha\beta}^2 & 0 & 0 \\ 0 & u_0 u_\alpha & -u_0 u_\beta \\ 0 & u_0 u_\beta & u_0 u_\alpha \end{bmatrix} \begin{bmatrix} p_{L0} \\ \tilde{p}_L - P_{L0} \\ q_L \end{bmatrix}. \tag{21}$$

Therefore, the p–q theory compensates the oscillating component of the total instantaneous real power and the total instantaneous imaginary power. Moreover, it eliminates the neutral current and the active power exchanged by the compensator is null.

III. Instantaneous Reactive Power Formulation in Phase Coordinates System

In this section, an alternative formulation is presented. It is equivalent to the derived in the previous section from the instantaneous reactive power theory, although its formulation is simpler than the other. This simplicity makes possible the achievement of modified compensation strategies which obtains balanced and sinusoidal source current in any voltage conditions.

Considering the voltage and current vectors in phase coordinates presented in (1) and (2), a zero-sequence voltage vector can be defined as follows:

$$\vec{v}_0 = \begin{bmatrix} \frac{v_0}{\sqrt{3}} & \frac{v_0}{\sqrt{3}} & \frac{v_0}{\sqrt{3}} \end{bmatrix}^t$$
$$v_0 = \frac{u_1 + u_2 + u_3}{\sqrt{3}}. \tag{22}$$

The zero-sequence axis is orthogonal to the plane $\alpha\beta$ and to the plane 123 according to the Clark transformation. Thus, applying vectorial algebra, the definition of a voltage vector without zero-sequence component $\vec{v}$ is possible

$$\vec{v} = \vec{u} - \vec{v}_0. \tag{23}$$

The zero sequence voltage vector and the voltage vector without zero-sequence component are orthogonal (Fig. 1). Thus, two current vectors can be defined as the projections of the current vector over both. Therefore, the zero-sequence current vector is defined as follows:

$$\vec{i}_0(t) = \frac{\vec{i} \cdot \vec{v}_0}{v_0^2} \vec{v}_0 = \frac{p_0(t)}{v_0^2} \vec{v}_0 \tag{24}$$

where $p_0(t)$ agrees with the zero-sequence instantaneous power defined in the p–q formulation. Equation (24) is based on the fact that one vector and the projection of another vector over the first are in the same direction. Therefore, considering that the only component of the load current that is in the $\vec{v}_0$ direction is $\vec{i}_0$, it is

$$\vec{i} \cdot \vec{v}_0 = \vec{i}_0 \cdot \vec{v}_0 = i_0 \cdot v_0. \tag{25}$$

From (25),

$$i_0 = \frac{\vec{i} \cdot \vec{v}_0}{v_0}. \tag{26}$$

Moreover, the product of (26) and the unitary vector corresponding to $\vec{v}_0$ is

$$\vec{i}_0 = i_0 \frac{\vec{v}_0}{v_0} = \frac{\vec{i} \cdot \vec{v}_0}{v_0} \frac{\vec{v}_0}{v_0} = \frac{\vec{i} \cdot \vec{v}_0}{v_0^2} \vec{v}_0. \tag{27}$$

The development follows is valid for any waveform and no constrains have been applied. Thus, it is good for unbalanced and/or nonsinusoidal voltages and currents.

In the same way, the instantaneous active current without zero-sequence component is defined as

$$\vec{i}_v(t) = \frac{\vec{i} \cdot \vec{v}}{v^2} \vec{v} = \frac{p_v(t)}{v^2} \vec{v} \tag{28}$$

where $p_v(t)$ agrees with the $\alpha\beta$ instantaneous real power defined in the p–q formulation. Total instantaneous real power can be calculated as follows:

$$p(t) = \vec{u} \cdot (\vec{i}_0 + \vec{i}_v) = p_0(t) + p_v(t). \tag{29}$$

The difference between load current and the sum of zero-sequence current and instantaneous active current without zero-sequence component is named instantaneous reactive current $\vec{i}_q$

$$\vec{i}_q = \vec{i} - \vec{i}_0 - \vec{i}_v. \tag{30}$$

As the first two current components, instantaneous reactive current can be calculated as the projection of the current vector over a new voltage vector, named orthogonal voltage vector $\vec{v}_q$, which is calculated as follows [11]:

$$\vec{v}_q = \frac{1}{\sqrt{3}} \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} u_2 - u_3 \\ u_3 - u_1 \\ u_1 - u_2 \end{bmatrix}. \tag{31}$$

The orthogonal voltage vector is orthogonal to the voltage vector and to the voltage vector without zero-sequence component.

