Universidad de Huelva

Departamento de Física Aplicada

Standard model extensions with massive neutrinos phenomenology and cosmological implications

Memoria para optar al grado de doctor
presentada por:

Pedro Naranjo Pérez

Fecha de lectura: 15 de julio de 2010

Bajo la dirección de los doctores:

José Rodríguez Quintero
Mario E. Gómez Santamaría
Smaragda Lola

Huelva, 2010

ISBN: 978-84-92944-28-6
D.L.: H 291- 2010

Standard Model Extensions with Massive Neutrinos

Phenomenology and Cosmological Implications

Dissertation by

Pedro Naranjo

Dpto. Física Aplicada, U. Huelva & Physics Dept., U. Patras

In partial fulfillment for the requirements

for the Degree of

Doctor of Philosophy

in the field of Physics

Supervisors:

Prof. M.E. Gómez,

Prof. S. Lola &

Prof. J. Rodríguez Quintero

The author strongly acknowledges the FP6 Marie Curie Excellence Grant MEXT-CT-2004-014297 for financial support

July 2010

Acknowledgements

The completion of a PhD thesis requires combined efforts from many people. As it is not possible to do justice to everyone, I will try to refer explicitly to those people who *must* be included.

First of all, I am very grateful to my three supervisors, hereafter cryptically referred to as *Ahab, Starbuck* and *kiria Lola.* To start with, Ahab and Starbuck gave me the opportunity to join their group. Throughout these years, they have tried hard to put me at my ease with programming. Well, they have absolutely failed! As regards of kiria Lola, with whom I got in touch thanks to my Moby-Dickian supervisors, I can but highly appreciate her care during my stay in Greece. Put it simpler: she has been my Greek mom. Needless to say, had it not been for them all, this dissertation would surely have not been accomplished.

Secondly, I would like to show my gratitude to Ivan, Pierre, Nikola and Costas for enlightening discussions (not necessarily about physics, though!) as well as for making the uncountable hours at the department much easier (even funny!).

Last, but not least, I would like to pay tribute to those two people who are always beside me (even when my rebelliousness faces them): my parents.

A thing is always something
that has such and such properties,
always something that is constituted
in such and such a way.
This something is the bearer of the properties;
the something, as it were,
that underlies the qualities

There are a number
of professional scientists
who have seen thousands of trees
but have never seen a forest

Why should anyone
join a group whose very rules
do nothing
but threaten freedom?

To my parents

Contents

List of Figures

List of Tables

Preface

The Standard Model (SM) of particle physics can be considered as a very successful theory in describing the plethora of phenomena taking place at energies up to ~ 200 GeV. It constitutes the triumph of a class of theories collectively referred to as *local gauge theories*. These are concrete realizations of the broader family of Quantum Field Theories (QFT), a remarkable scientific achievement whose predictive power stands at the top of physical theories constructed to date, Quantum ElectroDynamics (QED) being the prototype of these theories.
The foundations of a systematic QFT were laid by Dirac in 1927 in his celebrated paper on "The Quantum Theory of the Emission and Absorption of Radiation". From the quantisation of the electromagnetic field one is naturally led to the quantisation of any classical field, the quanta of the field being particles possessing well-defined properties. The interaction of these particles is brought about by other fields whose quanta are other particles. This way, one can think of the interaction between electrically charged particles, such as electrons and positrons, as being brought about by the electromagnetic field or as due to an exchange of photons. The electrons and positrons themselves can be thought of as different states of the electron-positron field.
However, it soon became clear that obtaining the *exact* solutions of the equations of quantum interacting fields was a forbidding task. As a result, only if the interaction happens to be sufficiently weak, one can employ perturbation theory, which in turn has been outstandingly successful when applied to particle physics. In this respect, the most prominent modern perturbative technique applies the intuitive approach put forward by Feynman, who first realized the one-to-one correspondence between the diagrammatic representation of a physical interaction and the terms arising in the perturbation series solution for collision processes. This framework was developed originally by Dyson and further improved by Wick. (For a pedagogical account of the Feynman approach, with many illuminating examples, see [Peskin and Schroeder, 1995]. For a more rigorous approach, see [Itzykson and Zuber, 2006]). Having arrived to such an achievement in the context of the electromagnetic field, physicists started applying the same techniques to the rest of the fundamental interactions. It became clear that weak interactions were also tackled by perturbative techniques, as it turned out in the nuclear β-decay.[1] The original theory, known as Intermediate Vector Boson (IVB) theory, suffered from a major difficulty, namely that it gave rise to infinities when comput-

[1] The perturbative approach fails when applied to the low-energy region of the strong interactions, as the coupling constant is not weak in that regime. We shall be commenting on this point later.

ing physical observables, once quantum corrections were taken into account. More precisely, IVB was *non-renormalisable*, *i.e.*, there was no way of cancelling out divergent terms in the expansion, due to the propagator of the massive bosons, by a *finite* number of additional constants. The problem was solved by 't Hooft ['t Hooft, 1971, 't Hooft and Veltman, 1972], who demonstrated that any ¡ *gauge* theory is renormalisable. Thus, the task was to formulate a gauge theory of weak interactions.
Generally, a theory is called *gauged* if any *local* transformation, namely one involving space-time-dependent parameters, leaves such a theory invariant. It was E. Noether who first realized the generality of the idea of "gauging" (making local) symmetries. The gauging procedure in its modern form was formulated by H. Weyl in the 1920's. The extension of gauge theories to non-Abelian theories, such the electro-weak theory and Quantum ChromoDynamics (QCD), was first made by Yang and Mills [Yang and Mills, 1954]. Soon, a gauge-invariant electroweak interaction was proposed; however, in its original form, all particles were massless. Thus, people first tried an *ad hoc* addition of mass terms to the Lagrangian. This was not successful, as such mass terms spoiled the gauge invariance of the theory. Thus, a mechanism for mass generation which retained gauge invariance was required. Such a mechanism is referred to as *Spontaneous Symmetry Breaking* (SSB) [2]. The essential feature of a spontaneously broken symmetry is the fact that the vacuum state does not exhibit the same symmetries as the Lagrangian. This difference arises due to scalar fields, generally known as *Goldstone bosons*, acquiring non-zero vacuum expectation values (VEV). The application of SSB to the electroweak theory led to the so-called *Higgs mechanism* for mass generation of both charged-leptons and the $W^{\pm}$, Z^0 gauge bosons, by invoking a massive, fundamental scalar field (known as the *Higgs boson*). The resulting theory is known as the *Standard Electroweak Theory*, independently developed by Weinberg, Salam and Glashow [Weinberg, 1967, Glashow, 1961], which triumphed when the predicted properties of the $W^{\pm}$, Z^0 gauge bosons were experimentally confirmed by the group headed by C. Rubbia at CERN. In this respect, it is worth noting that the prediction of the Higgs boson remains the only untested property of the SM.[3]
Let us comment a bit on the strong interactions. As noted, standard perturbative techniques are not suitable in the low-energy regime, where it is believed that quarks and gluons form hadrons. This is essentially a consequence of the fact that the strong coupling constant *increases* when quarks are far apart, a phenomenon referred to as *confinement*. The computation of observables in this regime remains a challenge. In this respect, *Lattice Gauge Theories* have proven to be very useful (see [Montvay and Munster, 1997] for a nice account). On the other hand, as shown by Gross, Wilczek and Politzer [Gross and Wilczek, 1973a, Gross and Wilczek, 1973b, Politzer, 1973], the strength of the interaction asymptotically tends to zero for very high energies, a feature known as *asymptotic freedom*. In this high-energy regime, standard perturbation theory does hold, as shown by the parton model, which has been successfully tested in deep inelastic scattering in hadron physics.

[2] An analogy comes from solid state physics. The laws describing ferromagnetic materials are invariant under spatial rotations, the magnetisation being the order parameter and the Curie temperature dictating whether or not there is symmetry breaking.

[3] This lack of experimental evidence for the Higgs boson has lately led physicists to build alternative versions of the SM, which do not require the existence of a Higgs particle. These theories are collectively known as *Little Higgs models* [Arkani-Hamed et al., 2001b, Arkani-Hamed et al., 2001a, Arkani-Hamed et al., 2002].

Therefore, we are left with the Standard Model of particle physics, a mathematical framework that encompasses both the Weinberg-Salam electro-weak theory and QCD for the strong interactions. As mentioned at the beginning of this Introduction, the SM possesses a very impressive predictive power, having described very accurately most non-gravitational phenomena to date.

Nevertheless, several open questions indicate that the SM cannot be the ultimate theory. On one hand, fundamental facts related to gravity, are not accommodated within the SM framework. On the other hand, there is now compelling evidence for several phenomena which lie beyond the SM frontiers, like neutrino masses or the mysterious Dark Matter and Dark Energy. For those reasons, physics beyond the SM is expected to exist.

Although the main research developed in this work focuses on the realm of particle physics, it inevitably takes into account the very important interaction between elementary particles and cosmology, since cosmological issues find a plausible explanation in terms of particle physics concepts. Thus, we will also be discussing the main features derived by our current understanding of cosmology.

In cosmological physics there also exists a paradigm, Standard Cosmology (SC), which stands on solid grounds when describing the observed Universe. Cosmic Microwave Background Radiation (CMBR) and Big Bang Nucleosynthesis (BBN) are among its most prominent triumphs. This picture is supported by recent observational data that provides stringent bounds on several cosmological parameters, mostly coming from the Wilkinson Microwave Anisotropy Probe (WMAP), the Sloan Digital Sky Survey (SDSS) and the 2dF Galaxy Redshift Survey (2dFGRS).

The SC is also known as the Hot Big Bang model, that claims that the Universe originated in a highly dense and hot explosion, after which everything we know came into existence. It was Alexander Friedman, in the early 1920's, who first derived a set of equations from Einstein General Relativity, and thought that the Universe might indeed be expanding, as opposed to the static model advocated until then. Soon after, in 1924, Edwin Hubble made the crucial observation that distant galaxies were receding from Earth. Further theoretical understanding came after Georges Lemaitre found independently, in 1927, the same set of equations implying the expansion of the Universe.

By 1950's, George Gamow further developed the Hot Big Bang model, who introduced the idea of Big Bang Nucleosynthesis (BBN) as a mechanism for producing light nuclei which eventually would form every light element. Two collaborators of Gamow, Alpher and Herman, predicted the Cosmic Microwave Background Radiation (CMBR), the radiation left over after the bang. The observational discovery of this radiation in 1964 by Penzias and Wilson strengthened the Big Bang model as the correct explanation for the evolution of the cosmos. Major refinements in telescope technology enabled the construction of more precise devices, such as the COsmic Background Explorer (COBE) and the Hubble Space Telescope (HST) in the 1990's, further constraining the allowed range of parameters. As stated above, the WMAP, the SDSS and the 2dFGRS cosmological probes (launched at the beginning of this millennium with the hope of shedding insights on the cosmological scenario) are revolutionising the field.

However, the Big Bang model clearly fails in accounting for, at least, two cosmological observations: Firstly, most of the matter in the Universe is not made of the ordinary particles (protons, neutrons and electrons) we know of. Secondly, cosmologists are trying to make sense of an unexpected discovery in 1998, when two teams claimed that Standard Supernovae data implied that the Universe expansion is accelerating. The former is referred to as the *Dark Matter problem*, while the latter is widely known as the *Dark Energy problem* [4] and they are two major challenges for our understanding of the deepest secrets of Nature.

In addition to these observational drawbacks, the SC scenario suffers from several difficulties concerning its basic structure. In particular, it accounts rather successfully for the observed Universe, *provided* some key assumptions are made. In this respect, although the observational data is nicely described by a Robertson-Walker-like metric, the Standard scenario says nothing about the origin of the large-scale isotropy and homogeneity, nor does it address the question of what happened in the earliest stages of the Universe.

One can find a very rich literature dealing with physics beyond the Standard Model of elementary particles. In this work, we will be focusing on supersymmetric aspects. Ideas concerning extra dimensions [Antoniadis, 1990, Antoniadis et al., 1998, Arkani-Hamed et al., 1999, Arkani-Hamed et al., 1998, Randall and Sundrum, 1999a, Randall and Sundrum, 1999b] or composite Higgs models are also very attractive. Supersymmetry (SUSY) has proven to be one of the most appealing extensions, since it is able to solve a number of deficiencies which the SM suffers from, such as the hierarchy problem. Moreover, combining SUSY with Grand Unified Theories (GUT) predicts the gauge coupling unification when extrapolating the observed values at low energies up to the GUT scale. Furthermore, SUSY provides solutions to some cosmological open issues, for it possesses nice Dark Matter candidates (which are, however, highly model-dependent), as well as new sources for baryon number violation, which may help clarify the matter-antimatter asymmetry.

In addition to the above nice characteristics, SUSY, when promoted to a local symmetry, leads to a theory of gravity, the so-called supergravity (SUGRA) theory (see [Nilles, 1984] for a thorough account of both SUSY and SUGRA. The mathematically inclined reader is referred to [Van Nieuwenhuizen, 1981] for a technical account of SUGRA). Fortunately, SUGRA not only conserves the desired features of the global case, but also provides some insights about a more fundamental theory, since SUGRA GUT models impose several constraints to model parameters, making such models very predictive.

Nevertheless, SUGRA cannot be the ultimate theory either and should thus be thought of as a low energy effective theory. Like any quantum theory based on point-like objects, that attempts to describe gravity, SUGRA suffers from dangerous non-renormalisations properties, which make SUGRA meaningless when approaching the gravitational scale, the so-called Planck scale, placed at $\sim 10^{19}$ GeV. This undesired fact led to postulate the existence of one-dimensional fundamental objects called strings, thus opening a very vast area of research that is still very active (see [Green et al., 1988, Polchinski, 2005] for a comprehensive discussion).

From the cosmological side, most of the difficulties inherent to the standard scenario are

[4] It must be stressed that *Dark Energy* is a possible *explanation* for the evidence, namely the accelerating expansion, and not the evidence itself. Despite being the most popular scenario within particle-physics, alternative ideas have been proposed, mainly invoking corrections to gravity.

solved within the *inflationary* scenario(see [Linde, 1990, Lyth and Liddle, 2009] for thorough introductions to inflationary cosmology), which basically invokes a huge accelerating phase at the very early Universe. Inflation explains those drawbacks nicely, and has become a paradigm in modern cosmology. Nevertheless, it still lacks an embedding into a realistic model.

The work in this dissertation is embedded within the framework of SUSY-GUT theories with massive neutrinos. The general scope is the analysis of Yukawa unification when both cosmological (mostly Dark Matter) and experimental constraints (including Lepton Flavour Violation) are properly taken into account. As such, this work is organised as follows. Chapter 1 provides an overall account of the current status of the SM as well as SC. It should be emphasised that the main research developed in this work deals with plausible scenarios beyond the SM. Thus, in the next chapter we shall be commenting on those issues that are not easily accommodated within the SM framework, together with the ones suggesting that new physics may be at work at a deeper level than that of the SM.

Regarding Standard Cosmology, since our research is linked to cosmology mainly through the Dark Matter problem, the discussion in the next chapter is intended to motivate the particle physics-cosmology interface, though no further insights will be offered, nor will any of the ideas pursued in this work be relying upon cosmological issues.

We shall be moving on Chapter 2 to briefly describe the experimental evidence for new physics as well as to motivate the ideas put forward for physics beyond the SM, focusing on those directly related to the present work. Thus, supersymmetric gauge field theories will be discussed in some detail, as one of the most promising directions towards new physics. Among others, supersymmetric extensions of the SM succeed in stabilising the electroweak scale against quantum corrections. In addition, gauge coupling unification is achieved to a great accuracy within supersymmetric theories, shedding new insights on the idea of GUT.

Along these lines, the idea of unification will also be addressed, as an appealing way to correlate in a fundamental manner the seemingly quite dissociated properties of elementary particles. As stated in the previous paragraph, Grand Unified Theories enhanced with supersymmetry (SUSY-GUT) may help towards providing a better understanding of Nature.

Having introduced the background required for this work, we shall be focusing in Chapter 3 on the specific topics within the context of SUSY-GUT models that are the focus of our research, namely phenomenological issues within Yukawa-unified models enhanced with massive neutrinos. Firstly, we will discuss one of the most popular mechanisms to account for the smallness of neutrino masses, known as the *see-saw mechanism.* Next, we shall comment on the embedding of neutrino masses within a unified framework taking all fermion mass bounds into account. The direction taken in this work makes use of *family symmetries*, which assume that the extremely hierarchical structure of fermion masses arises from the breaking of a hidden symmetry.

Next, we will address the implications of Yukawa-unified scenarios for Dark Matter. To this end, we will discuss the main candidates that SUSY-GUT models possess so as to provide an explanation, not only for the existence, but mostly for the dominance of Dark Matter over ordinary matter. For the sake of simplicity, we shall be addressing the topic within R-parity conserved models, whose most natural Dark Matter candidate is the Lightest Supersymmet-

ric Particle (LSP), which in this model happens to be the lightest *neutralino*.
As a final issue, searching for physics footprints beyond the SM of particle physics naturally leads to the study of interactions that violate Lepton Flavour, which are suppressed in the SM. Thus, any evidence for such these processes would be a major step towards the understanding of physics at the fundamental level. Our work is mainly focusing on the processes $\ell_i \to \ell_j \gamma$, which is the subject of intensive experimental study.
Having described to some extent the topics directly related to the research put forward in this work, we will present our results in Chapters 4 and 5 for the specific issues at hand. We shall first be arguing in Chapter 4 that lepton mixing arising in models with massive neutrinos enhances the allowed parameter space in bottom-tau Yukawa-unified scenarios, making it possible to consider a broader range for $\tan\beta$ as well as both signs for the μ parameter. Along these lines, we shall next be discussing the prediction for Dark Matter abundance assuming a neutralino LSP. We will show that the above scenario, compatible with low-energy constraints, allows a larger parameter space satisfying the striking WMAP Dark Matter constraints.
Equipped with these results, we shall end our discussion by showing in Chapter 5 that $b-\tau$ Yukawa-unified models with massive neutrinos may shed some light on the pattern of fermion masses once a family symmetry is also included. For definiteness, we shall be invoking an $U(1)_F$ symmetry that reproduces all low-energy data. In addition, it will be argued that the predictions for LFV rates may lie within current experimental bounds if the quantum corrections above the unification scale are properly taken into account, hence rendering the $SU(5)$ $b-\tau$ Yukawa-unified model feasible.
As a natural extension of this analysis, this chapter will end with a discussion of work in progress, namely LFV within the context of *left-right symmetric* models, whose structure, as argued, is compatible with a significantly larger supersymmetric parameter space.
Finally, Chapter 6 is devoted to draw the main conclusions inferred from our work as well as some future prospects.

Part I

Introduction

Chapter 1

Current Status of the Standard Model of Particle Physics and Standard Cosmology

We feel that any idea put forward must be properly motivated, making it clear why one should consider it in the first place. This chapter aims at fulfilling such a purpose. This dissertation deals with physics beyond the standard model of particle physics, thus one must explain what this model suffers from, along with realistic proposals to circumvent the problems.

The structure of this chapter is as follows[1]. We first discuss the main features of the standard model of particle physics, showing that it stands quite impressively when confronted with experimental data at energy scales up to $\simeq 200$ GeV. This may suggest an underlying structure of which the SM is a low energy effective theory. We move on to briefly comment on the main failures of the SM and the most promising ideas intended to address them.

Subsequently, we discuss the main concepts of the standard model of cosmology. This as well, is very successful in describing a wide range of astrophysical and cosmological phenomena. In the cosmological realm, problematic physics arises when going backwards in time to the very early Universe, an epoch when gravitational physics is expected to play a major role. This is essentially the same situation as the one in the SM of particle physics, since dealing with the infancy of the Universe amounts to facing very high energy phenomena. In this respect, an appealing interface between particle physics and cosmology naturally emerges.

Finally, we shall briefly discuss the most important ideas put forward to face the major drawbacks of the standard cosmology, related to the striking homogeneity and isotropy of the CMBR and the mysterious nature of the dark matter and the dark energy.

[1] As this dissertation mostly deals with particle physics, the discussion of physics beyond the SM deserves a chapter on its own and is deferred to chapter 2.