Therefore, the instantaneous reactive current can be expressed as

$$\vec{i}_q(t) = \frac{\vec{i} \cdot \vec{v}_q}{v_q^2} \vec{v}_q = \frac{q(t)}{v_q^2} \vec{v}_q \tag{32}$$

where the numerator $q(t)$ is the instantaneous imaginary power defined in the p–q theory.

Therefore, the load current can be expressed as the sum of the three components calculated above

$$\vec{i}_L(t) = \frac{p_{Lv}(t)}{v^2}\vec{v} + \frac{q_L(t)}{v_q^2}\vec{v}_q + \vec{i}_{L0}(t). \tag{33}$$

Now, the compensation strategy is developed.

A. Constant Power Compensation

In this new framework, it is possible to obtain constant power after compensation. In this case, the source current must be

$$\vec{i}_S = \frac{P_L}{u^2}\vec{u}. \tag{34}$$

Combining (33) and (34), the compensation current is

$$\vec{i}_C = \frac{p_{Lv}(t)}{v^2}\vec{v} + \frac{q_L(t)}{v_q^2}\vec{v}_q + \vec{i}_{L0}(t) - \frac{P_L}{u^2}\vec{u}. \tag{35}$$

Introducing (23) in (35), we obtain

$$\vec{i}_C = \left(\frac{p_{Lv}(t)}{v^2} - \frac{P_L}{u^2}\right)\vec{v} + \frac{q_L(t)}{v_q^2}\vec{v}_q + \vec{i}_{L0}(t) - \frac{P_L}{u^2}\vec{v}_0. \tag{36}$$

The compensation current formal expression divided in three components (zero-sequence component, component without zero sequence, and orthogonal component) is as follows:

$$\vec{i}_C = \frac{p_C(t)}{v^2}\vec{v} + \frac{q_C(t)}{v_q^2}\vec{v}_q + \vec{i}_{C0}(t). \tag{37}$$

From (36) and (37), the formal term related to the instantaneous power without zero-sequence component has the following expression

$$\frac{p_C(t)}{v^2} = \left(\frac{p_{Lv}(t)}{v^2} - \frac{P_L}{u^2}\right). \tag{38}$$

The term related to the instantaneous reactive current

$$\frac{q_C(t)}{v_q^2} = \frac{q_L(t)}{v_q^2}. \tag{39}$$

Moreover, the compensation current zero-sequence component is

$$\vec{i}_{C0}(t) = \vec{i}_{L0}(t) - \frac{P_L}{u^2}. \tag{40}$$

Therefore, from (38), the compensation instantaneous power without zero-sequence component is

$$p_C(t) = p_{Lv}(t) - \frac{P_L}{u^2}v^2. \tag{41}$$

The compensation instantaneous reactive power

$$q_C(t) = q_L(t). \tag{42}$$

Moreover, the compensation zero-sequence instantaneous power

$$p_{C0}(t) = p_{L0}(t) - \frac{P_L}{u^2}v_0^2. \tag{43}$$

B. Constant Power Compensation Eliminating the Zero-Sequence Component Current

If, besides constant power after compensation, the neutral current must be eliminated, the source current should be

$$\vec{i}_S = \frac{P_L}{v^2}\vec{v}. \tag{44}$$

Following the development presented in the previous section, compensation current corresponding to this new variant has the next value

$$\begin{aligned}\vec{i}_C &= \frac{p_{Lv}(t)}{v^2}\vec{v} + \frac{q_L(t)}{v_q^2}\vec{v}_q + \vec{i}_{L0}(t) - \frac{P_L}{v^2}\vec{v}\\ &= \frac{p_{Lv}(t) - P_L}{v^2}\vec{v} + \frac{q_L(t)}{v_q^2}\vec{v}_q + \vec{i}_{L0}(t)\end{aligned} \tag{45}$$

where the term related to the instantaneous power without zero-sequence component has the following expression

$$\frac{p_C(t)}{v^2} = \frac{p_{Lv}(t) - P_L}{v^2}. \tag{46}$$

The term related to the instantaneous reactive power

$$\frac{q_C(t)}{v_q^2} = \frac{q_L(t)}{v_q^2}. \tag{47}$$

Moreover, the compensation current zero-sequence component is

$$\vec{i}_{C0}(t) = \vec{i}_{L0}(t). \tag{48}$$

Therefore, the compensation instantaneous power without zero-sequence component is