1.1 Standard Model of Particle Physics

1.1.1 Overview

The Standard Model provides an excellent account of *almost* all non-gravitational physics up to energies $\sim$ 200 GeV. Mathematically, it is an example of a local or *gauged* quantum field theory. Its main technical features are the following:

- *Gauge invariance.* It means that physics remains unchanged under any transformation related to the gauge group.
- *Lorentz invariance.* It refers to the non-existence of any preferred frame in describing the physical laws.
- *Renormalisability.* It accounts for the fact that the strengths of the interactions depend on energy.
- *Non-chirality.* Asymmetry in the behaviour of *left-* and *right*-handed states.

The SM gauge group is $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, where C stands for *colour* (internal degree of freedom of quarks and gluons), L for *left-handed* and Y for the quantum number called *weak hypercharge*. The matter field content is displayed in Table 1.1.

Field	$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ quantum numbers
Q	$(3,2,\frac{1}{6})$
L	$(1,2,-\frac{1}{2})$
u^c_R	$(3,1,-\frac{2}{3})$
d^c_R	$(3,1,-\frac{1}{3})$
e^c_R	$(1,1,1)$

Table 1.1: Matter content within the SM.

From a phenomenological viewpoint, the SM has succeeded in correctly predicting the following observables:

- *Existence and masses of the weak $W^\pm$ and Z^0 bosons.*
- *Existence of the coloured gluon bosons, which are massless.*
- *Existence and masses of the charm and top quarks.*
- *Prediction of the asymptotic freedom for the strong interaction, successfully confirmed in hadron physics by measurements of α_s.*

Thus, the SM description of the three interactions it accommodates relies upon the exchange of the so-called *gauge bosons*, which are spin-1 fields, as displayed in Table 1.2.

Interaction	Strength	Symmetry (massless case)	Symmetry (massive case)	Gauge Boson
Strong	1	$SU(3)_C$	$SU(3)_C$	Gluons
Electromagnetic	10^{-2}	$U(1)_Y$	$U(1)_{EM}$	Photon
Weak	10^{-13}	$SU(2)_L$	No symmetry	$W^{\pm}$, Z^0

Table 1.2: Fundamental interactions within the SM.

Next, we will briefly discuss some key issues regarding the mathematical structure of the SM, namely the role played by gauge invariance and spontaneous symmetry breaking (which, as stated in the Preface, ensures the existence of massive fields).

1.1.2 Gauge invariance

We shall analyse the symmetry aspects of the SM. We will first address the Abelian case (exemplified by Quantum ElectroDynamics (QED)) and next the non-Abelian case (appearing in the Weak Interactions and in Quantum ChromoDynamics (QCD)).

Abelian Case

Electrodynamics was the first of the gauge field theories. It illustrates very nicely how symmetry aspects are incorporated in particle physics.
Consider a spin-1/2, positively charged fermion represented by the field ψ. The classical Lagrangian, describing its electromagnetic properties, is

$$\mathcal{L}_{\text{CED}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}\left(i\not{D} - m\right)\psi\,, \tag{1.1}$$

where $\not{D} \equiv \gamma_\mu D^\mu$, $D_\mu\psi = (\partial_\mu + ieA_\mu)\,\psi$ stands for the *covariant* derivative, m and e are the mass and the electric charge of ψ, respectively, A_μ is the gauge field for electromagnetism and $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the gauge-invariant field strength. This Lagrangian is invariant under the local (Abelian) $U(1)$ transformations

$$\psi\,(x) \to\ e^{-i\alpha(x)}\psi\,(x)\ , \tag{1.2}$$

$$A_\mu\,(x) \to\ A_\mu\,(x) + \frac{1}{e}\partial_\mu\alpha\,(x)\ . \tag{1.3}$$

The associated equations of motion are the Dirac equation

$$\left(i\not{\partial} - m - e\not{A}\right)\psi = 0\,, \tag{1.4}$$

and the Maxwell equation

$$\partial_\mu F^{\mu\nu} = e\bar{\psi}\gamma^\nu\psi\,. \tag{1.5}$$

Let us make a few remarks on $U(1)$ gauge invariance.

- *Universality of electric charge.* It is worth noting that $U(1)$ gauge invariance also holds for a fermion with charge parameter βe, with β any arbitrary integer. Thus, $U(1)$ symmetry says nothing about the seemingly equality between electron and proton electric charges.

- *Quantum Lagrangian.* The extension of (1.1) to the quantum realm requires the promotion of functions to operators. Moreover, it turns out that this quantum version is the most general Lorentz-invariant, hermitian and renormalisable which is also $U(1)$ invariant.

- *Renormalisability.* $U(1)$ symmetry, by itself, would admit a larger set of interaction terms than the one displayed in (1.1). However, such contributions are in turn ruled out by renormalisability, as they have dimension $d \leq 4$.

- *Massless gauge boson.* Note that no contribution corresponding to a gauge boson mass is allowed by the $U(1)$ symmetry. Such a term would be proportional to $A^\mu A_\mu$, which is not invariant under the gauge transformation, Eq. (1.3).

Non-Abelian Case

Chromodynamics is the $SU(3)$ non-Abelian gauge theory of *colour* charge. The fermions which carry colour charge are the *quarks*, each with field q_j^α, where $\alpha = u,\, d,\, s,\, \ldots$ is the flavour label and $j = 1,\, 2,\, 3$ is the colour index. The gauge bosons, which also carry colour, are the *gluons*, each with field G_μ^a, $a = 1,\, \ldots,\, 8$. Classical chromodynamics is defined by the following Lagrangian:

$$\mathcal{L}_{\mathrm{CCD}} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \sum_\alpha \bar{q}_j^\alpha \left(i\not{D}_{jk} - m^\alpha \delta_{jk} \right) q_k^\alpha \,, \tag{1.6}$$

where the repeated colour indices are summed over. The gauge field strength tensor is now

$$F_{\mu\nu}^a \equiv \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_3 f^{abc} G_\mu^b G_\nu^c \,, \tag{1.7}$$

with g_3 being the $SU(3)$ gauge coupling parameter and f^{abc} the structure constants of $SU(3)$. The quark covariant derivative takes the form

$$\mathbf{D}_\mu\, q = \left(\partial_\mu + i g_3 G_\mu^a \frac{\lambda_a}{2} \right) q \,. \tag{1.8}$$

where $\frac{\lambda_a}{2}$ are the $SU(3)$ group generators. The Lagrangian of Eq. (1.6) is invariant under local $SU(3)$ transformations of the colour degree of freedom, namely

$$q_j^\alpha\,(x) \to \mathbf{U}\,(\beta)\; q_j^\alpha\,(x) \;, \tag{1.9}$$

where $\mathbf{U}$ is a 3×3 matrix in colour space, $\beta = \beta\,(x)$ are the local $SU(3)$ parameters, and

$$G_\mu^a \to \mathbf{U}_a G_\mu^a \mathbf{U}_a^{-1} + \frac{i}{g_3} \partial_\mu \mathbf{U}_a \cdot \mathbf{U}_a^{-1} \,. \tag{1.10}$$

Note that, like in the case of $U(1)$ invariance, the gauge bosons corresponding to $SU(3)$ symmetry (gluons) are massless, as no gauge mass term is allowed by Eq. (1.10).
The equations of motion of the quark and gluon fields are

$$\left(i\not{D} - m^{\alpha}\right) q^{\alpha} = 0\,, \tag{1.11}$$

$$D^{\mu} F^{a}_{\mu\nu} = g_3 \sum_{\alpha} \bar{q}^{\alpha} \frac{\lambda_a}{2} \gamma_{\nu} q^{\alpha}\,. \tag{1.12}$$

A hallmark of QCD is *asymptotic freedom*, which, unlike electromagnetism, reveals the phenomenon of *confinement*, *i.e.*, the fact that quarks cannot exist in isolation at low energies, $170 < \Lambda < 360$ MeV.
As a final remark, let us mention that, unlike the case of QED, the most general QCD Lagrangian *does* include an additional term which cannot be reduced to any contribution in 1.6. This is the so-called θ-term, a CP-violating contribution that would in principle lead to a non-vanishing neutron electric dipole moment, but to date no signal has been observed experimentally.

1.1.3 Electroweak theory

We have seen thus far how symmetry aspects can be successfully implemented in particle physics. In particular, both electromagnetic and strong interactions are nicely described by symmetry principles. Yet, there is one fundamental force, included within the framework of the Standard Model of particle physics, which demands another requirement: electroweak interactions add a new concept, that of *spontaneous symmetry breaking*. The reason is simply the fact that electroweak gauge invariance is lost when *chirality* is introduced.
The Weinberg-Salam-Glashow (WSG) model of electroweak interactions is a gauge theory whose fermionic degrees of freedom are *massless* spin-1/2 chiral particles. Its group structure is $SU(2)_L \otimes U(1)_Y$, where the former stands for *weak isospin*, while the latter for *weak hypercharge*. The subscript "L" indicates that, among fermions, only left-handed states transform non-trivially under weak isospin.

Weak isospin and weak hypercharge assignments

Initially, we shall discuss how the fermionic weak isospin (T_W, T_{W3}) and weak hypercharge (Y_W) quantum numbers are assigned. The fermion generations are taken to obey a "template" pattern (we assume that each succeeding generation differs from the first only in mass). Thus, it will suffice to consider only the lightest fermions for the remainder of this discussion. The electroweak assignments corresponding to the first generation are displayed in Table 1.3.

For weak isospin, experience indicates that left-handed fermions belong to weak isodoublets,

Particle	T_W	T_{W3}	Y_W
ν_e	1/2	1/2	-1
e_L	1/2	-1/2	-1
e_R	0	0	-2
u_L	1/2	1/2	1/3
d_L	1/2	-1/2	1/3
u_R	0	0	4/3
d_R	0	0	-2/3

Table 1.3: $SU(2)_L \otimes U(1)_Y$ assignments.

while right-handed ones are placed in weak isosinglets, as in

$$\begin{aligned} \text{leptons :} \quad \ell_L \equiv \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad & e_R\,, \\ \text{quarks :} \quad q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \quad & u_R,\, d_R\,. \end{aligned} \tag{1.13}$$

Each of the degrees of freedom shown above must be assigned a weak hypercharge. There are *a priori* five in all,[2]

$$Y(\ell_L) \equiv Y_\ell\,,\ Y(q_L) \equiv Y_q\,,\ Y(e_R) \equiv Y_e\,,\ Y(u_R) \equiv Y_u\,,\ Y(d_R) \equiv Y_d\,. \tag{1.14}$$

Shortly, we shall see that the $\{Y_i\}$ are related to the fermion electric charges. One might therefore attempt to proceed by using the latter to determine the former. However, given the weak isospin assignments already made, it turns out to be both possible and worthwhile to use the theoretical apparatus of the WSG model to *predict* the $\{Y_i\}$.

Several constraints can be obtained by employing the anomaly cancellation conditions. We shall not discuss this phenomenon here, but rather use it to impose relationships between the $\{Y_i\}$. In particular, the conditions

$$\begin{aligned} \mathrm{Tr}\, F_3^2 Y_W &= 0\,, \\ \mathrm{Tr}\, T_{W3}^2 Y_W &= 0\,, \\ \mathrm{Tr}\, Y_W^3 &= 0\,, \end{aligned} \tag{1.15}$$

which arise from anomaly cancellation requirements, imply the following:

$$\begin{aligned} 2Y_q &= Y_u + Y_d\,, \\ Y_q &= -\frac{1}{3} Y_\ell\,, \\ 6Y_q^3 - 3Y_u^3 &= 3Y_d^3 - 2Y_\ell^3 + Y_e^3\,, \end{aligned} \tag{1.16}$$

[2] The reason that weak hypercharge is related to so many free parameters, in contrast to weak isospin, lies in the difference between an Abelian gauge structure (like weak hypercharge) and a non-Abelian one (like weak isospin). All doublets have the same weak isospin properties irrespective of their other properties, analogous to flavour independence in QCD. For weak hypercharge, the group structure by itself provides no guidelines for assigning the weak hypercharge quantum number. Like the electric charge, it is *a priori* an arbitrary quantity.

where, in the first equation in (1.15), F_3 stands for the third generator of the octet of colour charges, and "Tr" represents a sum over fermions.
To obtain further constraints, we note the linear relation between the electric charge Q and the $SU(2)_L \otimes U(1)_Y$ quantum numbers T_{W3} and Y_W,

$$aQ = T_{W3} + bY_W \,, \tag{1.17}$$

where a, b are constants. We can use the freedom in assigning the scale of the electric charge Q to choose $a = 1$. Ultimately, of course, the left- and right-handed components of the charged chiral fermions must unite to form the physical states themselves. Consistency demands that the electric charges of the chiral components of each such charged fermion be the same, whatever value that charge might have. From this, we learn

$$Y_q = Y_u - \frac{1}{2b} \,, \; Y_q = Y_d + \frac{1}{2b} \,, \; Y_\ell = Y_e + \frac{1}{2b} \,. \tag{1.18}$$

Thus, all products bY_i (*cf.* Eq. (1.17)) are determined, resulting in the observed electric charge values.
Let us make a couple of remarks regarding the above analysis. First, it reveals that, once the weak isospin is chosen as in Eqs. (1.13), (1.14) and all possible chiral anomalies are arranged to cancel, one obtains the observed quantisation of electric charge. Also, we see that any attempt to determine weak hypercharge values from the known fermion electric charges is affected by an arbitrariness associated with the value of b. This accounts for the variety of conventions seen in the literature. For definiteness, we have taken $b = 1/2$ in Table 1.3.

$SU(2)_L \otimes U(1)_Y$ gauge-invariant Lagrangian

Having assigned quantum numbers, we turn next to the electroweak interactions. The WSG Lagrangian divides naturally into three additive parts, gauge (G), fermion (F) and Higgs (H):

$$\mathcal{L}_{\text{WSG}} = \mathcal{L}_{\text{G}} + \mathcal{L}_{\text{F}} + \mathcal{L}_{\text{H}} \,. \tag{1.19}$$

The gauge boson fields which couple to the weak isospin and weak hypercharge are W_μ, $i = 1, 2, 3$ and B_μ, respectively. These contribute to the purely gauge part as

$$\mathcal{L}_{\text{G}} = -\frac{1}{4} F_i^{\mu\nu} F^i_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \,, \tag{1.20}$$

where the $SU(2)_L$ and the $U(1)_Y$ field strength tensors are,

$$F^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_2 \epsilon^{ijk} W^j_\mu W^k_\nu \,, \tag{1.21}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \,. \tag{1.22}$$

The fermionic sector includes both the left- and right-handed chiralities. Summing over left-handed weak isodoublets ψ_L and right-handed weak isosinglets ψ_R, we have

$$\mathcal{L}_{\text{F}} = \sum_{\psi_L} \bar{\psi}_L i \not{D} \psi_L + \sum_{\psi_R} \bar{\psi}_R i \not{D} \psi_R \,. \tag{1.23}$$

It is understood in Eq. (1.23) that, when applied to quark fields, a colour sum must be included to account for every quark internal degree of freedom.
Since right-handed chiral fermions do not couple to weak isospin, their covariant derivative has the simple form

$$D_\mu \psi_R = \left(\partial_\mu + i\frac{g_1}{2} Y_W B_\mu\right) \psi_R \,. \tag{1.24}$$

This expression serves to define the $U(1)_Y$ gauge coupling g_1. Its normalisation is dictated by our convention for weak hypercharge Y_W. The corresponding covariant derivative for the $SU(2)_L$ doublet ψ_L is

$$D_\mu \psi_L = \left(\mathbf{I}\left(\partial_\mu + i\frac{g_1}{2} Y_W B_\mu\right) + ig_2 \frac{\tau}{2} W_\mu\right) \psi_L \,, \tag{1.25}$$

where g_2 is the $SU(2)_L$ gauge coupling, τ stands for the 2×2 Pauli matrices and $\mathbf{I}$ is the 2×2 identity matrix. (Similarly, Eqs. (1.24), (1.25) hold for each distinct internal colour state when applied to quark fields).
The above equations define a mathematically consistent gauge theory of weak isospin and weak hypercharge. However, it is not a physically acceptable electroweak theory of Nature because the fermions and gauge bosons are massless. A Higgs sector must be added to arrive at the full WSG model. Thus, we introduce into the theory a complex doublet,

$$\mathbf{\Phi} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} , \tag{1.26}$$

of spin-0 (scalar) Higgs fields with electric charge assignments as indicated. The quanta of these fields each carry one unit of weak hypercharge. The Higgs Lagrangian $\mathcal{L}_{\mathrm{H}}$ is the sum of two kinds of terms, $\mathcal{L}_{\mathrm{HG}}$ and $\mathcal{L}_{\mathrm{HF}}$, which contain the Higgs-gauge and Higgs-fermion couplings, respectively. The former is written as

$$\mathcal{L}_{\mathrm{HG}} = \left(D^\mu \mathbf{\Phi}\right)^\star D_\mu \mathbf{\Phi} - V\left(\mathbf{\Phi}\right) \,, \tag{1.27}$$

where

$$D_\mu \mathbf{\Phi} = \left(\mathbf{I}\left(\partial_\mu + i\frac{g_1}{2} B_\mu\right) + ig_2 \frac{\tau}{2} W_\mu\right) \mathbf{\Phi} \,, \tag{1.28}$$

and V is the Higgs self-interaction,

$$V\left(\mathbf{\Phi}\right) = -\mu^2 \mathbf{\Phi}^\dagger \mathbf{\Phi} + \lambda \left(\mathbf{\Phi}^\dagger \mathbf{\Phi}\right)^2 \,. \tag{1.29}$$

The parameters μ^2 and λ are positive, but otherwise arbitrary. For simplicity, as at the beginning of this discussion, we just consider the first generation of fermions, hence write the Higgs-fermion interaction as

$$\mathcal{L}_{\mathrm{HF}} = -f_u \bar{q}_L \tilde{\mathbf{\Phi}} u_R - f_d \bar{q}_L \mathbf{\Phi} d_R - f_e \bar{\ell}_L \mathbf{\Phi} e_R + \mathrm{hc} \,, \tag{1.30}$$

where the couplings constants f_i are arbitrary and we employ the charge conjugate to $\mathbf{\Phi}$,

$$\tilde{\mathbf{\Phi}} \equiv i\tau_2 \mathbf{\Phi}^\star \,, \tag{1.31}$$

with τ_2 a Pauli matrix.
There is no term in Eq. (1.30) containing a right-handed neutrino because we assume such

particles do not exist. In a sense, the Higgs potential V and Higgs-fermion coupling $\mathcal{L}_{\mathrm{HF}}$ lie outside our guiding principle of gauge invariance because neither contains a gauge field. However, there is no principle which forbids such contributions, and their presence is phenomenologically required. Moreover, note that each is written in an $SU(2)_L \otimes U(1)_Y$ invariant form.

Spontaneous Symmetry Breaking

As stated above, mass terms for *both* fermions and gauge bosons are forbidden in an exact $SU(2)_L \otimes U(1)_Y$ model. The former are not allowed by chirality (as opposed to what happens in QED and QCD), while it is gauge invariance, like in the cases of QED and QCD, that precludes the latter. However, the observed spectrum is far from being massless, so a mechanism for mass generation must be implemented. This is accomplished by means of *spontaneous breaking* of the $SU(2)_L \otimes U(1)_Y$ symmetry. To begin, we obtain the ground state Higgs configuration by minimising the potential V to give

$$\mathbf{\Phi}\left(-\mu^2 + 2\lambda\mathbf{\Phi}^\dagger\mathbf{\Phi}\right) = 0 \,. \tag{1.32}$$

We interpret this ground state relation in terms of vacuum expectation values (vev), denoted by a zero subscript. Eq. (1.32) has two solutions, the trivial solution, $\langle\mathbf{\Phi}\rangle = 0$, and the non-trivial solution,

$$\langle\mathbf{\Phi}^\dagger\mathbf{\Phi}\rangle_0 = \frac{v^2}{2}\,, \tag{1.33}$$

with

$$v \equiv \sqrt{\frac{\mu^2}{\lambda}}\,. \tag{1.34}$$

Let us focus on the latter alternative. A non-trivial vacuum Higgs configuration, which obeys the constraint Eq. (1.33), respects conservation of electric charge, and describes the spontaneous breaking of the original $SU(2)_L \otimes U(1)_Y$ symmetry, is

$$\langle\mathbf{\Phi}\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}\,. \tag{1.35}$$

In one interpretation, it is the order parameter for the WSG model, playing a role analogous to the magnetisation in a ferromagnet. Group theoretically, it is seen to transform as a component of a weak isodoublet. The energy scale, v, of the effect is not predicted by the model and must be inferred from experiment.

The fermion and gauge boson masses are determined by employing Eq. (1.35) for the Higgs field everywhere in the Lagrangian $\mathcal{L}_{\mathrm{H}}$. We first define charged fields $W_\mu^\pm$,

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)\,, \tag{1.36}$$

corresponding to the gauge bosons $W^\pm$. By substitution, we find for the mass contribution to the Lagrangian

$$\begin{aligned}\mathcal{L}_{\mathrm{mass}} &= -\frac{v}{\sqrt{2}}\left(f_u\bar{u}u + f_d\bar{d}d + f_e\bar{e}e\right) + \left(\frac{vg_2}{2}\right)^2 W_\mu^+W_-^\mu \\ &\quad + \frac{v^2}{8}\left(W_\mu^3\,B_\mu\right)\begin{pmatrix} g_2^2 & -g_1g_2 \\ -g_1g_2 & g_1^2 \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}\,.\end{aligned} \tag{1.37}$$

The fermion masses are given by

$$m_\alpha = \frac{v}{\sqrt{2}} f_\alpha \quad (\alpha = u,\, d,\, e,\, \cdots)\ . \tag{1.38}$$

Although the theory can accommodate fermions of any mass, it does not predict the mass values. Instead, the measured fermion masses are used to fix the arbitrary Higgs-fermion couplings. The charged $W^\pm$ gauge boson masses can be read off directly from Eq. (1.37),

$$M_W = \frac{1}{2} v g_2\, . \tag{1.39}$$

Symmetry breaking induces mixing in the neutral gauge bosons, as shown in Eq. (1.37). Their mass matrix is not diagonal in the basis of the states $\left(W_\mu^3,\, B_\mu\right)$. Diagonalisation occurs in the basis

$$\begin{aligned} Z_\mu &= \cos\theta_W\, W_\mu^3 - \sin\theta_W\, B_\mu\,, \\ A_\mu &= \sin\theta_W\, W_\mu^3 + \cos\theta_W\, B_\mu\,, \end{aligned} \tag{1.40}$$

where the weak mixing angle, also referred to as the *Weinberg mixing angle*, θ_W, is defined by

$$\tan\theta_W \equiv \frac{g_1}{g_2}\, . \tag{1.41}$$

The neutral gauge boson masses are then found to be

$$M_\gamma = 0, \qquad M_Z = \frac{v}{2}\sqrt{g_1^2 + g_2^2}\,, \tag{1.42}$$

and the fields A_μ and Z_μ stand for the massless photon and massive Z^0 boson, respectively. Observe that the $W^\pm$-to-Z^0 mass ratio is fixed by

$$\frac{M_W}{M_Z} = \cos\theta_W\, . \tag{1.43}$$

1.1.4 Some Theoretical Drawbacks of the Standard Model

Despite its tremendous success in accounting for virtually all known micro-physical non-gravitational phenomena, the SM is not recognised as the ultimate theory. The reasons are twofold. On the one hand, theoretical inconsistencies accompanied by some dose of aesthetics suggest the existence of an underlying theory. On the other hand, both experimental and observational evidences point to physics beyond the SM. Let us briefly summarise both sets, the former in this section and the latter in the next section.

- *Large amount of free parameters.* A well-defined theory should be capable to adequately describe a range as wide as possible with the lowest number of free parameters, making the theory predictive. The large number of free parameters in the SM should be considered as a hint for an underlying theory that provides correlations among its parameters.

- *Gauge coupling unification.* The idea of unifying different phenomena has always been a very appealing one in the development of physics. The gauge structure of the SM may suggest the existence of a more fundamental theory which embeds the SM gauge group into a single, larger group. However, precise measurements of the low energy values of the gauge couplings demonstrated that the SM cannot describe gauge coupling unification accurately enough to imply it is more than a mere coincidence.

  Grand Unified Theories (GUT) do consider such unification, which takes place at a scale of $\simeq 2 \times 10^{16}$ GeV. Thus, we might expect a cutoff of the SM at this very high energy scale.

- *Charge quantisation.* There is no explanation within the SM for the fractional values of the quark charges. GUTs provide an elegant description by accommodating both quarks and leptons into the same multiplets.

- *Family structure and fermion masses.* Neither the reason of the existence of three families nor the values of the fermion masses are explained within the SM. Recall that the Yukawa couplings (which provide a mass for the fermions after the Higgs boson has acquired a non zero vacuum expectation value) are completely free in the SM. GUT theories enhanced with family symmetries might shed some light on this issue.

- *Hierarchy problem.* For the electroweak symmetry breaking to be triggered appropriately, the SM Higgs boson must acquire a mass in the electroweak range. Nevertheless, without any symmetry protecting the masses of fundamental scalar fields, radiative corrections induce a quadratic dependence of these masses on the Ultra-Violet (UV) cutoff, Λ, which may be taken as the GUT scale. The natural value of the Higgs mass, therefore, should be of $\mathcal{O}(\Lambda)$ rather than $\mathcal{O}(100\,\text{GeV})$, leading to a destabilisation of the hierarchy of the mass scales in the SM.

- *Electroweak symmetry breaking (EWSB).* The SM does not possess any principle that explains why electroweak symmetry should be broken at low energies. It only accommodates by hand the necessary Higgs sector for EWSB to happen.

- *Gravity.* The SM does not take into account the gravitational degrees of freedom. From an aesthetic viewpoint, it is extremely appealing to build a single framework where all interactions are embedded.

1.1.5 Some Evidences for Physics Beyond the Standard Model

- *Neutrino flavour oscillations.* There exists by now convincing evidence suggesting the existence of neutrino masses to account for neutrino flavour oscillations through the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix, in the same fashion as the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix for quarks. Within the SM, neutrinos are *massless* because of the lack of right-handed neutrino states, which would provide a *Dirac* mass term, and the non-conservation of $B - L$ (B, L being the total baryon and lepton numbers, respectively), which would allow a *Majorana* mass term.

- *Baryon asymmetry.* The SM cannot account for the observed net baryon number either. Although the Sakharov criteria [Sakharov, 1967] are fulfilled within the SM, the baryon asymmetry generated at the EW phase transition is too small [Shaposhnikov, 1986, Shaposhnikov, 1987].

1.1.6 Solving the Standard Model drawbacks

In order to address the above shortcomings of the SM, several possible extensions have been proposed, each one aiming to address at least one of the mentioned failures. Let us briefly point out the ones more relevant for this work, commenting why a given model/theory may be of any relevance for solving the SM drawbacks.

- *Grand Unified Theories.* Grand Unified Theories (GUT) are based on the appealing idea of unification of all forces in a single gauge group. By imposing correlations among the parameters, GUTs remedy the problem of the excessive number of free parameters that the SM was suffering from.

- *Supersymmetry.* Supersymmetry (SUSY) refers to a symmetry relating bosonic to fermionic states. The main virtue of supersymmetric theories relies on its ability to stabilise the electroweak scale, that is, to prevent fundamental scalar fields from acquiring dangerous radiative corrections, thus solving the hierarchy problem.
  Not only the solution of the hierarchy problem makes SUSY so appealing, but also the remarkable fact that *almost* exact gauge coupling unification is achieved, when extrapolating to high scales, once the supersymmetric degrees of freedom are added to those of the SM.
  In addition to the above features, *local* SUSY is a generally covariant theory that might shed further light on gravity. Besides, the lightest supersymmetric particle is a natural Dark Matter candidate.

1.2 Standard Cosmology

As emphasised at the beginning of this Chapter, this dissertation focuses on those Standard Model extensions with massive neutrinos that also satisfy the WMAP constraints on Dark Matter abundance. For the sake of completeness, we shall give a brief account of the current status of the field of cosmology, which will help to understand claims related to WMAP in subsequent sections.
Standard Cosmology accounts for our current understanding of the Universe based on three main pillars. We will briefly review these key features and then discuss the theoretical framework within which they are embodied (see [Kolb and Turner, 1994] for a comprehensive introduction; see [Trodden and Carroll, 2004] for a friendly account).

1.2.1 The Universe Observed

The main properties of the Universe based on our current understanding, are as follows:

Expansion

One of the most fundamental features of the Universe is its expansion, that is, objects are separating from each other at a given velocity. The evidence comes from the red shift observed by E. Hubble [Hubble, 1929] in the galaxy spectra. The relationship between the luminosity distance, $d_L \equiv (\mathcal{L}/(4\pi\mathcal{F}))^{1/2}$ ($\mathcal{L}$ = luminosity of the object, $\mathcal{F}$ = measured flux), and the red shift, z, of a galaxy can be written as

$$z = H_0 d_L + \frac{1}{2}(q_0 - 1)(H_0 d_L)^2 + \cdots, \tag{1.44}$$

where the Hubble parameter is the present expansion rate of the Universe, $H_0 \equiv \dot{R}(t_0)/R(t_0)$, and the deceleration parameter serves as a measure of the deceleration of the expansion, $q_0 \equiv -\ddot{R}(t_0)/\left(R(t_0)H_0^2\right)$, $R(t)$ being the scale factor, to be defined properly in the next section; the subscript 0 denotes the present value of a quantity.
At modest red shifts, say $z < 1$, the linear relationship between d_L and z is quite clear and convincing. Thus, the study of galaxies at relatively modest red shifts ($z \ll 1$) one may determine H_0. The immediate consequence of Eq. (1.44) is that the more distant an object is, the higher the velocity of the expansion.

Large-Scale Isotropy and Homogeneity

Another remarkable property of our Universe is the high degree of both isotropy and homogeneity.
The best evidence for the observed isotropy is the uniformity of the temperature of the CMBR [Penzias and Wilson, 1965]: if the Universe were not isotropic, the expansion anisotropy would lead to a temperature anisotropy in the CMBR of similar magnitude, which is ruled out from observations. Likewise, inhomogeneities in the density of the Universe on the last scattering surface would lead to temperature anisotropies. The remarkable uniformity of the CMBR indicates that at the epoch the CMBR last scattered (about 2×10^5 yr after the big bang) the Universe was to a high degree of precision ($\frac{\delta T}{T} \simeq 10^{-4}$) isotropic and homogeneous [Bennett et al., 1996].
As we shall be discussing in subsequent sections, the recent precise measurements reported by the WMAP satellite provide stringent constraints on cosmological parameters, and the CMBR spectrum is accurately matched by what is known as the Λ-Cold Dark Matter (ΛCDM) model.

Abundances of light nuclei

Observations of primordial nebulae reveal abundances of the light elements unexplained by stellar nucleosynthesis. We shall discuss how the study of nuclear processes in the background of an expanding cooling Universe yields a remarkable agreement between theory and experiment.

At temperatures below 1 MeV, the weak interactions are frozen out and neutrons and protons cease to interconvert. The equilibrium abundance of neutrons at this temperature is about 1/6 of that of protons (due to the slightly larger neutron mass). The neutrons have a finite lifetime ($\tau_n \approx 890$ sec) that is somewhat larger than the age of the Universe at this epoch, $t\,(1\,\mathrm{MeV}) \approx 1$ sec, but they begin to gradually decay into protons and leptons. Soon thereafter, however, we reach a temperature somewhat below 100 keV, and Big Bang Nucleosynthesis (BBN) begins (The nuclear binding energy per nucleon is typically of order 1 MeV, so one might expect that nucleosynthesis would occur earlier; however, the large number of photons per nucleon prevents nucleosynthesis from taking place until the temperature drops below 100 keV). At that point the neutron/proton ratio is approximately 1/7. Of all the light nuclei, it is energetically favourable for the nucleons to combine in ^{4}He, and indeed that is what most of the free neutrons are converted into; for every two neutrons and fourteen protons, we end up with one helium nucleus and twelve protons. Thus, about 25% of the baryons by mass are converted to helium. In addition, there are traces of deuterium (approximately 10^{-5} deuterons per proton), ^{3}He (also $\simeq 10^{-5}$) and ^{7}Li ($\simeq 10^{-10}$).

The most amazing fact about nucleosynthesis is that, given that the Universe is radiation dominated during the relevant epoch, the relative abundances of the light elements depend essentially on only one parameter, known as the *baryon to entropy ratio*

$$\eta \equiv \frac{n_B}{s}, \tag{1.45}$$

where $n_B = n_b - n_{\bar{b}}$ is the difference between the number of baryons and antibaryons per unit volume. The WMAP satellite has determined the value of η [Spergel et al., 2007],

$$\eta = 6.1 \times 10^{-10}{}^{+0.3\times 10^{-10}}_{-0.2\times 10^{-10}}\,, \tag{1.46}$$

from precise measurements of the relative heights of the first two CMBR acoustic peaks.

Let us also comment on a major result derived from the BBN scenario. If the number of light neutrino species exceeded three, the radiation energy density would increase, which in turn would decrease the time scales associated with a given temperature (since $t \simeq H^{-1} \propto \rho_R^{-1/2}$). Nucleosynthesis would therefore happen somewhat earlier, resulting in a higher abundance of neutrons, and hence in a larger abundance of ^{4}He. Observations of the primordial helium abundance, which are consistent with the Standard Model prediction, provided the first evidence that the number of light neutrinos is actually three.

1.2.2 Fundamentals of the Standard Cosmology

The theoretical framework that accounts for the observational facts described above is the following:

Homogeneity and Isotropy: The Robertson-Walker Metric

As stressed previously, the distribution of matter and radiation in the observable Universe is homogeneous and isotropic. While this by no means guarantees that the *whole* Universe is smooth, it does imply that a region at least as large as our present Hubble volume is smooth. For the sake of simplicity, we *assume* that the *entire* Universe is homogeneous and isotropic. The metric for a space with homogeneous and isotropic spatial sections is the maximally-symmetric Robertson-Walker (RW) metric, which can be expressed as

$$ds^2 = dt^2 - R^2(t)\Big\{\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\Big\}. \tag{1.47}$$

Here $(t,\, r,\, \theta,\, \varphi)$ are coordinates (referred to as *comoving* coordinates), $R(t)$ is the cosmic scale factor and, with an appropriate rescaling of the coordinates, the parameter k can be chosen to be $+1$, -1 or 0 for spaces of constant positive, negative or zero spatial curvature, respectively. Note that the coordinate r in Eq. (1.47) is dimensionless, hence $R(t)$ has dimensions of length. On the other hand, the coordinate t refers to the proper time measured by an observer at rest in a frame with $(r,\, \theta,\, \varphi) =$ constant (referred to as the *comoving* frame; an observer at rest in such a comoving frame remains at rest, and an observer initially moving with respect to this frame will eventually come to rest in it).

Dynamics: The Friedman Equations

So far, we have dealt with the kinematics of a Universe described by a RW metric. The dynamics of the expanding Universe only appear through the scale factor $R(t)$. We now proceed to the analysis of the main dynamical features of such a metric.
Let us consider the Einstein field equations,

$$R_{\mu\nu} - \frac{1}{2}\mathcal{R} g_{\mu\nu} \equiv G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}\,, \tag{1.48}$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, $\mathcal{R}$ is the Ricci scalar, $g_{\mu\nu}$ refers to the metric, $T_{\mu\nu}$ stands for the momentum-energy tensor and Λ is a cosmological constant.
Given the metric involved, the momentum-energy tensor $T_{\mu\nu}$ must be diagonal, and the isotropy requires the spatial components to be equal. The simplest realization of such a momentum-energy tensor is that of a perfect fluid characterised by a time-dependent energy density $\rho(t)$ and pressure $p(t)$:

$$T_{\mu\nu} = \mathrm{diag}(\rho, -p, -p, -p)\,. \tag{1.49}$$

The $\mu = 0$ component of the conservation of the momentum-energy tensor ($T^{\mu\nu}_{;\nu} = 0$) gives the 1^{st} law of thermodynamics

$$d(\rho R^3) = -p\, d(R^3)\,. \tag{1.50}$$

To go further, we must specify the equation of state of the Universe. The most general equation of state compatible with a RW metric is of the form $p = w\rho$, w being an arbitrary parameter. In the following, we shall assume $w =$ constant. With this equation of state, the energy density evolves as $\rho \propto R^{-3(1+w)}$, implying the following:

$$\begin{aligned} \text{RADIATION}\ \ (p = \tfrac{1}{3}\rho) \quad &\Longrightarrow \quad \rho \propto R^{-4}\,, \\ \text{MATTER}\ \ (p = 0) \quad &\Longrightarrow \quad \rho \propto R^{-3}\,, \\ \text{VACUUM}\ \ (p = -\rho) \quad &\Longrightarrow \quad \rho \propto \text{constant}\,. \end{aligned} \tag{1.51}$$

The $0-0$ component of Eq. (1.48) leads to the so-called Friedman equation[3]

$$\frac{\dot{R}^2}{R^2}+\frac{k}{R^2}=\frac{8\pi G}{3}\rho\,. \tag{1.52}$$

Recalling the definition of the Hubble constant, $H_0 \equiv \dot{R}_0/R_0$, we may define the Hubble *parameter* as $H \equiv \dot{R}/R$. Thus, the Friedman equation can be written as

$$\frac{k}{H^2R^2}=\Omega-1\,, \tag{1.53}$$

where Ω is the ratio of the density to the *critical* density ρ_C:

$$\Omega \equiv \frac{\rho}{\rho_C}\,,\quad \rho_C \equiv \frac{3H^2}{8\pi G}\,. \tag{1.54}$$

Since $H^2R^2 \geq 0$, there is a correspondence between the signs of k and $\Omega-1$:

$$\begin{aligned} k=+1 &\implies \Omega>1 \;\; \text{CLOSED}\,,\\ k=0 &\implies \Omega=1 \;\; \text{FLAT}\,,\\ k=-1 &\implies \Omega<1 \;\; \text{OPEN}\,. \end{aligned} \tag{1.55}$$

1.2.3 Theoretical Drawbacks of the Standard Cosmology

Despite the success of the Standard Cosmology, several difficulties still remain.

The Horizon Problem

The horizon problem reflects the fact that the CMBR is isotropic to an outstanding degree of precision, even though widely separated points on the last scattering surface are completely outside each others' horizons, *i.e.*, distant patches of the CMBR were causally disconnected at recombination. If we are to explain the high isotropic nature of the CMBR, therefore, we must somehow modify the causal structure of the Friedman-RW (FRW) cosmology.

Small-Scale Inhomogeneity

While the Universe is apparently very smooth on large scales, there is a plethora structure at smaller scales: stars, galaxies, clusters, voids and superclusters ranging in size from much less than 1 Mpc to 100 Mpc (and perhaps larger). While such a rich abundance of small-scale structure can be accommodated within the standard cosmology as the result of the growth of *assumed* small, primeval inhomogeneities (Jeans instability), an immediate question arises: what is the origin of the primeval inhomogeneities?

[3]Because of the Bianchi identities, the field equation associated to the $i-i$ component can be expressed in terms of Eqs. (1.50) and (1.52).

The Flatness Problem

The Friedman equation may be cast into the form

$$\Omega - 1 = \frac{k}{H^2 R^2} \,. \tag{1.56}$$

Differentiating this with respect to the scale factor yields

$$\frac{d\Omega}{dR} = (1+3w)\frac{\Omega(\Omega-1)}{R} \,. \tag{1.57}$$

It can be shown easily that this differential equation has three fixed points, depending on the sign of $1+3w$, as shown in Table 1.4.

Fixed Point	$1+3w>0$	$1+3w<0$
$\Omega=0$	Attractor	Repeller
$\Omega=1$	Repeller	Attractor
$\Omega=\infty$	Attractor	Repeller

Table 1.4: Behaviour of the density parameter near fixed points.

Observationally, we know that $\Omega \simeq 1$ today, *i.e.*, we are very close to the repeller of this differential equation for a Universe dominated by ordinary matter and radiation ($w > -1/3$). Even if we only took account of the luminous matter in the Universe, we would clearly live in a Universe that was far from the attractor points. It is already puzzling that the Universe has not reached yet any of its attractor points, given the long time it has been evolving. A rough estimate leads to

$$|\Omega(10^{-43}\,\mathrm{s}) - 1| \leq \mathcal{O}(10^{-60}), \quad |\Omega(1\,\mathrm{s}) - 1| \leq \mathcal{O}(10^{-16}) \,. \tag{1.58}$$

This remarkable degree of fine tuning is the flatness problem. Within the context of the standard cosmology there is no known explanation of this fact.