$$p_C(t) = \tilde{p}_{Lv}(t) - P_{L0}. \tag{49}$$

The compensation instantaneous reactive power

$$q_C(t) = q_L(t). \tag{50}$$

Moreover, the compensation instantaneous power zero-sequence component is

$$p_{C0}(t) = p_{L0}(t). \tag{51}$$

The compensation current calculated in (45) to obtain constant power eliminating neutral current, can be expressed in the $\alpha\beta$ coordinates system. In this way, and taking into account that

$$\begin{aligned} v^2 &= u_{\alpha\beta}^2\\ \vec{v} &= [0 \quad u_\alpha \quad u_\beta]^T \\ \vec{v}_q &= [0 \quad -u_\beta \quad u_\alpha]^T\\ \vec{i}_0 &= [0 \quad 0 \quad i_0] \end{aligned}$$

(52) (53)

(45) can be expressed as

$$\vec{i}_C = \frac{\tilde{p}_{L\alpha\beta}(t) - P_{L0}}{u_{\alpha\beta}^2}\begin{bmatrix}0\\u_\alpha\\u_\beta\end{bmatrix} + \frac{q_{L\alpha\beta}(t)}{u_{\alpha\beta}^2}\begin{bmatrix}0\\-u_\beta\\u_\alpha\end{bmatrix} + \begin{bmatrix}i_0\\0\\0\end{bmatrix} \tag{54}$$

where the following equalities have been considered about the parameter in both coordinate systems

$$p_{L\alpha\beta} = p_{Lv}$$
$$q_{L\alpha\beta} = q_L. \tag{55}$$

From (24), (54) is as follows:

$$\vec{i}_C = \frac{\tilde{p}_{L\alpha\beta}(t) - P_{L0}}{u_{\alpha\beta}^2}\begin{bmatrix}0\\u_\alpha\\u_\beta\end{bmatrix} + \frac{q_{L\alpha\beta}(t)}{u_{\alpha\beta}^2}\begin{bmatrix}0\\-u_\beta\\u_\alpha\end{bmatrix} + \frac{p_{L0}(t)}{u_0^2}\begin{bmatrix}u_0\\0\\0\end{bmatrix}. \tag{56}$$

Finally, (56) can be expressed in matrix form as

$$\begin{bmatrix}i_{C0}\\i_{C\alpha}\\i_{C\beta}\end{bmatrix} = \frac{1}{u_0 u_{\alpha\beta}^2}\begin{bmatrix}u_{\alpha\beta}^2 & 0 & 0\\0 & u_0u_\alpha & -u_0u_\beta\\0 & u_0u_\beta & u_0u_\alpha\end{bmatrix}\begin{bmatrix}p_{L0}(t)\\p_{L\alpha\beta}(t) - P_{L0}\\q_{L\alpha\beta}(t)\end{bmatrix}. \tag{57}$$

AQ2 This strategy is the same as the one presented in (21). It obtains constant power after compensation and eliminates neutral current. The result (57) shows that compensation currents i_{C0}, $i_{C\alpha}$, $i_{C\beta}$ obtained according to the development presented in this section are the same as the ones obtained according to the development corresponding to the original p–q formulation.

C. *Simulation Results*

This compensation objective (constant power) develops the compensation of total instantaneous imaginary power and variable part of instantaneous power.

Notice that to reduce the line losses as much as possible without altering the instantaneous power (or the instantaneous active current), i.e., without using energy storage, the imaginary power, or equivalently the instantaneous reactive current, should be annihilated. The magnitude of instantaneous imaginary power or the length of the instantaneous reactive current characterizes the instantaneous line loss component which can be reduced by elements without energy storage. The compensation with energy storage corresponds to reducing the average loss, without altering the average power transfer. This is the case of constant power after compensation.

This strategy has been applied to the power system shown in Fig. 2. It is a three-phase four-wire system whose load is made up of three face-to-face SRCs with a star connected resistor AQ3 on the right-hand side. The source impedance is 1 Ω in each phase, and the values of the load resistors are 10, 5, and 15 Ω corresponding to phases 1, 2, and 3, respectively. It makes the load unbalanced.

The results of applying the compensation strategy to the system shown in Fig. 2 when the source is balanced and sinusoidal with a rms value of 100 V are shown in Figs. 3 and 4. In fact, the instantaneous power after compensation is constant. In addition, the source current is balanced and sinusoidal.

If source voltage is unbalanced with rms values of 100, 80, and 110 V corresponding to phases 1, 2, and 3, respectively, the instantaneous power after applying the compensation strategy to the same power system is constant (688 W) although the source voltage is not balanced and sinusoidal. The source current is shown in Fig. 5. It is not sinusoidal, although the distortion presented by the waveform after compensation is much lower than the one presented before.