Unwanted Relics

Generally, any particle species with a very large mass typically possesses a very small annihilation cross-section, σ_A. Thus, as thermal relics, X, (a property featured by virtually every particle at sufficiently early times) satisfy $\Omega_X \propto \langle \sigma_A |v| \rangle^{-1}$ ($|v|$ being the average velocity), any very massive, stable particle species is very likely to dominate the Universe.
A prominent example is monopole production within unified gauge theories. Standard cosmology provides us with no mechanism of eliminating relics that are overproduced early in the history of the Universe.

1.2.4 Evidence for Physics Beyond the Standard Cosmology

The nature of the energy content of the Universe today, seems to point to new physics that SC cannot account for.

Accelerating Universe

A quite remarkable discovery took place over a decade ago. In 1998, analysis of the data reported by two different teams [Riess et al., 1998, Perlmutter et al., 1999] from Type Ia supernovae observations seems to confirm the fact that matter does not dominate our current Universe, as it appears to be undergoing an accelerating phase.
There exists a vast literature on possible attempts to account for this phenomenon, ranging from a *cosmological constant* (see [Padmanabhan, 2003, Peebles and Ratra, 2003] for comprehensive reviews) to quite exotic models, like *quintessence* [Zlatev et al., 1999] or *phantom cosmology* [Caldwell, 2002].
Let us briefly discuss the evidence for an accelerating Universe. Two independent groups undertook searches for distant supernovae in order to measure cosmological parameters. Their data are much better fitted by a Universe dominated by a cosmological constant than by a matter-dominated one. Taking into account predictions coming from BBN, $\Omega_M \simeq 0.3$, and $\Omega_\Lambda \simeq 0.7$. This corresponds to a vacuum energy density $\rho_\Lambda \simeq (10^{-3}\,\mathrm{eV})^4$. This non-zero value for the vacuum energy has become one of the leading contemporary research areas in cosmology.

Dark Matter

The first evidence for the existence of non-luminous matter dates back to the pioneering work by Fritz Zwicky [Zwicky, 1933], who claimed that the mass of luminous matter (stars) in the Coma cluster, which consists of around 1000 galaxies, was much smaller than the total mass implied by the motion of cluster member galaxies. However, only in the 1970s the existence of Dark Matter began to be seriously considered, as it turned out to be the most plausible explanation of the anomalous rotation curves of spiral galaxies.
Let us briefly discuss the three most important sources of evidence for Dark Matter.

- *Galactic rotation curves in spiral galaxies.* Considering that virial equilibrium holds for galaxies, we may expect the following expression for the rotational velocity, v,

$$v^2 \propto \frac{M}{r}, \tag{1.59}$$

  where M is the mass of the galaxy at distance r from the centre. Many samples of rotation curves have been reported, and the surprising conclusion is that all curves appear to be flat *beyond* the point where most of the light ceases, outside the core of the galaxy, which implies by (1.59) that $M \propto r$ beyond the point where most of the light stops.

- *X-ray emission from elliptical galaxies and clusters of galaxies.* Considering hydrostatic equilibrium for the galaxies, one is able to estimate the overall mass distribution in the galaxy once the profiles for the temperature and density of the hot X-ray emitting gas are provided. In doing so, an isothermal model is usually assumed. The results reported showed that the visible mass contributed only 1%.

- *Gravitational lensing.* Gravitational lensing is due to the bending of radiation when passing near a massive object. Thus, by measuring the deflection of the radiation coming from a given source it is possible to estimate the mass responsible for the deflection. It turns out that, for the observed angular distribution to be explained, a new source other than bright matter is required.

It should be emphasised that our approach to Dark Matter is almost entirely from the viewpoint of supersymmetric particle-physics models. In this section we have only pointed out the most relevant evidence for the existence of Dark Matter from astrophysical and cosmological observations.
The most widely believed source of Dark Matter is referred to as *Weakly Interacting Massive Particles* (WIMPs) and has the following properties:

- *Cold.* Dark Matter must be non-relativistic; if Dark Matter was *hot*, *i.e.*, relativistic, it would have free-streamed out of overdense regions, suppressing the formation of galaxies.
- *Weakly interacting.* Having escaped detection this far, Dark Matter must be very weakly interacting with ordinary matter.
- *Stable.* Since we "observe" Dark Matter today, it has to be stable. More precisely, the lifetime of Dark Matter must be larger than the current age of the Universe.

1.3 Models for Physics Beyond the Standard Cosmology

Having discussed the different reasons for considering a cosmological model beyond SC, in this section we shall be commenting, for the sake of completeness, on the main ideas intended to address such difficulties (see [Linde, 1990, Kolb and Turner, 1994] for details).

1.3.1 Inflation

Among the quoted difficulties inherent to SC, the horizon problem is especially serious, because at its heart it is simply reflecting causality. Any solution to this problem will almost certainly require an important modification to how information propagated in the earliest stages of the Universe. Cosmological inflation may provide some clues [Guth, 1981, Starobinsky, 1980].
The fundamental idea in the inflationary paradigm is that the Universe underwent a period of accelerated expansion, *i.e.*, $\ddot{R} > 0$, at early times. The effect of this acceleration is to quickly expand a small region of space to a huge size, diminishing spatial curvature in the process, and making the Universe extremely close to flat. In addition, the horizon is greatly increased, so the distant points on the CMBR actually were in causal contact and unwanted relics were tremendously diluted, solving the monopole problem (*i.e.*, the fact that GUT models generally predict the production of monopoles in the early universe, with an abundance fairly higher than the allowed by cosmological observations). As an unexpected bonus, quantum

fluctuations make it impossible for inflation to smooth out the Universe with perfect precision, so there is a spectrum of remnant density perturbations; this spectrum turns out to be approximately scale-invariant, in good agreement with observations.

Slowly-Rolling Scalar Fields

For the sake of clarity, we shall discuss the *chaotic* inflation model [Linde, 1983]. Let us consider modelling matter in the early Universe by a real scalar field ϕ, with potential $V(\phi)$. For simplicity, we will focus on the homogeneous case, in which all quantities depend only on the cosmological time t, and set $k = 0$. A homogeneous real scalar field behaves as a perfect fluid with

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,. \tag{1.60}$$

The equation of motion of the scalar field is given by

$$\ddot{\phi} + 3\frac{\dot{R}}{R}\dot{\phi} + \frac{dV}{d\phi} = 0, \tag{1.61}$$

which can be thought of as the usual equation for a scalar field in Minkowski space, but with a friction term due to the expansion of the universe. The Friedman equation with such a field as the sole energy source is

$$H^2 = \frac{8}{3}\pi G\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]\,. \tag{1.62}$$

The *slow-roll* approximation for inflation involves neglecting the $\ddot{\phi}$ term in Eq. (1.61) as well as neglecting the kinetic energy of ϕ compared to the potential energy in Eq. (1.62). The scalar field equation of motion and the Friedman equation then become

$$\dot{\phi} \simeq -\frac{V'(\phi)}{3H}, \quad H^2 \simeq \frac{8}{3}\pi G V(\phi)\,. \tag{1.63}$$

These conditions will hold if the two *slow-roll conditions* are satisfied, namely

$$|\varepsilon| \ll 1, \quad |\eta| \ll 1, \tag{1.64}$$

where the *slow-roll parameters* are defined as

$$\varepsilon \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta \equiv M_p^2\frac{V''}{V}\,. \tag{1.65}$$

To see how the slow-roll conditions yield inflation, we express the Hubble parameter in the form

$$\frac{\ddot{R}}{R} = \dot{H} + H^2, \tag{1.66}$$

indicating that inflation (which requires $\ddot{R}/R > 0$)in principle occurs if

$$\frac{\dot{H}}{H^2} > -1\,. \tag{1.67}$$

However, in slow-roll, $\dot{H}/H^2 \simeq -\varepsilon$, which is small; it is the smallness of the second parameter η that eventually helps to ensure that inflation will continue for a sufficient period.

Solving the Problems of the Standard Cosmology

How does inflation solve (some of) the difficulties the standard cosmology suffers from? A typical inflationary solution for the scale factor reads

$$R(t) = R(0)\exp\left[\frac{2\pi}{M_p^2}(\phi_0^2 - \phi^2)\right], \tag{1.68}$$

which, at the time interval that inflation lasts, can be written as

$$R(\Delta t) \simeq R(0)\exp\left(\frac{2\pi}{\xi^2}\right), \tag{1.69}$$

where $\xi \equiv m/M_p \ll 1$, m being the mass of the field ϕ and M_P the Planck mass. Taking a typical value for m, for which $\xi < 10^{-4}$, we obtain

$$R(\Delta t) \simeq R(0) \times 10^{10^8}\,. \tag{1.70}$$

This has a remarkable consequence. A proper distance L_P at $t = 0$ will inflate to a size 10^{10^8} cm after a time $\Delta t \simeq 5 \times 10^{-36}$ s. It is important at this point to know that the size of the observable universe today is $H_0^{-1} \simeq 10^{28}$ cm. Consequently, only a small fraction of the original Planck length comprises today's entire visible universe. Homogeneity over a patch less than (or of order of) the Planck length at the onset of inflation is all that is required to solve the horizon problem. Likewise, any unwanted relics are diluted by the tremendous expansion; so long as the GUT phase transition happens before inflation, monopoles will have an extremely low density.

Concerning the flatness problem, recall the discussion about the fixed points of the Friedman equation. For a universe dominated by energy with $w < -1/3$, $\Omega = 1$ is an *attractor*. Thus, if the inflationary period lasts long enough, the density parameter would not have had time to stray away from unity.

1.3.2 Dynamical Dark Energy

Strictly speaking, the data reported by the supernovae observations are consistent with smoothly-distributed sources of dark energy that vary slowly with time.

Let us argue for a dynamical dark energy on the basis of the Friedman equation (1.52), which implies

$$\dot{R}^2 \propto R^2\rho + \text{constant}\,. \tag{1.71}$$

From this relation, it is clear that the only way to get acceleration ($\dot{R}$ increasing) in an expanding universe is if ρ falls off more slowly than R^{-2}; neither matter ($\rho_M \propto R^{-3}$) nor radiation ($\rho_R \propto R^{-4}$) will have this feature. Thus, a source of dark energy evolving very slowly with time may shed some light.

The simplest possibility along these lines involves the same kind of source typically invoked in inflationary scenarios: a scalar field ϕ rolling slowly in a potential, sometimes referred to as *quintessence*. The energy density of a scalar field can be written as

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)\,. \tag{1.72}$$

For a homogeneous field, the gradient contribution can be neglected, leaving us with the following equation of motion in an expanding universe:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0\,. \tag{1.73}$$

If the slope of the potential V is quite flat, we will have solutions for which ϕ is nearly constant throughout space and only evolving very slowly with time; the energy density in such a configuration is

$$\rho_\phi \approx V(\phi) \approx \text{constant}\,. \tag{1.74}$$

Thus, a slowly-rolling scalar field is an appropriate candidate for dark energy.
However, introducing dynamics opens up severe difficulties. Most quintessence models are built with scalar fields ϕ with masses of order the current Hubble scale, $m_\phi \simeq H_0 \simeq 10^{-33}$ eV. In QFT, light scalar fields are unnatural, as there is no symmetry preventing them from acquiring large masses due to RGE effects. On top of that, light scalar fields give rise to long-range forces and time-dependent couplings that should be observable even if couplings to ordinary matter are suppressed. We therefore need to invoke additional fine-tuning to explain why the quintessence field has not already been experimentally detected.

1.3.3 The Concordance Model: ΛCDM

We shall end our cosmological discussion by commenting on the widely accepted cosmological model that best fits the data. It is called the ΛCDM model, as it contains a non-vanishing cosmological constant, Λ, and *Cold* Dark Matter [Komatsu et al., 2009, Hinshaw et al., 2009]. The ΛCDM model has the following properties:

- *Spatially flat, homogeneous and isotropic universe on large scales.*
- *It is composed of ordinary matter, radiation, Cold non-baryonic Dark Matter and a non-zero cosmological constant.* Recall that the reason why Dark Matter must be "cold" (non-relativistic) is because "hot" Dark Matter would have free-streamed out of overdense regions, suppressing the formation of galaxies. Likewise, baryonic Dark Matter is ruled out simply because it does not provide the necessary density assigned to Dark Matter.
- *The primordial fluctuations are adiabatic, nearly scale-invariant Gaussian random fluctuations.*
- *It possesses six free parameters*:
    1. Matter density $\Omega_M\, h^2$
    2. Baryon density $\Omega_B\, h^2$
    3. Hubble constant H_0
    4. Amplitude of fluctuations σ_8
    5. Optical depth τ

6. Slope of the scalar perturbation spectrum n_s.

This set of parameters is sufficient to predict not only the statistical properties of the microwave sky, measured by WMAP at several hundreds points on the sky [Spergel et al., 2007, Dunkley et al., 2009], but also the large-scale distribution of matter and galaxies, mapped by the Sloan Digital Sky Survey (SDSS) [Abazajian et al., 2009] and the 2dF Galaxy Redshift Survey (2dFGRS) [Cole et al., 2005]. The best-matching values of the above parameters are [Dunkley et al., 2009]:

$$\begin{cases} \Omega_M \, h^2 \simeq 0.128 \\ \Omega_B \, h^2 \simeq 0.022 \\ h \simeq 0.732 \end{cases} \Rightarrow \begin{cases} \Omega_{DM} \, h^2 \simeq 0.106 \\ \Omega_\Lambda \, h^2 \simeq 0.408 \, , \end{cases} \tag{1.75}$$

$$n_s \simeq 0.958, \quad \tau \simeq 0.089, \quad \sigma_8 \simeq 0.761 \, . \tag{1.76}$$

Figs. 1.1 and 1.2 show the main results reported by the WMAP team. Fig. 1.1 is the temperature profile at the earliest stages of the universe. One should notice the high level of isotropy at large scales. However, the small spots (anisotropies) become crucial. Had these anisotropies not been present, structure formation would not have taken place. Fig. 1.2 shows the CMBR power spectrum. The highest peak stands for the maximum anisotropy, which happens on a scale that had just enough time to collapse but not to come to equilibrium, $R \simeq H^{-1}_{CMB}$. As a result, there is a peak at an angular size corresponding to the horizon size at last scattering. For a flat universe, the prediction is $l \simeq 220$, which is in impressive agreement with observations.

The data are so constraining that there is little room for significant modifications of the basic ΛCDM model. The combination of WMAP and other astronomical measurements places significant limits on the geometry of the universe, the nature of dark energy, and even neutrino properties. While allowing for a running spectral index slightly improves the fit to the WMAP data, the improvement in the fit is not significant enough to introduce a new parameter.

Cosmology requires new physics beyond the Standard Model of particle physics, providing sources of Dark Matter and Dark Energy as well as a mechanism for the generation of primordial fluctuations. The WMAP data provide insights into all three of these fundamental problems:

- The clear detection of the predicted acoustic peak structure implies that the Dark Matter is non-baryonic.

- The WMAP data are consistent with a nearly flat universe in which Dark Energy has an equation of state close to that of a cosmological constant, $w = -1$. The combination of WMAP data with measurements of the Hubble constant, baryon oscillations, supernovae data and large-scale structure observations all reinforce the evidence for Dark Energy.

- The simplest model for structure formation, a scale-invariant spectrum of fluctuations, is not a good fit to the WMAP data, as in order to fit the angular spectrum, it requires either tensor modes or a spectral index with $n_s < 1$. These observations match the basic inflationary predictions and are well fitted by the predictions of the simple $m^2\phi^2$ model.

Further WMAP observations and future analysis will test the inflationary paradigm. While there is by now no convincing evidence for significant non-Gaussianities, an alternative model that better fits the low ℓ data would be an exciting development. Within the context of inflationary models, measurements of the spectral index as a function of the scale and measurements of tensor modes will provide a direct probe of physics at the earliest moments of the universe.

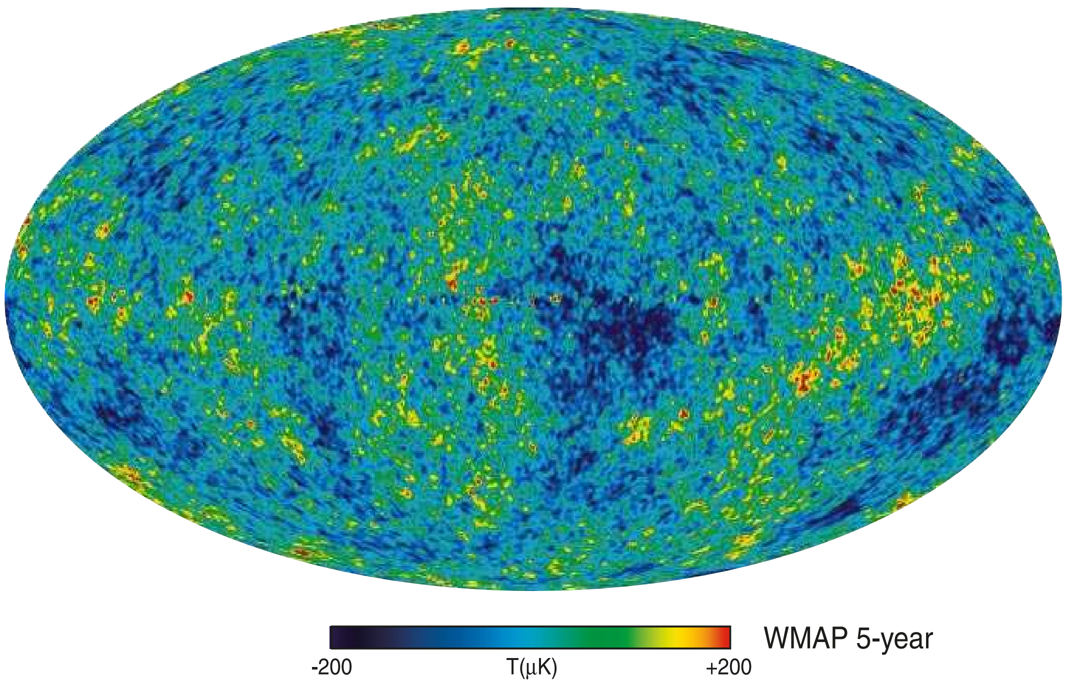

Figure 1.1: *Temperature profile of the early universe. The global homogeneity is evident; note however the anisotropies that are the seeds for structure formation.*

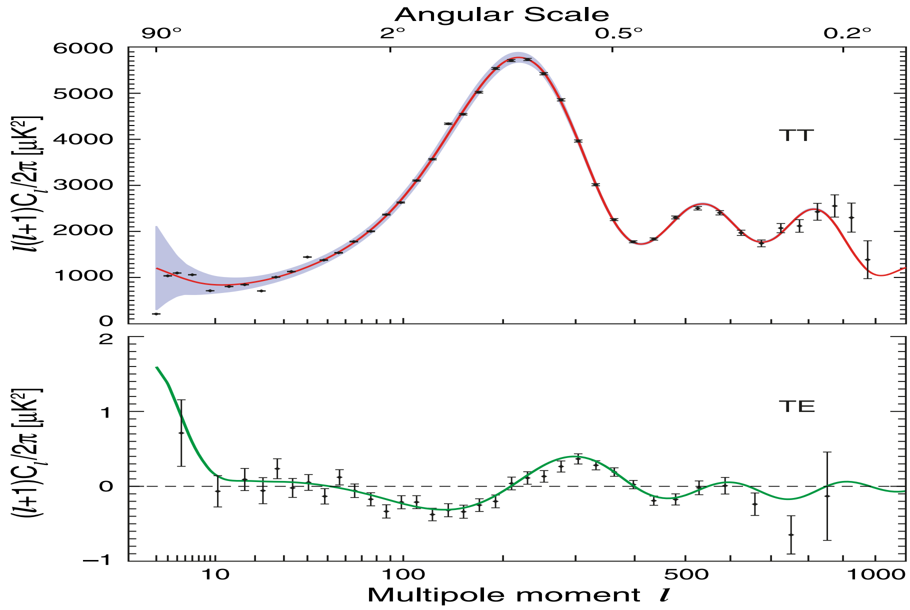

Figure 1.2: *Power spectrum of CMBR.*

Chapter 2

Beyond the Standard Model of Particle Physics

So far, we have provided an overview of the widely accepted phenomena the SM can accommodate. However, there exist both theoretical and experimental reasons for expecting new physics. Since the bulk of this work focuses on possible scenarios beyond the SM, in the next section we shall be discussing in some detail what this evidence is. The second section of this chapter will deal with the theoretical ideas pursued in order to remedy (some of) the SM deficiencies that are directly related to the work presented here. A word is in order here. It should be emphasised that the number of experimental facts not described by the SM, such as neutrino oscillations, is very limited. On the other hand, there are some *aesthetic* arguments which render the SM not sufficiently appealing, including the non-unification of gravity with the rest of the interactions.
Along these lines, the ideas discussed in the second section of this chapter need not have a purely experimental motivation, but may mostly rely upon theoretical grounds to be tested.

2.1 Experimental Evidence for Physics Beyond the Standard Model

We discuss some of the strongest experimental evidence pointing to new physics. This includes the by now convincing evidence for neutrino masses and the matter-antimatter asymmetry.

2.1.1 Neutrino Flavour Oscillations

There exist two main sources of evidence for neutrino masses, namely the *atmospheric* and *solar* neutrino problems.

- *Atmospheric neutrino problem.* The situation is the following: Atmospheric neutrinos are electron and muon neutrinos and their antineutrinos which are produced in the hadronic showers induced by primary cosmic rays in the Earth's atmosphere. The main mechanism of production of atmospheric neutrinos is given by the following chain of reactions:

$$\begin{aligned} p(\alpha) + \text{air} \;\rightarrow\; \pi^{\pm}(K^{\pm}) + \;\; & X \\ \pi^{\pm}(K^{\pm}) \;\rightarrow\; & \mu^{\pm} + \nu_\mu(\bar{\nu}_\mu) \\ & \mu^{\pm} \rightarrow e^{\pm} + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu) \end{aligned} \tag{2.1}$$

Naively, from the reaction chain one would expect to have two atmospheric muon neutrinos (antineutrinos) for every electron neutrino (antineutrino). However, the Super-Kamiokande zenith angle distributions for e and μ-like events reveals an striking pattern. For e-like events the measured zenith angle distributions agree very well with the Monte Carlo predictions, for both sub-GeV and multi-GeV energy ranges, while for μ-like events both samples exhibit zenith-angle dependent deficiency of event numbers as compared to expectations. The deficit of muon neutrinos is stronger for upward going neutrinos which have larger path lengths. In the multi-GeV sample, there is practically no deficit of events caused by muon neutrinos coming from the upper hemisphere ($\cos\theta > 0$), whereas in the sub-GeV sample all μ-like events show a deficit decreasing with $\cos\theta$. This pattern is perfectly consistent with oscillations $\nu_\mu \leftrightarrow \nu_\tau$ or $\nu_\mu \leftrightarrow \nu_s$, where ν_s is a sterile neutrino.

The idea of neutrino oscillations was first introduced by Pontecorvo [Pontecorvo, 1968]. More precisely, neutrinos are produced by the charged-current weak interactions and therefore are weak-eigenstates neutrinos ν_e, ν_μ, ν_τ. However, the neutrino mass matrix in this (flavour) basis is in general not diagonal. Thus, the neutrino mass eigenstates ν_1, ν_2, ν_3 (the states that diagonalise the neutrino mass matrix,*i.e.*, the free propagation eigenstates) are in general different from the flavour eigenstates. Therefore, the probability of finding a neutrino created in a given flavour state to be in the same state (or any other flavour state) oscillates with time.

We shall first consider neutrino oscillations in the case of a Dirac neutrino mass term, and then will comment on the Majorana as well as Dirac+Majorana cases. The part of the Lagrangian that describes the lepton masses and charged-current interactions is

$$-\mathcal{L}_{W+m} = \frac{g}{\sqrt{2}} \bar{e}'_{aL}\gamma^\mu \nu'_{aL} W^-_\mu + (m_\ell)_{ab}\, \bar{e}'_{aL} e'_{bR} + (m_D)_{ab}\, \bar{\nu}'_{aL}\nu'_{bR} + \text{hc}, \tag{2.2}$$

where the primes denote the flavour eigenstates fields. It follows from this expression that the individual lepton flavours L_e, L_μ, L_τ are not conserved when the Majorana mass term is present, but the total lepton number $L = L_e + L_\mu + L_\tau$ is still conserved. The mass matrix of charged leptons m_ℓ and the Dirac neutrino mass matrix m_D are general complex matrices which can be diagonalised by bi-unitary transformations. Let us write

$$e'_L = V_L e_L, \quad e'_R = V_R e_R, \quad \nu'_L = U_L \nu_L, \quad \nu'_R = U_R \nu_R\,, \tag{2.3}$$

and choose the unitary matrices of charged leptons and neutrinos

$$V_L^\dagger m_\ell V_R = m_\ell^\delta, \quad U_L^\dagger m_D U_R = m_D^\delta\,. \tag{2.4}$$

The "unprimed" fields e_{iL}, e_{iR}, ν_{iL}, ν_{iR} are then the components of the Dirac mass eigenstates fields $e_i = e_{iL} + e_{iR}$ and $\nu_i = \nu_{iL} + \nu_{iR}$. The Lagrangian in Eq. (2.2) can thus be expressed in the mass eigenstate basis as

$$-\mathcal{L}_{W+m} = \frac{g}{\sqrt{2}}\bar{e}_{iL}\gamma^{\mu}\left(V_L^{\dagger}U_L\right)\nu_{jL}W_{\mu}^{-} + (m_\ell)_i\,\bar{e}_{iL}e_{iR} + (m_D)_i\,\bar{\nu}_{iL}\nu_{iR} + \mathrm{hc}\,. \tag{2.5}$$

The matrix $U = V_L^{\dagger}U_L$ is known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [Maki et al., 1962]. It is the leptonic analog of the CKM quark mixing matrix. It relates a neutrino flavour eigenstate $|\nu_a'\rangle$, produced or absorbed alongside with the corresponding charged lepton, to the mass eigenstates $|\nu_i\rangle$:

$$|\nu_a'\rangle = U_{ai}^{*}|\nu_i\rangle\,. \tag{2.6}$$

In what follows, for the sake of simplicity, we shall omit the primes and distinguish between the flavour and mass eigenstates just by using the indices from the beginning of the Latin alphabet for the former and from its middle for the latter. Assume that at time $t = 0$ the flavour eigenstate $|\nu_a\rangle$ was produced. What is the probability of finding the neutrino in a state $|\nu_b\rangle$ at a later time? Time evolution of the flavour eigenstates is not simple, so we shall follow the evolution of the system in the mass eigenstate basis. The initial state at $t = 0$ is $|\nu(0)\rangle = |\nu_a\rangle = U_{aj}^{*}|\nu_j\rangle$. The neutrino state at a later time t is then

$$|\nu(t)\rangle = U_{aj}^{*}e^{-iE_jt}|\nu_j\rangle\,. \tag{2.7}$$

The probability amplitude of finding the neutrino at the time t in a flavour state $|\nu_b\rangle$ is

$$A\left(\nu_a \to \nu_b; t\right) = \langle\nu_b|\nu(t)\rangle = U_{aj}^{*}e^{-iE_jt}\langle\nu_b|\nu_j\rangle = U_{bi}U_{aj}^{*}e^{-iE_jt}\langle\nu_i|\nu_j\rangle = U_{bj}e^{-iE_jt}U_{aj}^{*}, \tag{2.8}$$

where a sum over all intermediate states j is implied. The last expression here has a very simple physical meaning. The factor $U_{aj}^{*} = U_{ja}^{\dagger}$ is the amplitude of transformation of the initial flavour eigenstate neutrino ν_a into a mass eigenstate one ν_j; the factor e^{-iE_jt} is simply the propagator describing the time evolution of the mass eigenstate neutrino ν_j in the energy representation; finally, the factor U_{bj} converts the time-evolved mass eigenstate ν_j into the flavour eigenstate ν_b. The neutrino oscillation probability, *i.e.*, the probability of the transformation of a flavour eigenstate neutrino ν_a into another one ν_b becomes then

$$P\left(\nu_a \to \nu_b; t\right) = |A\left(\nu_a \to \nu_b; t\right)|^2 = |U_{bj}e^{-iE_jt}U_{aj}^{*}|^2\,. \tag{2.9}$$

We have discussed neutrino oscillations in the case of Dirac neutrinos. Let us consider that a Majorana mass term is present instead. Eq. (2.2) has now to be modified: the term $(m_D)_{ab}\bar{\nu}_{aL}'\nu_{bR}' + \mathrm{hc}$ has to be replaced by $(m_M)_{ab}\bar{\nu}^{c\prime}_{aL}\nu_{bR}' + \mathrm{hc} = (m_M)_{ab}\bar{\nu}_{aL}'^{T}C\nu_{bR}' + \mathrm{hc}$, C being the charge-conjugation operator. This mass term breaks not only the individual lepton flavours, but also the total lepton number. The symmetric Majorana mass matrix $(m_M)_{ab}$ is diagonalised by the transformation $U_L^T m_M U_L = m_M^{\delta}$, so one can again use the field transformations Eq. (2.3). Therefore, the structure of the charged-current interactions is the same as in the case of the Dirac neutrinos, and the diagonalisation of the neutrino mass matrix in the case of n fermion generations yields n mass eigenstates as well. Thus, the oscillation probabilities in the case of the Majorana mass term are the same as that of a Dirac one. This, in particular, means that one cannot distinguish between Dirac and Majorana neutrinos by studying neutrino oscillations. Essentially, this is because the total lepton number is not violated by the neutrino flavour oscillations.

The situation is different in the case when we consider the most general Dirac + Majorana mass term (which, like in the pure Dirac case, requires the existence of the electroweak singlet neutrinos ν_R). In particular, in the case of n ν_L and n ν_R species, the neutrino mass matrix is $2n \times 2n$-dimensional, leading to $2n$ massive Majorana neutrino states. The total lepton number conservation is violated by the Majorana mass term. Unlike the pure Dirac and Majorana cases, the existence of both Dirac and Majorana masses allows a new type of neutrino oscillations: in addition to the usual flavour oscillations $\nu_a \to \nu_b$, oscillations into "sterile" states, $\nu_a \to \nu_b^c$, can occur. These oscillations violate the total lepton number L. Thus, the general Dirac+ Majorana case can in principle be distinguished from the pure Dirac and Majorana cases through neutrino oscillations experiments.

Let us now move on to describe how neutrino oscillations in vacuum may shed some light on the atmospheric neutrino problem. The neutrino flavour and mass eigenstates are related through

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = U \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}, \tag{2.10}$$

where the lepton mixing matrix U depends in general on three mixing angles θ_{12}, θ_{13}, θ_{23} and one CP-violating phase δ in the Dirac case (Majorana masses introduce two additional phases). It is convenient to use the parameterisation of the mixing matrix which coincides with the standard parameterisation of the quark mixing matrix:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{2.11}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$ (in the 2-flavour case, the mixing matrix would be just

$$U = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}, \tag{2.12}$$

where $c \equiv \cos\theta_0$, $s \equiv \sin\theta_0$, θ_0 being the vacuum mixing angle). The oscillation probabilities between various flavour states are given by the general expression Eq. (2.9). They are generally hard to cast in terms of elementary functions. There are, however, several important limiting cases in which one can obtain very simple approximate expressions for the oscillation probabilities in terms of those of the 2-flavour case.

Assume first that the neutrino mass squared differences $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ have a hierarchy

$$|\Delta m_{21}^2| \ll |\Delta m_{31}^2| \simeq |\Delta m_{32}^2| \,. \tag{2.13}$$

This means that either $m_1 \approx m_2 \ll m_3$ (direct hierarchy) or $m_3 \ll m_1 \approx m_2$ (inverse hierarchy). These cases are of practical interest since the solar neutrino data indicate that one needs a small mass squared difference $\Delta m^2_{solar} \simeq 10^{-5}$ eV^2 for the solution of the solar neutrino problem through matter-enhanced neutrino oscillations (see below), whereas the explanation of the atmospheric neutrino experiments through neutrino oscillations requires $\Delta m^2_{atm} \simeq 10^{-3}$ eV^2, much larger than that for the solar problem. Consider first the oscillations over the baselines L for which

$$\frac{\Delta m^2_{21}}{2E} L \ll 1\,. \tag{2.14}$$

This case is relevant for atmospheric, reactor and accelerator neutrino experiments. It follows from Eq. (2.14) that the oscillations due to the small mass difference Δm^2_{21} are effectively frozen, and one can safely take the limit $\Delta m^2_{21} \to 0$. The probability of $\nu_a \to \nu_b$ oscillations then takes a very simple form:

$$P(\nu_a \to \nu_b;\, L) = 4|U_{a3}|^2|U_{b3}|^2 \sin^2\left(\frac{\Delta m^2_{31}}{4E} L\right), \tag{2.15}$$

where it has been taken into account that for relativistic neutrinos of momentum p, one has

$$E_i = \sqrt{p^2 + m_i^2} \approx p + \frac{m_i^2}{2p} \approx p + \frac{m_i^2}{2E} \tag{2.16}$$

as well as $L \approx t$, L being the distance travelled by the neutrinos during the time t.
Let us consider now another limiting case, which is relevant for the solar neutrino oscillations and also for very long baselines reactor experiments (such as KamLAND). We shall be again assuming the hierarchy Eq. (2.13) and in addition

$$\frac{\Delta m^2_{31}}{2E} L \approx \frac{\Delta m^2_{32}}{2E} L \gg 1, \tag{2.17}$$

the condition Eq. (2.14) being no longer necessary. In this case, the oscillations due to the mass squared differences Δm^2_{31} and Δm^2_{32} are very fast and lead to an averaged effect.

- *Solar neutrino problem.* It can be summarised as follows. The Super-Kamiokande experiment detected neutrinos coming from the sun through the reaction

$$\nu_a + e^- \to \nu_a + e^-, \tag{2.18}$$

where a stands for the families. The question that came up was that the detection rate of electron neutrinos was *smaller* than expected. The most natural solution to the problem is the existence of *neutrino oscillations*, which can convert a fraction of solar ν_e into $\nu_{\mu,\tau}$ (or their combination).
Let us briefly discuss neutrino oscillations in matter. Generally, neutrinos of all three flavours interact with the electrons, protons and neutrons of matter through neutral current (NC) interactions mediated by Z^0 bosons. Besides, electron neutrinos also have charged-current (CC) interactions mediated by $W^\pm$ boson exchange.
Let us now consider the evolution of a system of oscillating neutrinos in matter. In

this case, the evolution is more easily followed in the flavour basis, because the effective potentials of neutrinos are diagonal in such a basis. For the sake of simplicity, we shall consider the two-flavour case. As usual, $\nu_f = U\nu_m$, where ν_f and ν_m are two-component vectors of neutrino fields in the flavour and mass eigenstate bases, respectively, and the mixing matrix U is given by Eq. (2.12). In the absence of matter, the evolution equation in the mass eigenstate basis is $i(d/dt)|\nu_m\rangle = H_m|\nu_m\rangle$, where $H_m = \mathrm{diag}(E_1,\, E_2)$. This gives the evolution equation in the flavour basis: $i(d/dt)|\nu_f\rangle = H_f|\nu_f\rangle = UH_mU^\dagger$. For relativistic neutrinos, $E_i \simeq p + m_i^2/2E$, and we thus obtain

$$i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} p + \frac{m_1^2+m_2^2}{4E} - \frac{\Delta m^2}{4E}\cos 2\theta_0 & \frac{\Delta m^2}{4E}\sin 2\theta_0 \\ \frac{\Delta m^2}{4E}\sin 2\theta_0 & p + \frac{m_1^2+m_2^2}{4E} + \frac{\Delta m^2}{4E}\cos 2\theta_0 \end{pmatrix}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. \tag{2.19}$$

Here ν_e and ν_μ stand for time-dependent amplitudes of finding the electron and muon neutrino, respectively. Note that the first terms in the diagonal entries above coincide. They can only modify the common phase of the neutrino states and therefore have no effect on neutrino oscillations, as these depend on phase differences. Thus, we shall omit them. To derive the neutrino evolution equation in matter one has to add the matter-induced potentials V_e and V_μ to the diagonal elements of the effective Hamiltonian in Eq. (2.19). Notice that V_e and V_μ contain a common term due to NC interactions. Again, such common diagonal terms will be omitted. Thus, we are left with

$$i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta_0 + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta_0 \\ \frac{\Delta m^2}{4E}\sin 2\theta_0 & \frac{\Delta m^2}{4E}\cos 2\theta_0 \end{pmatrix}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. \tag{2.20}$$

Recall that only electron neutrinos interact through CC interactions, hence the extra term in the (1,1) entry $V_e^{CC} = \sqrt{2}G_F N_e$, where G_F is the Fermi constant and N_e stands for the electron number density. Eq. (2.20) is the evolution equation which describes $\nu_e \leftrightarrow \nu_\mu$ oscillations in matter. As the extra term $\sqrt{2}G_F N_e$ does not appear for either ν_μ or ν_τ, $\nu_\mu \leftrightarrow \nu_\tau$ oscillations in matter exhibit the same form as the vacuum case in this two-flavour case. In the full three-flavour case, mixing with ν_e does modify the results in matter.

For our purposes (describe the solar anomaly), it suffices to consider the case of constant matter density. Diagonalisation of the effective Hamiltonian in Eq. (2.20) gives the following neutrino eigenstates in matter:

$$\nu_A = \nu_e \cos\theta + \nu_\mu \sin\theta \tag{2.21}$$

$$\nu_B = -\nu_e \sin\theta + \nu_\mu \cos\theta\,, \tag{2.22}$$

where the mixing angle θ is given by

$$\tan 2\theta = \frac{\frac{\Delta m^2}{2E}\sin 2\theta_0}{\frac{\Delta m^2}{2E}\cos 2\theta_0 - \sqrt{2}G_F N_e}\,. \tag{2.23}$$

This differs from the vacuum mixing angle θ_0 and therefore the matter eigenstates ν_A and ν_B do not coincide with ν_1 and ν_2 for the vacuum. The difference of neutrino eigenenergies in matter is

$$E_A - E_B = \sqrt{\left(\frac{\Delta m^2}{2E}\cos 2\theta_0 - \sqrt{2}G_F N_e\right)^2 + \left(\frac{\Delta m^2}{2E}\sin 2\theta_0\right)^2}\,. \tag{2.24}$$

It is now easy to obtain the probability of $\nu_e \leftrightarrow \nu_\mu$ oscillations in matter:

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\pi\frac{L}{\ell_m}\right), \tag{2.25}$$

where

$$\ell_m \equiv \frac{2\pi}{E_A - E_B} = \frac{2\pi}{\sqrt{\left(\frac{\Delta m^2}{2E}\cos 2\theta_0 - \sqrt{2}G_F N_e\right)^2 + \left(\frac{\Delta m^2}{2E}\sin 2\theta_0\right)^2}}\,. \tag{2.26}$$

It exhibits the same form as the probability of oscillations in vacuum, the mixing angle and the oscillation length in vacuum being replaced by those of matter. The oscillation amplitude takes the form

$$\sin^2 2\theta = \frac{\left(\frac{\Delta m^2}{2E}\sin 2\theta_0\right)^2}{\left(\frac{\Delta m^2}{2E}\cos 2\theta_0 - \sqrt{2}G_F N_e\right)^2 + \left(\frac{\Delta m^2}{2E}\sin 2\theta_0\right)^2}\,. \tag{2.27}$$

Eq. (2.27) exhibits a typical resonance form, with the maximum value, $\sin^2 2\theta = 1$, achieved when the condition

$$\frac{\Delta m^2}{2E}\cos 2\theta_0 = \sqrt{2}G_F N_e \tag{2.28}$$

holds. Condition Eq. (2.28) is generally referred to as the Mikheev-Smirnov-Wolfenstein (MSW) matter effect [Wolfenstein, 1978, Wolfenstein, 1979, Mikheev and Smirnov, 1985, Mikheev and Smirnov, 1986]. From Eq. (2.27) it follows that when the MSW condition is satisfied, mixing in matter is maximal ($\theta = \pi/4$), independently from the vacuum mixing angle θ_0. Thus, the probability of neutrino flavour transition in matter can be large even with a small vacuum mixing angle. Thus, the observed small θ_{13} mixing angle is balanced by the MSW enhancement in the probability of $\nu_e \leftrightarrow \nu_\tau$. This solves the solar neutrino problem.

Summarising, the experimental evidence for the existence of neutrino oscillations and masses is widely accepted, pointing for the first time to physics beyond the Standard Model of particle physics [Fukuda et al., 1998, Fukuda et al., 1999a, Fukuda et al., 1999b] (see also [Ahmad et al., 2001, Ahmad et al., 2002, Eguchi et al., 2003, Ahn et al., 2003, Araki et al., 2005, Michael et al., 2006]. By now, it has been established that the atmospheric and solar mixing angles θ_{23} and θ_{12} are large and that the squared mass differences are $\Delta m^2_{atm} \simeq 2.5\times10^{-3}$ eV^2 and $\Delta m^2_{sol} \simeq 8\times10^{-5}$ eV^2, respectively (for a global discussion of neutrino parameters, see [Maltoni et al., 2004, Gonzalez-Garcia, 2005]).