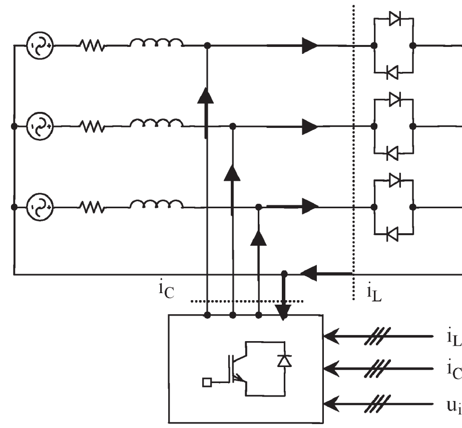


Fig. 2. Experimental prototype.

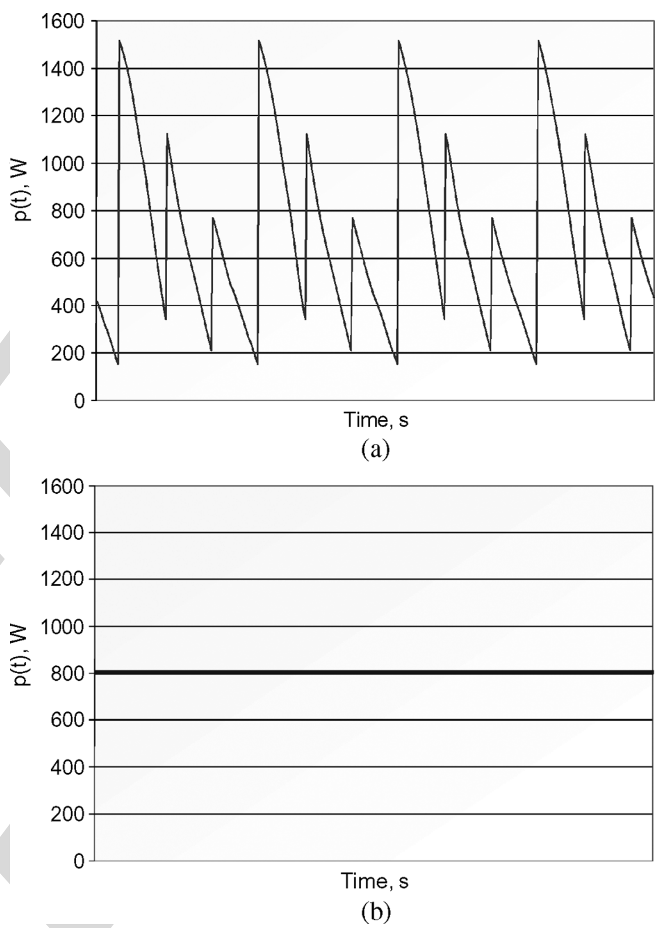


Fig. 3. Instantaneous power (two periods, 0.04 s). (a) Before compensation. (b) After compensation with balanced sinusoidal voltage supply. AQ4