2.1.2 Baryogenesis

Another observational evidence not accounted for by the SM is the domination of matter over antimatter. In the earliest stages of the Universe *almost* equal amounts of matter and

antimatter were supposed to coexist, raising questions on why the observed Universe does exhibit such a large asymmetry.
Since the baryon number density, $n_B \equiv \frac{n_b - n_{\bar{b}}}{V}$, V being the volume, has evolved with time, it is useful to define the baryon asymmetry as

$$\eta \equiv \frac{n_B}{s}, \tag{2.29}$$

where $s \equiv S/V$ is the entropy density.
The key ingredients for the baryon asymmetry to emerge are summarised in the Sakharov conditions [Sakharov, 1967], which we briefly discuss next.

- *Baryon number violation.* Starting from a nearly baryon symmetric Universe, $\eta \simeq 0$, baryon number violation must take place to evolve into a Universe with $\eta \neq 0$.
- *C and CP violation.* If both C (charge conjugation symmetry) and CP (symmetry under the composition of parity and C) were exact, then the total rate for any process which produces an excess of baryons would be equal to the rate of the complementary process which produces an excess of antibaryons, hence no net baryon number would have been created.
- *Departure from thermal equilibrium.* In thermal equilibrium, any process that may create baryon number is balanced by the reversed reaction yielding *anti*baryons, hence keeping the net baryon number zero. More precisely, the equilibrium average of the baryon number operator, B, at a temperature $T = 1/\beta$, vanishes:

$$\begin{aligned} \langle B \rangle_T &= \mathrm{Tr}\big(e^{-\beta H} B\big) = \mathrm{Tr}\big[(CPT)\,(CPT)^{-1}\,e^{-\beta H} B\big] \\ &= \mathrm{Tr}\big[e^{-\beta H}\,(CPT)^{-1}\,B\,(CPT)\big] = -\mathrm{Tr}\big(e^{-\beta H} B\big), \end{aligned} \tag{2.30}$$

  where we have used that the Hamiltonian H commutes with CPT. Thus, no net baryon number is generated in thermal equilibrium.

2.2 Models for Physics Beyond the Standard Model

As stated in the Preface, from a purely theoretical point of view, there is a number of motivations to look for physics beyond the SM. Among them, the absence of gravity within the SM, the hierarchy problem of GUT models and the unification of couplings have generated a lot of work in the last decades.
As mentioned, we shall simply focus on supersymmetric extensions of the SM embedded into a GUT model.

2.2.1 Grand Unified Theories

The basic idea of Grand Unified Theories (GUT) consists of encompassing both the fundamental interactions and the matter content (quarks and leptons) in a single theoretical framework (see [Langacker, 1981] for an in-depth account of GUT phenomenology). More precisely, the first goal is to find a Lie Group G such that $G \supset G_{SM}$, where $G_{SM} \equiv SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is the gauge group of the SM. Of course, a symmetry-breaking pattern to reduce G down to G_{SM} must be at work, since the SM, as emphasised in the Preface, has been experimentally verified to a high degree in the low energy sector.

A typical consequence of this embedding is that the new symmetry generators and their associated gauge bosons involve both colour and flavour. Thus, the new interactions generally violate baryon number conservation, hence leading to a possible proton decay. We will return to this point later.

At first sight, such a unification might seem implausible, as the intrinsic strengths of the three interactions differ substantially. In terms of their coupling strengths, at laboratory energies $\alpha_S \gg \alpha_2 \gg \alpha_{EM}$, where $\alpha_S \equiv g_S^2/4\pi$ is the $SU(3)_C$ colour "fine structure constant", $\alpha_2 \equiv g^2/4\pi$ is the weak "fine structure constant" and $\alpha_{EM} \equiv e^2/4\pi$ is the usual fine structure constant. However, due to quantum corrections, coupling constants are *not* constant, but "run" (evolve) with energy, after a process referred to as *renormalisation* is implemented. The variation of a coupling constant with energy is given by its so-called Renormalisation Group Equation (RGE):

$$\beta \equiv \frac{\partial g}{\partial \ln \sqrt{-q^2/\mu^2}} ,$$

$$\beta = -b_0 \frac{g^3}{16\pi^2} - b_1 \frac{g^5}{(16\pi^2)^2} - \dots , \tag{2.31}$$

where $q^2 < 0$ is the momentum transfer (which sets the energy scale) and μ is some convenient energy scale. (Let us stress that the coefficients in Eq. (2.31) can be obtained perturbatively). Using this, it follows that

$$\alpha_{EM}(q^2) = \frac{\alpha_{EM}(m_e^2)}{1 - [\alpha_{EM}(m_e^2)/3\pi] \ln(-q^2/m_e^2)}, \tag{2.32}$$

from which we see that α_{EM} *increases* logarithmically with energy, *i.e.*, at short distances (large energies) the electromagnetic interaction becomes stronger.

The situation for non-Abelian gauge theories can be very different. For QCD, the evolution of the coupling strength is given by

$$\alpha_S(q^2) = \frac{\alpha_S(\mu^2)}{1 + [(33 - 2N_f)\alpha_S(\mu^2)/12\pi] \ln(-q^2/\mu^2)}, \tag{2.33}$$

where N_f is the number of quark flavours. The sense of the evolution is opposite to that of α_{EM} (provided that $N_f < 17$): $\alpha_S(q^2)$ tends to zero as $-q^2 \to \infty$ (known as *asymptotic freedom*) and diverges for some small value of $-q^2$ (referred to as *infrared slavery*). This qualitatively different behaviour originates from the fact that in non-Abelian gauge theories the gauge bosons themselves carry the charge of the interaction and thus contribute to vacuum polarisation.

Since the coupling constants in gauge theories run with energy, and at different rates, it is possible for the three coupling constants to evolve to a common value. It turns out that by means of solely the fields of the SM, such a convergence does *not* arise. However, if one takes into account the new degrees of freedom that supersymmetric theories bring in, the couplings *do* converge to a high precision, at an energy of about 2×10^{16} GeV and value

$\alpha_{GUT} \simeq 1/27$, as shown in Fig. 2.1 [1]. Moreover, since the weak mixing angle θ_W is given by $\tan^2\theta_W = \alpha_1/\alpha_2$, it also evolves with energy, increasing to a value of $\sin^2\theta_W = 3/8$ at the unification scale. The convergence is independent of the unification group, and it turns out that several simple groups G may do the trick, such as $SU(5)$, $SO(10)$, E_6, $E_8,\ldots$. For the sake of simplicity, we shall focus on the most economical unification scheme: $SU(5)$ (We shall not discuss supersymmetric GUT models here, our purpose being to illustrate the accommodation of the SM fields into a single group. In the supersymmetric version, one should *double* the SM modes so as to account for the additional (supersymmetric) degrees of freedom).

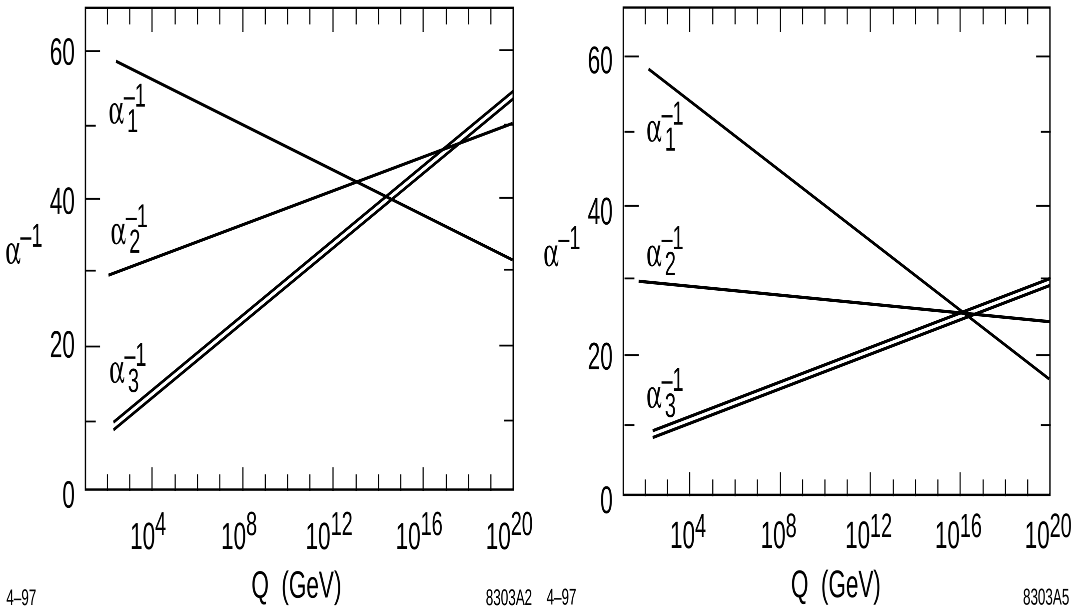

Figure 2.1: *Running of the gauge couplings. On the left, within the SM. On the right, within the MSSM.*

In $SU(5)$, each generation of quarks and leptons is assigned to the reducible representation $\mathbf{10}+\bar{\mathbf{5}}$, where $\mathbf{5}$ is the fundamental representation. The quarks and leptons of the first family are accommodated as follows:

$$\begin{aligned} \mathbf{10} &\supset u^i_L,\, d^i_L,\, \bar{u}^i_L,\, \bar{e}_L, \\ \bar{\mathbf{5}} &\supset \bar{d}^i_L,\, e^-_L,\, \nu_{eL}, \end{aligned} \tag{2.34}$$

where $i = 1, 2, 3$ is the colour index and $\bar{e} = e^+$. The fifteen right-handed CP conjugate fermion fields are accommodated in a $\overline{\mathbf{10}} + \mathbf{5}$ representation.

The gauge bosons are assigned to the 24-dimensional adjoint representation. Since the SM possesses only twelve gauge bosons, there are twelve additional gauge bosons in $SU(5)$, denoted by X, Y, and sometimes referred to as *diquarks* or *leptoquarks*. Classified according to

[1] Here, we focus on supersymmetric theories, but there may certainly be other approaches to GUT models not relying upon supersymmetry.

their $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ quantum numbers, they are

$$\begin{pmatrix} Y^{-1/3} \\ X^{-4/3} \end{pmatrix} \qquad \text{isodoublet, colour triplet, } Y = -5/3, \tag{2.35}$$
$$\begin{pmatrix} \bar{X}^{4/3} \\ \bar{Y}^{1/3} \end{pmatrix} \qquad \text{isodoublet, colour anti} - \text{triplet, } Y = +5/3\,.$$

These new gauge bosons have fractional electric charge and carry colour. They are known as leptoquarks or diquarks because they mediate transitions between quarks and leptons or quarks and antiquarks within a given multiplet, as

$$\begin{aligned} e^-_L &\to \bar{d}_L + X^{-4/3}, \\ \bar{u}_L &\to u_L + X^{-4/3}, \\ \nu_{eL} &\to \bar{d}_L + Y^{-1/3}, \\ \bar{d}_L &\to u_L + Y^{-1/3}\,. \end{aligned} \tag{2.36}$$

It is impossible to assign the X, Y gauge bosons lepton and baryon numbers so that these new interactions conserve B and L; thus, the new interactions in $SU(5)$ violate both B and L. In fact, this prediction is generic to GUT models: because quarks and leptons sit in the same representation, there must be new gauge bosons that mediate processes that violate B and L conservation.

In our $SU(5)$ case, processes mediated by X, Y exchange like

$$\begin{aligned} \bar{u} + \bar{u} &\to d + e^- \quad (\Delta B = \Delta L = 1), \\ \bar{u} + \bar{d} &\to d + \nu_e \quad (\Delta B = \Delta L = 1) \end{aligned} \tag{2.37}$$

violate B and L by one unit (Due to an accidental global symmetry, $SU(5)$ interactions do conserve $B - L$). Because of these new interactions the proton is unstable. It can decay through X exchange by

$$p \to \pi^0 + e^+, \tag{2.38}$$

which at the quark level is $uu(d) \to e^+\bar{d}(d)$, the quark in parentheses going along as a "spectator".

Naturally, the full GUT symmetry cannot be manifest; if it were, the proton would decay in 10^{-24} sec. The gauge group G must be spontaneously broken to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. For $SU(5)$, this is accomplished by a 24-dimensional Higgs representation, which leads to masses of order the unification scale for the twelve X, Y gauge bosons. Thus, at energies below $\simeq 10^{16}$ GeV the processes mediated by X, Y boson exchange can be treated as an effective four-fermion interaction (although *non*-renormalisable) with strength $G_{\Delta B} \simeq \alpha_{GUT}/M^2_{GUT} \simeq 10^{-27}$ GeV^{-2}. Since $G_{\Delta B} \ll G_F$, these new B, L violating interactions are extremely weak at energies below the GUT scale. The decay rate of the proton must be proportional to $G^2_{\Delta B}$, so that on dimensional grounds the proton lifetime must be

$$\tau_p = \Gamma_p^{-1} \propto (G^2_{\Delta B} m_p^5)^{-1} \simeq 10^{31}\,\text{yr}\,. \tag{2.39}$$

The current limits to proton longevity are in excess of 10^{32} yr, which seems to rule out (minimal) $SU(5)$.

Additional Higgs bosons are required in order to give masses to the quarks and leptons, at the very least one complex 5-dimensional Higgs. This Higgs multiplet contains the usual doublet Higgs required for electroweak SSB and a colour triplet Higgs $H^{\pm 1/3}$, which can also mediate B, L violation. The triplet component must acquire a mass comparable to the GUT scale so as to guarantee the proton's longevity, while the doublet component must acquire a mass of order of few 100 GeV to trigger electroweak SSB at the appropriate scale. This is referred to as the *doublet-triplet splitting problem*, and continues to be a major task facing GUT models.
In GUT models the quarks and leptons are assigned to common multiplets and so relations among the fermion masses are predicted; as before, the overall Yukawa couplings still remain arbitrary. In $SU(5)$ there is an apparently successful prediction for the b-quark mass. In § 4.1 we will discuss the relation between the b-quark mass and the lepton mixing. A link between this b-quark mass prediction and the Dark Matter abundance will be discussed in § 4.2.
In the simplest version of $SU(5)$, neutrinos remain massless, as the right-handed neutrino is a singlet. However, one can embed the see-saw mechanism in the GUT model in order to provide neutrinos with a mass (It is worth noting that in more general GUT groups, like $SO(10)$, the right-handed neutrino is accommodated within a non-trivial representation (the 16-dimensional multiplet in $SO(10)$), predicting *naturally* a non-vanishing neutrino mass).
In sum, the successes of grand unification are:

- The unification of the strong and electroweak interactions.
- The unification of the quarks and leptons, thereby explaining why $|q(e^-)| = |q(p)|$.
- The successful prediction of $\sin^2\theta_W$.
- The successful prediction for the b-quark mass.

Moreover, GUT models have important cosmological consequences, such as the origin of baryon asymmetry and the creation of topological defects, including monopoles.
On the other hand, GUT models provide no particular insight whatsoever as to the resolution of the generation problem, the origin of CP violation, the existence of gravity, nor is the number of required "input parameters" substantially reduced. In addition, there is no specific prediction for the unification group.

2.2.2 Supersymmetry

Among the different extensions of the SM, supersymmetry (SUSY) is one of the most beautiful and appealing ones (see [Haber and Kane, 1985, Nilles, 1984] for a thorough discussion of phenomenological aspects; see [Fayet and Ferrara, 1977, Gates et al., 1983, Sohnius, 1985] for technical issues; see [Martin, 1997] for a pedagogical account). This is true, despite the lack of even the slightest *direct* evidence for supersymmetry being realized in Nature. The motivation for supersymmetry is purely theoretical, since

- SUSY relates matter (quarks and leptons) and interactions (gauge bosons).
- Local SUSY implies gravity.

- Both global and local SUSY divergences are much better controlled, thus it might be of relevance in constructing finite quantum gravity theories.

- SUSY provides a nice solution to the hierarchy problem.

Supersymmetry Algebra

Since SUSY relates particles with *different* spin, the generator, Q, of the transformations

$$Q|b\rangle = |f\rangle, \quad Q|f\rangle = |b\rangle, \tag{2.40}$$

where b and f are boson and fermion states, respectively, must be an anti-commuting spinor. Since spinors are intrinsically complex objects, $Q^\dagger$ is also a symmetry generator. Due to their spinor nature, SUSY generators should be labelled with a spinor index, $\alpha = 1, 2$. The corresponding algebra obeyed by Q_α, $Q^\dagger_{\dot\alpha}$ is:

$$\{Q_\alpha, Q_\beta\} = \{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\} = 0, \tag{2.41}$$

$$\{Q_\alpha, Q^\dagger_{\dot\beta}\} = 2\,(\sigma^\mu)_{\alpha\dot\beta}\, P_\mu, \tag{2.42}$$

where $\sigma^\mu \equiv \left(1, \sigma^i\right)$, σ^i being the Pauli matrices. In addition to the above relations, the commutators with the generators of the Poincaré group read

$$[P^\mu, Q_\alpha] = 0, \quad [M^{\mu\nu}, Q_\alpha] = -i\,(\sigma^{\mu\nu})_\alpha^{\ \beta}\, Q_\beta, \tag{2.43}$$

where P^μ is the four-momentum generator of space-time translations and $M^{\mu\nu}$ the generator of the space-time rotations.
The supersymmetric algebra has two important consequences:

- *The number of bosonic degrees of freedom equals that of fermionic degrees of freedom for each supermultiplet* (*supermultiplet* refers to any irreducible representation of the supersymmetric algebra where single-particle states are accommodated).

- *The vacuum energy is always positive in a supersymmetric theory.*

The first consequence implies, when applied to the SM, that supersymmetric theories *double* the number of degrees of freedom, since no bosonic (fermionic) superpartners of the SM fermions (bosons) have been found in the spectrum.
Next, we will discuss in some detail the characteristics of a supersymmetric model, taking the Minimal Supersymmetric extension of the SM (MSSM) as an example.

Supersymmetric Lagrangians

We shall provide the steps to build a supersymmetric model containing matter fields and gauge interactions.

Free Chiral Supermultiplets The matter content contained in the chiral supermultiplet consists of a collection of left-handed two-component Weyl fermions ψ_i and a set of complex scalar fields ϕ_i, i running over all flavour degrees of freedom. The simplest Lagrangian just involves kinetic terms for each set of fields:

$$\mathcal{L}_{\text{chiral}}^{\text{free}} = \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{auxiliary}}, \tag{2.44}$$

$$\mathcal{L}_{\text{scalar}} = -\partial^\mu \phi^{*\,i} \partial_\mu \phi_i, \quad \mathcal{L}_{\text{fermion}} = -i\psi^{\dagger\,i}\bar{\sigma}^\mu \partial_\mu \psi_i, \;\; \mathcal{L}_{\text{auxiliary}} = F^{*\,i} F_i\,, \tag{2.45}$$

where F_i is a collection of complex scalar fields introduced to guarantee that the SUSY algebra closes off-shell. We should stress that the auxiliary fields F_i do not propagate physically and can be removed by their equations of motion.

Interacting Chiral Supermultiplets The most general set of supersymmetric renormalisable interactions for the chiral supermultiplets reads

$$\mathcal{L}_{\text{chiral}}^{\text{int}} = -\frac{1}{2} W^{ij} \psi_i \psi_j - F^{*\,i} F_i + \text{hc}\,. \tag{2.46}$$

W^{ij} is an analytic polynomial in the complex fields ϕ_k (that is, they do not depend on $\phi^{*\,k}$) which, for the sake of convenience, is usually written as

$$W^{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}\,, \tag{2.47}$$

where W is referred to as the *superpotential*, and is an holomorphic function of the scalar fields ϕ_i treated as complex variables.

Gauge Supermultiplets The propagating degrees of freedom in a gauge supermultiplet are a massless gauge boson field A_μ^a and a two-component Weyl fermion gaugino λ^a, a running over the adjoint representation of the gauge group ($a = 1, \ldots, 8$ for $SU(3)_C$; $a = 1, 2, 3$ for $SU(2)_L$; $a = 1$ for $U(1)_Y$). The kinetic and self-interaction terms become

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} - i\lambda^{\dagger a} \bar{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a\,, \tag{2.48}$$

where D^a is a real bosonic auxiliary field which, like its chiral version F_i, can be eliminated via its equation of motion.

Supersymmetric gauge interactions Since the MSSM contains all the SM interactions, the coupling of the gauge and chiral supermultiplets is achieved through the *covariant* derivative. However, we also have to consider the gauge-invariant interactions involving the gaugino λ^a and D^a fields. It turns out that there are three such possible interaction terms that are renormalisable, namely

$$-\sqrt{2}g\left(\phi^* T^a \psi\right)\lambda^a, \;\; -\sqrt{2}g\lambda^{\dagger a}\left(\psi^\dagger T^a \phi\right), \;\; +g\left(\phi^* T^a \phi\right) D^a\,. \tag{2.49}$$

The full Lagrangian density for a renormalisable supersymmetric theory is just the sum of (2.44), (2.45), (2.46), (2.48) and (2.49). From this, we can express the full scalar potential as follows

$$V(\phi, \phi^*) = F^{*i} F_i + \frac{1}{2} \sum_a D^a D^a \,, \tag{2.50}$$

where

$$F_i = \frac{\partial W}{\partial \phi_i}, \quad D^a = -g\left(\phi^* T^a \phi\right) . \tag{2.51}$$

The terms in Eq. (2.51) are referred to as F- and D-term contributions, respectively. Such terms play a crucial role when discussing *spontaneous* supersymmetry breaking.

Soft Supersymmetry Breaking Interactions As pointed out, supersymmetry implies mass degeneracy among the fields within the same supermultiplet, namely the SM fields and their superpartners. Lacking any experimental evidence on the existence of such superparticles with the expected masses, we must consider that for SUSY to be of any relevance in the description of Nature, it must be broken. However, since the main virtue of SUSY comes from its ability to stabilise the EW scale, the superparticle masses should not fall significantly below such a scale. For this reason, we would like to consider the most general set of terms that break SUSY *without* inducing quadratic divergences. Such a set is referred to as *soft terms*. The possible soft SUSY-breaking Lagrangian becomes

$$\mathcal{L}_{\text{soft}} = -\left(\frac{1}{2} M_a \lambda^a \lambda^a + \frac{1}{6} a^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + t^i \phi_i\right) + \text{hc} - \left(m^2\right)^i_j \phi^{j*} \phi_i \,, \tag{2.52}$$

and contains the gaugino masses for each group,M_a, scalar squared-mass terms $\left(m^2\right)^i_j$ and b^{ij}, (scalar)^3 couplings a^{ijk} and tadpole couplings t^i. Since no scalar gauge singlets appear in the MSSM spectrum, the last term (which only arises if ϕ_i is a gauge singlet) is absent from the MSSM.

Minimal Supersymmetric Standard Model

We now discuss the matter content of the MSSM, which is displayed in Table 2.1.

As emphasised in the previous sections, a supersymmetric model is completely defined once the superpotential W is specified. After the discussion of SUSY-breaking, it is also clear that the soft SUSY-breaking Lagrangian must be specified as well. For the MSSM, they read

$$\mathcal{W}_{\text{MSSM}} = \bar{u} \mathbf{Y_u} Q H_u - \bar{d} \mathbf{Y_d} Q H_d - \bar{e} \mathbf{Y_e} L H_d + \mu H_u H_d \,, \tag{2.53}$$

$$\begin{aligned}
\mathcal{L}^{\text{MSSM}}_{\text{soft}} \;=\; & -\frac{1}{2}\left(M_3 \tilde{g}\tilde{g} + M_2 \tilde{W}\tilde{W} + M_1 \tilde{B}\tilde{B} + \text{hc}\right) \\
& - \left(\tilde{\bar{u}} \mathbf{A_u} \tilde{Q} H_u - \tilde{\bar{d}} \mathbf{A_d} \tilde{Q} H_d - \tilde{\bar{e}} \mathbf{A_e} \tilde{L} H_d + \text{hc}\right) \\
& - \tilde{Q}^\dagger \mathbf{m^2_Q} \tilde{Q} - \tilde{L}^\dagger \mathbf{m^2_L} \tilde{L} - \tilde{\bar{u}}^\dagger \mathbf{m^2_{\bar{u}}} \tilde{\bar{u}}^\dagger - \tilde{\bar{d}}^\dagger \mathbf{m^2_{\bar{d}}} \tilde{\bar{d}}^\dagger - \tilde{\bar{e}}^\dagger \mathbf{m^2_{\bar{e}}} \tilde{\bar{e}}^\dagger \\
& - m^2_{H_u} H^*_u H_u - m^2_{H_d} H^*_d H_d - \left(b H_u H_d + \text{hc}\right) .
\end{aligned} \tag{2.54}$$

In Eq. (2.53) the objects H_u, $H_d\, Q$, L, $\bar{u}$, $\bar{d}$, $\bar{e}$ are chiral superfields corresponding to the chiral supermultiplets in Table 2.1. The dimensionless Yukawa coupling parameters $\mathbf{Y_u}$, $\mathbf{Y_d}$, $\mathbf{Y_e}$

Superfields		Spin-0	Spin-1/2	$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ quantum numbers
squarks, quarks	Q	$(\tilde{u}_L\, \tilde{d}_L)$	$(u_L\, d_L)$	$(\mathbf{3},\, \mathbf{2},\, \frac{1}{6})$
(×3 families)	$\bar{u}$	$\tilde{u}^*_R$	$u^\dagger_R$	$(\bar{\mathbf{3}},\, \mathbf{1},\, -\frac{2}{3})$
	$\bar{d}$	$\tilde{d}^*_R$	$d^\dagger_R$	$(\bar{\mathbf{3}},\, \mathbf{1},\, \frac{1}{3})$
sleptons, leptons	L	$(\tilde{\nu}_L\, \tilde{e}_L)$	$(\nu_L\, e_L)$	$(\mathbf{1},\, \mathbf{2},\, -\frac{1}{2})$
(×3 families)	$\bar{e}$	$\tilde{e}^*_R$	$e^\dagger_R$	$(\mathbf{1},\, \mathbf{1},\, 1)$
Higgs, Higgsinos	H_u	$(H_u^+\, H_u^0)$	$(\tilde{H}_u^+\, \tilde{H}_u^0)$	$(\mathbf{1},\, \mathbf{2},\, \frac{1}{2})$
	H_d	$(H_d^0\, H_d^-)$	$(\tilde{H}_d^0\, \tilde{H}_d^-)$	$(\mathbf{1},\, \mathbf{2},\, -\frac{1}{2})$

Table 2.1: Chiral supermultiplets in the MSSM. The spin-0 fields are complex scalars, while the spin-1/2 fields are left-handed two-component Weyl fermions.

are 3×3 matrices in flavour space. For the sake of simplicity, all of the gauge ($SU(3)_C$ colour and $SU(2)_L$ weak isospin) and family indices in Eq. (2.53) are suppressed. Some comments are in order here.

- The last term in Eq. (2.53), the so-called μ term, is the supersymmetric version of the Higgs boson mass in the SM.
- *Both* H_u and H_d are required in order to provide masses to all of the fermions. To satisfy the correct quantum numbers, the term $\bar{u}QH_u$ for example, would be replaced by something like $\bar{u}QH_d^*$, but this term is forbidden by the analyticity of the superpotential.
- Since the Yukawa interactions y^{ijk} must be completely symmetric under interchange of i, j, k, the matrices $\mathbf{Y_u}$, $\mathbf{Y_d}$, $\mathbf{Y_e}$ appearing in Eq. (2.53) not only imply Higgs-fermion-fermion couplings as in the SM, but also sfermion-Higgsino-fermion interactions. Generally, for every SM vertex there is a supersymmetric analogue which consists of replacing two of the SM particles by two superparticles.

On the other hand, in Eq. (2.54) the parameters M_3, M_2 and M_1 are the gluino, wino and bino mass terms (the adjoint representation gauge indices on the wino and gluino fields are suppressed). $\mathbf{A_u}$, $\mathbf{A_d}$, $\mathbf{A_e}$ are complex 3×3 matrices in flavour space that correspond to the (scalar)^3 couplings. The third line in Eq. (2.54) consists of squark and slepton mass terms. Again, $\mathbf{m^2_Q}$, $\mathbf{m^2_L}$, $\mathbf{m^2_{\bar{u}}}$, $\mathbf{m^2_{\bar{d}}}$, $\mathbf{m^2_{\bar{e}}}$ are 3×3 matrices in flavour space; these can have complex entries, but they must be hermitian for the Lagrangian to be real. Finally, the last line of Eq. (2.54) shows the SUSY-breaking contributions to the Higgs potential: $m^2_{H_u}$ and $m^2_{H_d}$ are squared-mass terms, while b is often written in the literature as $B\mu$. It should be stressed again that the parameters appearing in Eq. (2.54) are constrained to have a mass scale not much larger than $\mathcal{O}(1\,\mathrm{TeV})$ for the supersymmetric features not to be spoilt.

***R*-parity** The MSSM is referred to as *minimal* because the superpotential (2.53) does not include all the terms that are both gauge-invariant and analytic in the chiral superfields. These extra terms violate either baryon (B) or lepton (L) numbers or both, symmetries that are *exact* in the MSSM. In fact, it turns out that the most general gauge-invariant and renormalisable superpotential is Eq. (2.53) *plus* the following terms:

$$\begin{aligned} W_{\Delta L=1} &= \frac{1}{2}\lambda_1^{ijk} L_i L_j \bar{e}_k + \lambda_2^{ijk} L_i Q_j \bar{d}_k + \lambda_3^i L_i H_u \,, \\ W_{\Delta B=1} &= \frac{1}{2}\lambda_4^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k \,, \end{aligned} \tag{2.55}$$

where $i = 1, 2, 3$ is a family index.
The presence of the terms appearing in Eq. (2.55) is highly constrained by the lack of any evidence for B and L number violation. The most stringent constrain comes from the non-observation of proton decay, since the *simultaneous* presence of the λ_2 and λ_4 couplings would induce proton decay with a lifetime much shorter than the current experimental bound.
To avoid this difficulty, a multiplicatively conserved quantum number defined as

$$P_R = (-1)^{3(B-L)+2s} \tag{2.56}$$

is invoked. The quantity defined by Eq. (2.56) is referred to as R-parity, s being the spin of the particle. It should be noticed that particles within the same supermultiplet do not have the same R-parity, thus, SM particles do not share the same R-parity transformations with their superpartners. The conventional assignments for B and L lead to even R-parity ($P_R = +1$) for the SM fields and odd R-parity ($P_R = -1$) for their superpartners. Such a discrimination between particles and sparticles due to R-parity implies three important phenomenological consequences:

- The lightest particle with $P_R = -1$, called the *lightest supersymmetric particle* (LSP), must be absolutely stable.
- Each sparticle other than the LSP must eventually decay into a state containing an odd number of LSPs (usually just one).
- Sparticles can only be produced in even numbers (usually two-at-a-time).

Before ending the discussion of R-parity, we would like to stress one point. R-parity conservation is assumed in the MSSM to avoid a too rapid proton decay. Nevertheless, as stated above, *both* B and L numbers must be violated for the proton to decay. Put it in other words, it is possible to retain a subset of couplings in Eq. (2.55) which violate B *or* L numbers, breaking R-parity but maintaining proton stability.

Part II

Analysis

Chapter 3

Supersymmetry Phenomenology

So far, we have provided the general ideas that are currently investigated in the literature, with no special emphasis on any topic other than, mostly, supersymmetric gauge theories. Next, we shall be discussing the particular ideas among those hitherto addressed that directly connect to the present work, whose main results will be analysed in Chapters 4 and 5.
As massive neutrinos is a major motivation for this work, we will first address the issue of the smallness of neutrino masses in terms of the see-saw mechanism. Next, we shall be embedding the neutrino mass matrices into a broader framework, that of family symmetries, which also accommodates the rest of fermion masses, while providing some clues so as to describe its peculiar patterns, focusing on an SU(5) gauge group.
In § 3.3, we will address the link between particle physics and cosmology by means of the Dark Matter candidates within R-parity conserved models, where the lightest neutralino is the LSP. Next, we shall be discussing LFV issues. § 3.4 will provide brief remarks on the phenomenon of LFV, whereas § 3.5 comments on left-right symmetric models as compelling scenarios for acceptable LFV rates. Finally, we shall be commenting on some rare processes whose existence will give us definite evidence for physics beyond the SM.

3.1 Neutrino Phenomenology: See-saw Mechanism

As already mentioned, the SM demands massless neutrinos, a feature that must somehow be removed, as there is by now convincing evidence for neutrino oscillations. The observed oscillations require tiny masses for the neutrinos. Fermion masses within the SM can be accommodated through a proper choice of Yukawa couplings, leading to a mass difference within the same generation of $1-2$ orders of magnitude. Unlike charged fermions, the inclusion of neutrinos yields a huge disparity of the fermion masses *within* each generation.
To understand this, one should appeal to a mechanism which accounts for such small neutrino masses. The so-called *see-saw* mechanism claims to do so [Gell-Mann et al., 1979, Yanagida, 1979][1].
The see-saw mechanism provides a very simple and appealing explanation for the smallness

[1]It is a common practice to invoke symmetry arguments to make sense of an unnaturally small value of a given observable. The symmetry motivating the see-saw mechanism makes use of the existence of very heavy

of neutrino masses. It relates the smallness of m_ν to the existence of very large mass scales. Although the see-saw mechanism is most naturally accommodated within the framework of Grand Unified Theories (GUT), it also works for minimal extensions of the SM which solely add an electroweak-singlet right-handed neutrino field ν_R.
The most general mass term for n generations of both left- and right-handed neutrinos is given by

$$-\mathcal{L}_m = \frac{1}{2} n_L^T C \mathcal{M} n_L + \text{hc}\,, \tag{3.1}$$

where

$$\mathcal{M} = \begin{pmatrix} m_L & m_D \\ m_D^T & M_R \end{pmatrix} . \tag{3.2}$$

Here $n_L = (\nu_L, (\nu_R)^c) = (\nu_L, \nu_L^c)$ is the vector of $2n$ left-handed fields (written in a row-like style for simplicity), m_L and M_R are complex symmetric $n \times n$ matrices and m_D is a complex $n \times n$ matrix. Let us briefly see how the see-saw mechanism is implemented.
We would like to block-diagonalise the matrix $\mathcal{M}$, so as to decouple light and heavy neutrino degrees of freedom:

$$n_L = U \chi_L, \qquad U^T \mathcal{M} U = \begin{pmatrix} \tilde{m}_L & 0 \\ 0 & \tilde{M}_R \end{pmatrix} , \tag{3.3}$$

with U an unitary $2n \times 2n$ matrix. The physical masses (eigenvalues) become

$$\tilde{m}_L \simeq m_L - m_D M_R^{-1} m_D^T, \quad \tilde{M}_R \simeq M_R\,. \tag{3.4}$$

Thus, the diagonalisation of the effective mass matrix $\tilde{m}_L$ yields n light Majorana particles which are predominantly composed of the usual active neutrinos ν_L. Likewise, diagonalisation of $\tilde{M}_R$ produces n heavy Majorana neutrinos which are mainly composed of ν_R. It is worth noting that considering the largest Dirac mass of the order of the electroweak scale, $m_D \approx$ 200 GeV, one obtains the mass of the heaviest of the light neutrinos, $m_\nu \approx (10^{-2} - 10^{-1})$ eV, provided the right-handed scale is as high as $M_R \approx 10^{15} - 10^{14}$ GeV, which strikingly happens to be close to the GUT scale.
The see-saw mechanism throughout our work, though somewhat hidden, plays a key role when considering the energy evolution of the SUSY-GUT models, a fact that will be essential in our analysis, as will be shown in Chapters 4 and 5.

3.2 Neutrino Mass Matrices

The pattern of fermion masses and mixings is one of the most compelling mysteries in particle physics. The reasons for the large hierarchies in the fermion mass matrices and the origin of mixing terms remain unclear. The need for a convincing explanation of this issue has become urgent due to the clear evidence for neutrino oscillation and masses, which renders the pattern of fermion masses and mixings even more peculiar.
Along these lines, several attempts have been put forward in the literature. Among them, flavour or horizontal symmetries are particularly appealing (see [Froggatt and Nielsen, 1979]

right-handed neutrinos, which are missed within the SM.

for the pioneering work; see [Dudas et al., 1995, Binetruy et al., 1996, Barbieri et al., 1997, Irges et al., 1998] for early implementations; see [King and Ross, 2001, King and Ross, 2003, Altarelli and Feruglio, 2004, Antusch et al., 2008, Antusch et al., 2009] for more recent work). The basic idea is that only third generation entries are non-zero as long as the family symmetry remains unbroken, whereas the remaining entries are generated through non-renormalisable terms after symmetry breaking by fields acquiring non-zero vacuum expectation values (vevs).

In supersymmetric (SUSY) theories, the most general mass matrices (Yukawa and soft terms) involve off-diagonal entries and complex phases that lead to unacceptably large Flavour Changing Neutral Currents (FCNC) and CP-violating vertices. Two popular solutions usually adopted in the literature to solve this problem are either to consider universal soft terms at the high scale [Ellis and Nanopoulos, 1982] or to invoke some kind of alignment among the Yukawa textures and the soft terms [Nir and Seiberg, 1993]. Whichever option is taken, we should keep in mind that RGE evolution from the high scale down to low energies generates off-diagonal terms through the Yukawa couplings (from the fermion sector) as well as through the scalar sector.

Let us have a closer look at the general idea behind family symmetries (For the sake of simplicity, we shall focus on the Abelian case). The main idea is that the Yukawa couplings can be written as

$$(Y_f)_{ij} \propto \left(\frac{\langle\theta\rangle}{M}\right)^{\alpha^f_{ij}} , \tag{3.5}$$

where $\langle\theta\rangle$ is the non-zero vev acquired by scalar fields known as *flavons*, M is the family symmetry breaking scale and α^f_{ij} is a function of the $U(1)_F$ charges. As an illustration, let us consider the $U(1)_F$ charges assignments for the MSSM states, as shown in Table 3.1.

Fields	$\hat{Q}_i$	$\hat{U}_i$	$\hat{D}_i$	$\hat{L}_i$	$\hat{E}_i$	$\hat{N}_i$	$\hat{H}_d$	$\hat{H}_u$
$U(1)_F$	α_i	β_i	γ_i	δ_i	λ_i	ξ_i	μ	ν

Table 3.1: $U(1)F$ charge assignments for the MSSM fields.

with $i = 1, 2, 3$ being a generation index. Thus, a coupling of the form $Y^u_{13}u^c tH_u$ becomes $u^c tH_u(\langle\theta\rangle/M)^{\alpha_3+\beta_1+\nu}$. Thus, to ensure that the only non-vanishing element *before* the family symmetry is broken is the (3,3) entry ($u^c \to t^c$ above), we impose $\alpha_3 + \beta_3 + \nu = 0 \to \nu = -(\alpha_3 + \beta_3)$. Likewise, the charge for the down-type Higgs is obtained by imposing the vanishing of the (3,3) element involving either the b-quark or the τ lepton. This procedure allows one to build the full Yukawa matrices.