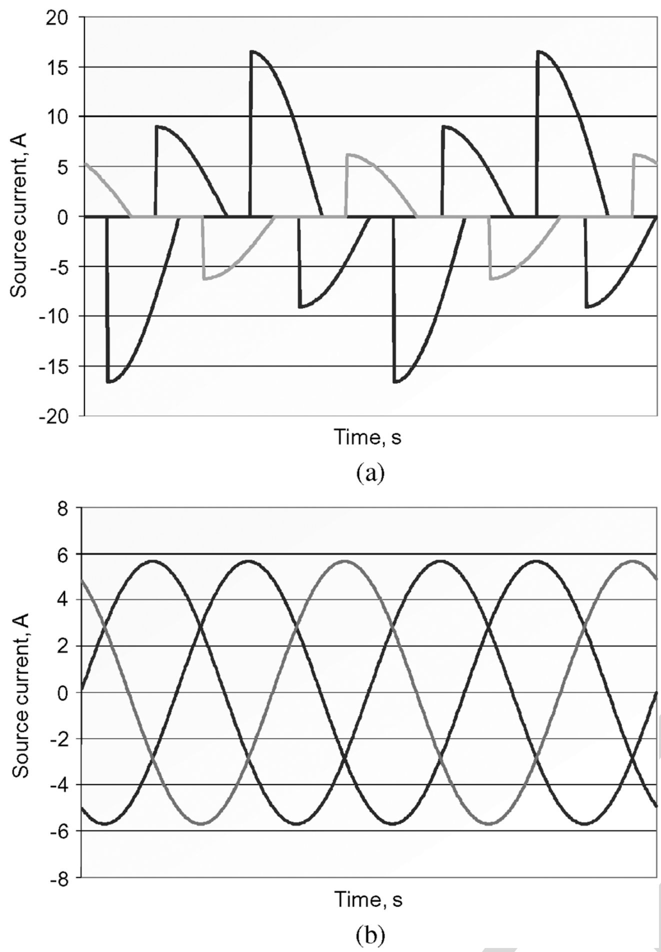


Fig. 4. Source current (two periods, 0.04 s). (a) Before compensation. (b) After compensation with balanced sinusoidal voltage supply.

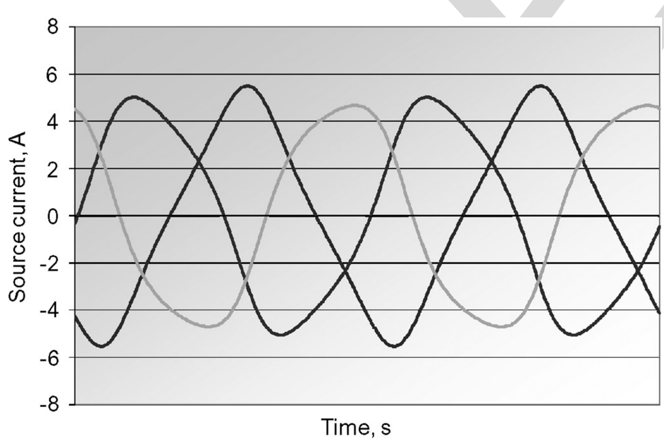


Fig. 5. Source current (two periods, 0.04 s) after compensation with unbalanced sinusoidal voltage supply.

In the case of distorted voltage supply, the instantaneous power after applying the compensation strategy to the same power system is constant (757 W) although the source voltage is not balanced and sinusoidal. The source current, as shown in Fig. 6, is not sinusoidal, although the distortion presented by the waveform after compensation is much lower than the one presented before.

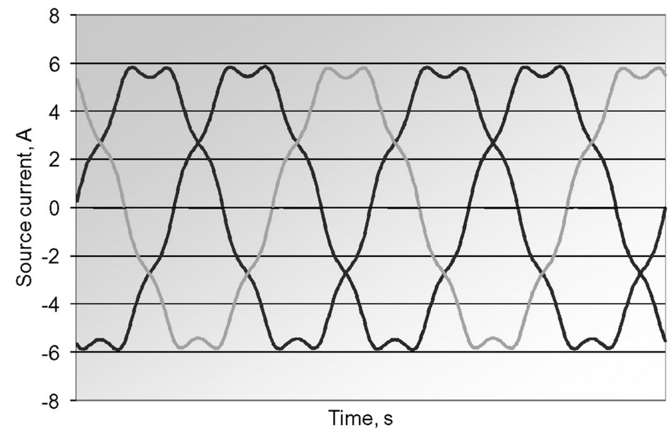


Fig. 6. Source current (two periods, 0.04 s) after compensation with unbalanced sinusoidal voltage supply.

The results shown in Figs. 3–6 are the same as the ones obtained by the original theory. It proves that the compensation strategy proposed is not a new theory but a new formulation whose expression is easier to obtain than the original theory's. On the other hand, to obtain sinusoidal source current, as some other authors consider [13]–[17], the instantaneous reactive power theory has to be submitted to a few modifications [4], as presented in next section. This calculation becomes easier from the phase coordinate expression.

IV. Balanced and Sinusoidal Source Current

The strategies presented in Section III obtain constant source power, unity power factor, and balanced and sinusoidal source currents when the voltage applied is balanced and sinusoidal. Nevertheless, if the voltage applied is unbalanced and sinusoidal, the source current after compensation is not balanced and sinusoidal. Besides, if the voltage applied is balanced nonsinusoidal, the source current is distorted, too. Therefore, in the case of unbalanced and/or nonsinusoidal voltage, the compensation strategy has to be modified to obtain balanced and sinusoidal source current [4].

Therefore, according to [14]–[17], the source current must be proportional to a balanced and sinusoidal voltage vector, i.e., the voltage vector positive-sequence phase component. The proportional constant value must guarantee a null active power supplied by the compensator. Thus, the source current is

$$\vec{i}_S = \frac{P}{U^{+2}}\vec{u}^+ \tag{58}$$

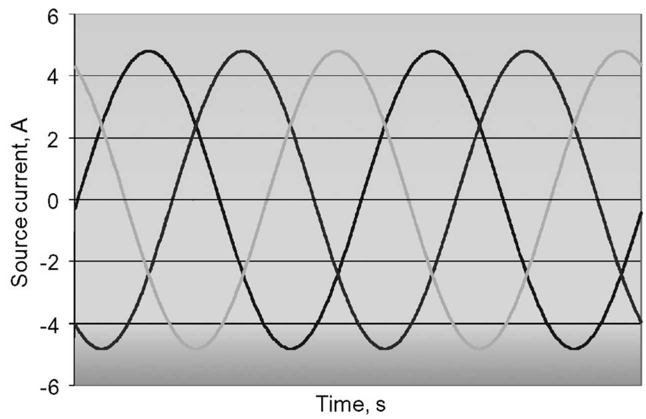
where U^+ is the voltage vector positive sequence component rms value

$$U^{+2} = \frac{1}{T}\int_T \left(u_1^{+2} + u_2^{+2} + u_3^{+2}\right) dt \tag{59}$$

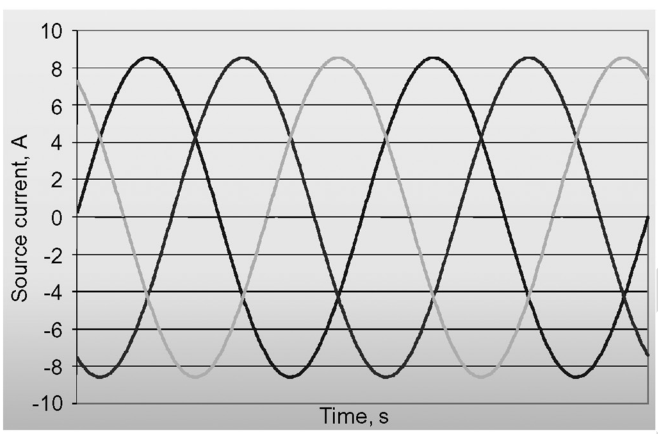
and u_1^+, u_2^+, and u_3^+ are the components of the positive sequence voltage vector $\vec{u}^+$.

The compensation current is

$$\vec{i}_C = \vec{i} - \vec{i}_S. \tag{60}$$

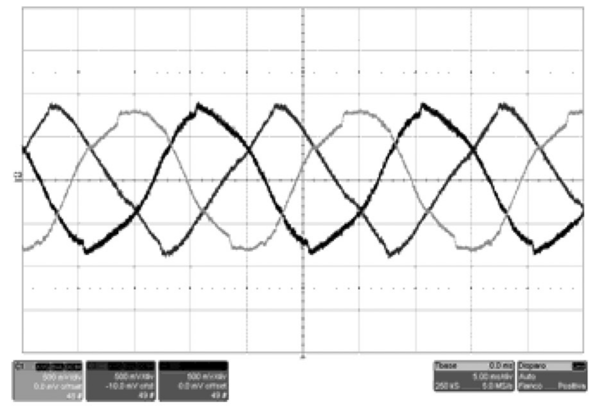


(a)

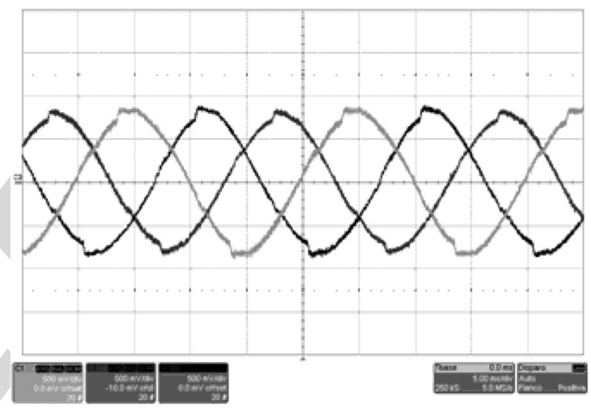


(b)

Fig. 7. Source current after compensation with (a) unbalanced and sinusoidal and (b) balanced and nonsinusoidal supply voltage.



(a)



(b)

Fig. 8. Source current after compensation in an experimental prototype with an unbalanced voltage supply applying (a) traditional strategy and (b) new strategy.

On the other hand, if the voltage is balanced nonsinusoidal, to obtain a sinusoidal source current and according to [14]–[17], the compensation strategy must be the following:

$$\vec{i}_S = \frac{P}{U_f^2}\vec{u}_f \tag{61}$$

where $\vec{u}_f$ is the voltage vector fundamental component and U_f its rms value

$$U_f^2 = \frac{1}{T}\int_T \left(u_{1f}^2 + u_{2f}^2 + u_{3f}^2\right) dt \tag{62}$$

and u_{1f}, u_{2f}, and u_{3f} are the components of the voltage vector fundamental component $\vec{u}_f$.

If the voltage applied is unbalanced and nonsinusoidal, the compensation strategy is a composition of the two previous ones

$$\vec{i}_S = \frac{P}{U_f^{+2}}\vec{u}_f^{+} \tag{63}$$

where U_1^+ is the voltage vector positive sequence fundamental component rms value

$$U_f^{+2} = \frac{1}{T}\int_T \left(u_{1f}^{+2} + u_{2f}^{+2} + u_{3f}^{+2}\right) dt \tag{64}$$

and u_{1f}^+, u_{2f}^+, and u_{3f}^+ are the components of the voltage vector positive-sequence fundamental component $\vec{u}_f^+$.

The compensation current is calculated as in (60). The global compensation strategy presented in (61)–(64) guarantees the achievement of balanced and sinusoidal source current with any voltage conditions.

Fig. 7 shows the results of applying this strategy to the system shown in Fig. 2. Fig. 7(a) corresponds to an unbalanced and sinusoidal voltage supply and Fig. 7(b) to a balanced and nonsinusoidal one. In both cases, the source current is balanced and sinusoidal, as can be seen.

An experimental prototype has been developed corresponding to the power system shown in Fig. 2. The trigger control, for power electronic devices that constitute the APF, has been implemented through a digital signal processor control board system. The constant power compensation strategy (proposed in [1] and [2]) and the new one (sinusoidal balanced compensation strategy) have been implemented in the control system. The experimental results corresponding to an unbalanced voltage supply are shown in Fig. 8. There, constant power compensation strategy presents a source current waveform [Fig. 8(a)] different from the sinusoidal waveform than that obtained applying the new strategy [Fig. 8(b)].

As reference, both waveforms total harmonic distortion measures are indicated. Thus, the corresponding to the constant power compensation is 16%, and the corresponding to the sinusoidal and balanced source current compensation is 6%. Besides, applying the last strategy, the source current after compensation is balanced. In any case, it is necessary to consider that these are experimental results where there is an unavoidable ripple due to the threshold band imposed by the pulsewidth modulation control.

V. Conclusion

The original *p*–*q* formulation has been analyzed. An equivalent development has been presented in phase coordinates, which allows compensation strategy in a simpler way than the corresponding to the original formulation to be obtained. The new development makes possible, in an easy way, alternative compensation objectives. Thus, the analysis of constant power compensation and constant power compensation eliminating neutral current have been carried out. On the other hand, alternative compensation strategies are derived that obtain balanced and sinusoidal source current in any supply voltage conditions. These new developments have been applied as simulation and experimental example to a three-phase four-wire power system, and the results are presented.

References

[1] H. Akagi, Y. Kanazawa, and A. Nabae, "Generalized theory of the instantaneous reactive power in three-phase circuits," in *Proc. IPEC*, Tokyo, Japan, 1983, pp. 1375–1386.

[2] H. Akagi, Y. Kanazawa, and A. Nabae, "Instantaneous reactive power compensators comprising switching devices without energy storage components," *IEEE Trans. Ind. Appl.*, vol. IA-20, no. 3, pp. 625–630, May/Jun. 1984.

[3] M. Depenbrock, V. Staudt, and H. Wrede, "A theoretical investigation of original and modified instantaneous power theory applied to four-wire systems," *IEEE Trans. Ind. Appl.*, vol. 39, no. 4, pp. 1160–1167, Jul./Aug. 2003.

[4] P. Salmeron, R. S. Herrera, and J. R. Vazquez, "Mapping matrices against vectorial frame in the instantaneous reactive power compensation," *IET—Elect. Power Appl.*, vol. 1, no. 5, pp. 727–736, Sep. 2007.

[5] A. Nabae, H. Nakano, and S. Togasawa, "An instantaneous distortion current compensator without any coordinate transformation," in *Proc. IEEJ IPEC*, Yokohama, Japan, 1995, pp. 1651–1655.

[6] F. Z. Peng and J.-S. Lai, "Generalized instantaneous reactive power theory for three-phase power systems," *IEEE Trans. Instrum. Meas.*, vol. 45, no. 1, pp. 293–297, Feb. 1996.

[7] J. L. Willems, "A new interpretation of the Akagi–Nabae power components for nonsinusoidal three-phase situations," *IEEE Trans. Instrum. Meas.*, vol. 41, no. 4, pp. 523–527, Aug. 1992.

[8] H. Akagi, S. Ogasawara, and H. Kim, "The theory of instantaneous power in three-phase four-wire systems: A comprehensive approach," in *Conf. Rec. IEEE IAS Annu. Meeting*, 1999, vol. 1, pp. 431–439.

[9] S. Bhattacharya, D. M. Divan, and B. Banerjec, "Syncronous frame harmonic isolator using active series filter," in *Proc. 4th Eur. Power Electron. Conf.*, 1991, pp. 30–35.

[10] V. Soares and P. Verdelho, "An instantaneous active and reactive current component method for active filtres," *IEEE Trans. Power Electron.*, vol. 15, no. 4, pp. 660–669, Jul. 2000.

[11] P. Salmerón and J. C. Montaño, "Instantaneous power components in polyphase systems under nonsinusoidal conditions," *Proc. Inst. Elect. Eng.—Sci. Meas. Technol.*, vol. 143, no. 2, pp. 151–155, Mar. 1996.

[12] P. Salmerón, J. C. Montaño, J. R. Vázquez, J. Prieto, and A. Pérez, "Practical application of the instantaneous power theory in the compensation of four-wire three-phase systems," in *Proc. IEEE 28th Annu. Conf. IECON*, Nov. 5–8, 2002, vol. 1, pp. 650–655.

[13] R. S. Herrera, P. Salmerón, and H. Kim, "Instantaneous reactive power theory applied to active power filter compensation: Different approaches, assessment, and experimental results," *IEEE Trans. Ind. Electron.*, vol. 55, no. 1, pp. 184–196, Jan. 2008.

[14] Z. Du, L. M. Tolbert, J. N. Chiasson, and B. Ozpineci, "Reduced switching-frequency active harmonic elimination for multilevel converters," *IEEE Trans. Ind. Electron.*, vol. 55, no. 4, pp. 1761–1770, Apr. 2008.

[15] A. Pigazo, V. Moreno, M. Liserre, and A. Dell'Aquila, "FBD-method based controller for shunt active power filters in marine vessels," in *Proc. IEEE ISIE*, Jun. 30–Jul. 2, 2008, pp. 2253–2257.

[16] K. L Wei, K. Dai, Q. Lei, D. Xiang, Y. Kang, F. Luo, and G. Zhu, "Performance analysis of three-phase three-wire shunt active power filter compensating for unbalanced loads," in *Proc. 33rd Annu. IEEE IECON*, Nov. 5–8, 2007, pp. 1875–1879.

[17] M. Abdusalam, P. Poure, and S. Saadate, "Study and experimental validation of harmonic isolation based on high selectivity filter for three-phase active filter," in *Proc. IEEE ISIE*, Jun. 30–Jul. 2, 2008, pp. 166–171.

Reyes S. Herrera was born in Huelva, Spain. She received the Industrial Engineering degree from the University of Seville, Seville, Spain, in 1995, and the Ph.D. degree in electrical engineering from the University of Huelva, Huelva, Spain, in 2007.

From 1996 to 2001, she was with Atlantic Copper Smelter, Huelva, Spain. Since 2001, she has been a Lecturer in the areas of electrotechnical and power electronics with the Department of Electrical Engineering, Escuela Politécnica Superior, University of Huelva. Currently, her research includes electrical power theory, electrical power systems, active power filters, and power conversion systems.

Patricio Salmerón was born in Huelva, Spain, in 1958. He received the Dr. degree from the Electrical Engineering Department, University of Seville, Seville, Spain, in 1993.

In 1983, he joined the University of Seville as an Assistant Professor, later becoming an Associate Professor in the Department of Electrical Engineering. Since 1993, he has been a Professor with the Escuela Politécnica Superior, University of Huelva, Huelva, where he is also currently Dean of the Escuela Politécnica Superior. He has joined various projects in connection with research on power theory of nonsinusoidal systems and power control in electrical systems. His research interests include electrical power theory, electrical power systems, active power filters, and power conversion systems.

AUTHOR QUERIES

AUTHOR PLEASE ANSWER ALL QUERIES

AQ1 = The acronym “APLCs” was defined as “active power line conditioners.” Please check if correct.
AQ2 = “Section 2” was deleted. Please check if OK.
AQ3 = Please provide the expanded form of the acronym “SRCs.”
AQ4 = All occurrences of “0,04” were changed to “0.04.” Please check if correct.

END OF ALL QUERIES