Let us now discuss the construction of the light neutrino mass matrix in a $SU(5)$ framework. In this model, the left-handed neutrino fields ν_L are accommodated in the $\bar{5}$ representation, while the right-handed ones N are singlets. Thus, the Dirac mass term $\nu_L N H_u$ becomes

$$m^D_\nu \propto \begin{pmatrix} \varepsilon^{\delta_1+\xi_1+\nu} & \varepsilon^{\delta_1+\xi_2+\nu} & \varepsilon^{\delta_1+\xi_3+\nu} \\ \varepsilon^{\delta_2+\xi_1+\nu} & \varepsilon^{\delta_2+\xi_2+\nu} & \varepsilon^{\delta_2+\xi_3+\nu} \\ \varepsilon^{\delta_3+\xi_1+\nu} & \varepsilon^{\delta_3+\xi_2+\nu} & \varepsilon^{\delta_3+\xi_3+\nu} \end{pmatrix} . \tag{3.6}$$

In addition, we also have the Majorana mass term $NN\Sigma$, where Σ is an $SU(3)_C \otimes SU(2)_L \otimes$

$U(1)_Y$ invariant Higgs field, σ being its $U(1)_F$ charge. Then, the right-handed Majorana neutrino mass matrix becomes

$$m_\nu^M \propto \begin{pmatrix} \varepsilon^{2\xi_1+\sigma} & \varepsilon^{\xi_1+\xi_2+\sigma} & \varepsilon^{\xi_1+\xi_3+\sigma} \\ \varepsilon^{\xi_2+\xi_1+\sigma} & \varepsilon^{\xi_2+\xi_2+\sigma} & \varepsilon^{\xi_2+\xi_3+\sigma} \\ \varepsilon^{\xi_3+\xi_1+\sigma} & \varepsilon^{\xi_3+\xi_2+\sigma} & \varepsilon^{\xi_3+\xi_3+\sigma} \end{pmatrix} . \tag{3.7}$$

Note that the neutrino mass matrices above do not exhibit a hierarquical structure, as it is the *effective* light neutrino mass matrix that is accessible to experiment, hence no unit element is *a priori* located at the (3,3) entries of either Dirac or Majorana matrices. Following the see-saw mechanism, the light neutrino mass matrix is

$$m_\nu^{eff} \approx m_\nu^D (m_\nu^M)^{-1} (m_\nu^D)^T . \tag{3.8}$$

For the sake of simplicity, we shall consider a vanishing $U(1)_F$ charge for the Higgs fields (not for the Σ Higgs-like field). The effective neutrino mass matrix can be written as

$$m_\nu^{eff} \propto \begin{pmatrix} \varepsilon^{2x} & \varepsilon^{x+y} & \varepsilon^{x} \\ \varepsilon^{x+y} & \varepsilon^{2y} & \varepsilon^{y} \\ \varepsilon^{x} & \varepsilon^{y} & 1 \end{pmatrix} . \tag{3.9}$$

This texture can potentially lead to large solar and atmospheric mixings. We shall be focusing on a particular set of $U(1)_F$ charges in Chapter 5, when we will discuss the implications for Lepton Flavour violating processes.

3.3 Neutralino and WMAP Dark Matter

One of the more striking problems in contemporary high-energy physics is the mysterious Dark Matter, whose abundance greatly exceeds that of bright matter. In fact, reconciling the CDM predictions of supersymmetric models with the stringent constraints from the WMAP data has been one of the challenges within the particle physics community in recent years. The amount of CDM deduced from the WMAP data is given by [Spergel et al., 2007]

$$\Omega_{CDM} h^2 = 0.111^{+0.011}_{-0.015} , \tag{3.10}$$

where h is the Hubble parameter in units of 100km/s/Mpc and $\Omega_{CDM} = \rho_{CDM}/\rho_c$ the ratio of the matter density of cold dark matter, ρ_{CDM}, over the critical density, ρ_c, that leads to a flat Universe. This puts severe constraints on the type and parameter space of unified theories with Dark Matter candidates. Recall that such a candidate must be electrically neutral and weakly interacting, as is the case for WIMPS.

One of the most promising frameworks in this respect is provided by supergravity (SUGRA) (*i.e., local* SUSY) models that conserve R-parity (for original works, [Chamseddine et al., 1982, Barbieri et al., 1982, Nath et al., 1983, Hall et al., 1983]; see [Nath, 2003] for a review), and thus predict a stable lightest supersymmetric particle (LSP). As a result, these models provide a natural candidate for Dark Matter, which is called the *neutralino* [Goldberg, 1983, Ellis et al., 1984] (It is worth saying that the scalar partner of the graviton, known as *gravitino*, is also well-motivated as a Dark Matter candidate [Ellis et al., 2004])[2]. As we will assume throughout the subsequent work R-parity conserved models, we shall focus on the lightest neutralino as a Dark Matter candidate. For completeness, let us briefly discuss their main properties.

The higgsinos and electroweak gauginos mix with each other because of electroweak symmetry breaking. The neutral higgsinos, $\tilde{H}_u^0$, $\tilde{H}_d^0$, and the neutral gauginos, $\tilde{B}$, $\tilde{W}^0$, combine to form four mass eigenstates called *neutralinos*. Likewise, the charged higgsinos, $\tilde{H}_u^+$, $\tilde{H}_d^-$, and winos, $\tilde{W}^+$, $\tilde{W}^-$, mix to form two mass eigenstates with charge ± 1 referred to as *charginos*. We will denote the neutralino and chargino mass eigenstates by $\tilde{N}_i$ $(i = 1, 2, 3, 4)$ and $\tilde{C}_i^\pm$ $(i = 1, 2)$. By convention, these are labelled in ascending order, so that $m_{\tilde{N}_1} < m_{\tilde{N}_2} < m_{\tilde{N}_3} < m_{\tilde{N}_4}$ and $m_{\tilde{C}_1} < m_{\tilde{C}_2}$.

In the gauge-eigenstate basis $\psi^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0)$, the neutralino mass part of the Lagrangian becomes

$$\mathcal{L}_N^m = -\frac{1}{2}(\psi^0)^T \mathcal{M}_{\tilde{N}} \psi^0 + \mathrm{hc}\,, \tag{3.11}$$

where

$$\mathcal{M}_{\tilde{N}} = \begin{pmatrix} M_1 & 0 & -g'v_d/\sqrt{2} & g'v_u/\sqrt{2} \\ 0 & M_2 & gv_d/\sqrt{2} & -gv_u/\sqrt{2} \\ -g'v_d/\sqrt{2} & gv_d/\sqrt{2} & 0 & -\mu \\ g'v_u/\sqrt{2} & -gv_u/\sqrt{2} & -\mu & 0 \end{pmatrix}. \tag{3.12}$$

The entries M_1 and M_2 in this matrix come directly from the MSSM soft Lagrangian [see Eq. 2.54], while the entries $-\mu$ are the supersymmetric higgsino mass terms [see Eq. 2.53]. The terms proportional to g, g' are the result of Higgs-higgsino-gaugino couplings [see Eq. 2.49], with the Higgs scalars replaced by their VEVs. In terms of physical masses, this matrix can be recast in the form

$$\mathcal{M}_{\tilde{N}} = \begin{pmatrix} M_1 & 0 & -c_\beta s_W M_Z & s_\beta s_W M_Z \\ 0 & M_2 & c_\beta c_W M_Z & -s_\beta c_W M_Z \\ -c_\beta s_W M_Z & c_\beta c_W M_Z & 0 & -\mu \\ s_\beta s_W M_Z & -s_\beta c_W M_Z & -\mu & 0 \end{pmatrix}, \tag{3.13}$$

where we have introduced the abbreviations $s_\beta \equiv \sin\beta, c_\beta \equiv \cos\beta, s_W \equiv \sin\theta_W, c_W \equiv \cos\theta_W$ (recall that $\tan\beta \equiv v_u/v_d$ and θ_W is the weak mixing angle). The mass matrix $\mathcal{M}_{\tilde{N}}$ can be diagonalised by a unitary matrix $\mathbf{N}$ so as to obtain mass eigenstates:

$$\tilde{N}_i = \mathbf{N}_{ij}\psi_j^0\,, \tag{3.14}$$

so that

$$\mathbf{N}^* \mathcal{M}_{\tilde{N}} \mathbf{N}^{-1} = \begin{pmatrix} m_{\tilde{N}_1} & 0 & 0 & 0 \\ 0 & m_{\tilde{N}_2} & 0 & 0 \\ 0 & 0 & m_{\tilde{N}_3} & 0 \\ 0 & 0 & 0 & m_{\tilde{N}_4} \end{pmatrix} \tag{3.15}$$

[2] In fact, gravitinos can be dark matter even in theories with R-parity violation, if their lifetime is larger than the age of the universe [Takayama and Yamaguchi, 2000, Buchmuller et al., 2007, Lola et al., 2007, Bomark et al., 2009].

has real positive entries on the diagonal.
In general, the parameters M_1, M_2 and μ in the equations above can have arbitrary complex phases. A redefinition of the phases of $\tilde{B}$ and $\tilde{W}$ always allows us to choose a convention in which both M_1 and M_2 are real and positive. The phase of μ within that convention is then really a physical parameter and cannot be rotated away. However, in this complex μ scenario there can be potentially disastrous CP-violating effects in low-energy physics, including electric dipole moments for both the electron and the neutron. Therefore, it is usual (and it is the choice made in the subsequent work) to assume μ to be real in the same set of conventions that make M_1, M_2, b, $\langle H_u^0 \rangle$ and $\langle H_d^0 \rangle$ real and positive. The sign of μ remains however unconstrained.
Let us now discuss the role of the lightest neutralino as a Dark Matter candidate (for a pedagogigal discussion, see [Trodden and Carroll, 2004]). We shall first recall the relic abundance of particles that decouple in an expanding universe (As we will be focusing on potential candidates for *Cold* Dark Matter, no discussion on *relativistic* particles will be provided).
In the early universe, matter and radiation were in thermal equilibrium. Because of the expansion, however, eventually the density becomes so low that interactions are no longer frequent enough, and the particles decouple or *freeze-out*. As mentioned above, we shall discuss the case when the particle is non-relativistic (cold) at the time of decoupling. Generally, any reliable account requires the detailed conditions of the freeze-out process, which amounts to numerically integrating the Boltzmann equation; however, a reasonable approximate expression is sufficient for most purposes. If σ_0 is the annihilation cross-section of the species X at a temperature $T = m_X$, the final number density in terms of the photon density is

$$n_X(T < T_f) \simeq \frac{n_\gamma}{\sigma_0 m_X M_P}\,, \tag{3.16}$$

M_P being the Planck scale.
Since the particles are non-relativistic when they decouple, they will certainly be non-relativistic today, and their energy density is

$$\rho_X = m_X n_X\,. \tag{3.17}$$

Thus, the density parameter becomes

$$\Omega_X \equiv \frac{\rho_X}{\rho_c} \simeq \frac{n_\gamma}{\sigma_0 M_P^3 H_0^2}\,. \tag{3.18}$$

In the case of WIMPs, which are weakly interacting, the annihilation cross-section yshould be $\sigma_0 \simeq \alpha_W^2 G_F$, where α_W is the weak coupling constant and G_F the Fermi constant. By plugging in numerical values, we find $\Omega_X \approx 1$; thus, a stable particle with a weak interaction cross-section naturally produces a relic density of order the critical density today.
Next, we will apply these ideas to the case of the lightest neutralino as the LSP. A recurring feature of many models of supersymmetry breaking is that the lightest neutralino is mostly bino *i.e.*, the supersymmetric partner of the neutral gauge boson of $U(1)$. It turns out that in much of the parameter space not already ruled out by LEP with a bino-like $\tilde{N}_1$, the predicted relic density is too high, either because the LSP couplings are too small or the sparticles are too heavy, or both, leading to a thermal-averaged annihilation cross-section that is too low.

To avoid this, there must be significant contributions to this cross-section. The possibilities can be classified qualitatively in terms of the diagrams that contribute most strongly to the annihilation.

First, if at least one sfermion is not too heavy, the process is effective in reducing the Dark Matter density. In models with a bino-like $\tilde{N}_1$, the most important contribution usually comes from $\tilde{e}_R$, $\tilde{\mu}_R$ and $\tilde{\tau}_1$ exchange. The region of parameter space where this works out correctly, is often referred to as "bulk region". In the minimal supergravity (mSUGRA) inspired framework, the viable bulk region usually requires $m_0 < 100$ GeV and $M_{1/2} < 250$ GeV [Baer et al., 2003, Baer et al., 2004].

A second way of annihilating excess bino-like LSPs to the correct density is obtained if $2m_{\tilde{N}_1} \approx m_{A^0}$, m_{h^0} or m_{H^0}, as the last diagram in Fig. 3.1, so that the cross-section is near a resonance pole. An A^0 resonance annihilation will be an s-wave, and therefore more efficient than a p-wave h^0, H^0 resonance. Since the $A^0 f\bar{f}$ coupling is proportional to $m_f \tan\beta$, this usually implies large values of $\tan\beta$. The region of parameter space where this happens is often called the "A-funnel" or "Higgs funnel" or "Higgs resonance" region.

A third effective annihilation mechanism is obtained if $\tilde{N}_1$ mixes to obtain a significant higgsino or wino admixture. In the "focus point" region of parameter space, where $|\mu|$ is not too large, an LSP with a significant higgsino content can yield the correct relic abundance even for heavy squarks and sleptons. It is also possible to arrange for just enough wino content in the LSP to do the job, by an appropriate choice of M_1/M_2.

A fourth possibility, the "sfermion co-annihilation region" of parameter space, is obtained if there is a sfermion that happens to be less than a few GeV heavier than the LSP. In many models, this is most naturally the lightest stau, but it could also be the lightest stop. A significant density of this sfermion will then coexist with the LSP around the freeze-out time; as a result, annihilations involving the sfermion with itself or with the LSP, including those in Fig. 3.1, will further dilute the number of sparticles and thus the eventual Dark Matter density.

In § 4.2 we will show that $b - \tau$ Yukawa-unified models with a mostly bino-like neutralino LSP are compatible with WMAP Dark Matter constraints.

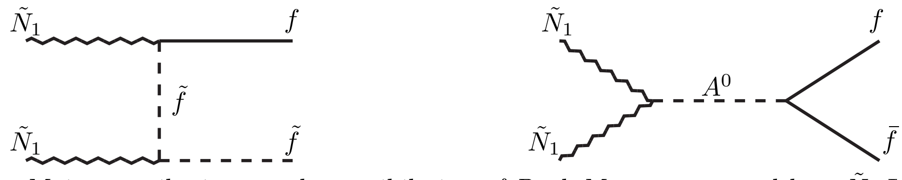

Figure 3.1: *Main contributions to the annihilation of Dark Matter composed by a* $\tilde{N}_1$ LSP from (a) t-channel slepton and squark exchange and (b) near-resonant annihilation through a Higgs boson.

3.4 Charged-Lepton Flavour Violation Processes

Charged-Lepton Flavour Violation (LFV) would provide us with an evidence for physics beyond the Standard Model, since within the latter, lepton-flavour number is exactly conserved (see [Chiang et al., 1993, Hisano et al., 1996]. Discussion of LFV processes within the see-saw model can be found in [Hisano et al., 1995, Deppisch and Valle, 2005, Deppisch et al., 2006]). One of the minimal extensions of the Standard Model with LFV is its enhancement with non-vanishing neutrino masses. If the masses of the neutrinos are induced by the see-saw mechanism, one has a new set of Yukawa couplings involving the right-handed neutrinos. Introduction of the new Yukawa couplings generally gives rise to flavour violation in the lepton sector, similar to its quark sector counterpart. In non-supersymmetric standard models, however, the amplitudes of the LFV processes are proportional to inverse powers of the right-handed neutrino mass scale, which is typically much higher than the electroweak scale. As a consequence such rates are highly suppressed.
In a supersymmetric SM, as MSSM, the situation becomes quite different. LFV in the right-handed neutrino Yukawa couplings leads to LFV in slepton masses through renormalisation-group effects. Then the LFV processes are only suppressed by powers of the supersymmetry breaking scale, which is assumed to be not far from the electroweak scale. A large left-right mixing of the slepton masses in particular, greatly enhances LFV rates, such as $\mu \to e\gamma$ and $\tau \to \mu\gamma$, whose dominant diagrams are shown in Fig. 3.2. Due to this effect, they can be within the reach of near future experiments even if the mixing angle of the lepton sector is as small as the respective one of the quark sector.
The structure of the SUSY breaking mass matrices for sleptons depends on the mechanism to generate the SUSY breaking terms in the MSSM. One of the most popular scenarios is that of minimal supergravity (mSUGRA), where one assumes that the supersymmetry breaking masses for squarks, sleptons and the Higgs bosons are *universal* at tree-level. This is the simplest way out to avoid unacceptable large Flavour Changing Neutral Current (FCNC) processes.
However, universality of the SUSY breaking masses for the scalar bosons is not stable under radiative corrections. Assuming that below a scale M_{grav}, where the gravitational interaction may be neglected, sources of LFV do exist, the low-energy mass matrices are sensitive to the supersymmetric corrections induced all the way down to the electroweak scale, as given by the MSSM RGEs (providing evidence for physics beyond the SM).
Nevertheless, the current experimental bounds on LFV processes are hard to implement. In fact, recent measurements provide the following bounds [Amsler et al., 2008]:

$$BR(\tau \to \mu\gamma) < 4.5 \times 10^{-8}, \quad BR(\tau \to e\gamma) < 1.1 \times 10^{-7}, \quad BR(\mu \to e\gamma) < 1.2 \times 10^{-11}\,. \tag{3.19}$$

We shall have a closer look at these LFV processes in Chapter 5 within the cosmologically favoured parameter space of Yukawa-unified models with lepton mixing, supplemented with an Abelian family symmetry.

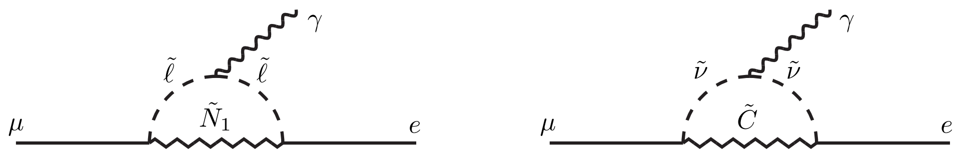

Figure 3.2: *Main contributions to the $\mu \to e\gamma$ decay in the MSSM.*

3.5 Left-right symmetric models

The marriage between SUSY and GUT provides a fertile arena beyond the SM to successfully account for low energy physics. For illustrative purposes, the simplest GUT model (based on $SU(5)$) was briefly described above. Despite its appeal, however, there is a somewhat awkward issue, since the fermion fields are not equally treated in $SU(5)$ (being accommodated within different representations). This is remedied in *left-right* models, $SO(10)$ being the most prominent candidate.

The aim of this section is to analyse realistic Yukawa textures arising in $SO(10)$-like models, incorporating the successful prediction of the $m_\tau/m_b \simeq 3$ ratio, which is easily accounted for in *non*-symmetric scenarios such as $SU(5)$, but which calls for a closer analysis in $SO(10)$. For the sake of simplicity, we shall focus on one of such realistic textures, known as the Georgi-Jarlskog texture, featuring the following mass matrices:

$$\mathcal{Y}_u \propto \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix}, \quad \mathcal{Y}_d \propto \begin{pmatrix} 0 & F & 0 \\ F & E & 0 \\ 0 & 0 & D \end{pmatrix}, \quad \mathcal{Y}_e \propto \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix}, \tag{3.20}$$

where the factor of -3 ensures acceptable low energy physics.

$SO(10)$-like models enable us to produce textures as in (3.20) if non-minimal Higgs structures belonging to either $\mathbf{10}$ or $\overline{\mathbf{126}}$ representations are also included. On general grounds, the textures (3.20) are generated by Yukawa couplings of the form

$$\mathbf{16} \begin{pmatrix} 0 & F\mathbf{10^d} & 0 \\ F\mathbf{10^d} & E\overline{\mathbf{126}}^{\mathbf{d}} & 0 \\ 0 & 0 & D\mathbf{10^d} \end{pmatrix} \mathbf{16}\,, \tag{3.21}$$

for down quarks and charged leptons, and

$$\mathbf{16} \begin{pmatrix} 0 & CS^u & 0 \\ CS^u & 0 & BR^u \\ 0 & BR^u & AS^u \end{pmatrix} \mathbf{16}\,, \tag{3.22}$$

for up quarks, where R, S can be either a $\mathbf{10}$ or a $\overline{\mathbf{126}}$.

As we are mostly interested in the phenomenological properties of (3.20), we refer the reader to [Babu and Mohapatra, 1995] for the explicit derivation of the Georgi-Jarlskog textures

from (3.21, 3.22).
Once the Georgi-Jarlskog textures have been presented, we are able to exploit them at the end of section 5.3, in order to enhance the parameter space compatible with LFV constraints as compared to $SU(5)$ (which is the main focus in this work).

3.6 Some Constraints on New Physics

This section aims at providing the bridge between the general topics belonging to SUSY phenomenology discussed in the previous sections and the original research put forward in Chapters 4 and 5.
Let us begin by enumerating the specific topics addressed in the present work, so as to easily recognise the need for a prior discussion of related ideas.

- *b-quark mass and Yukawa Unification.* Since the present work deals with supersymmetric extensions of the SM with massive neutrinos, one should first analyse how the introduction of non-zero neutrino masses alters the general idea of (Yukawa) unification in SUSY models. We shall be recalling the main ideas put forward in the area in § 3.6.1.
- *Yukawa Unification and Dark Matter.* The discussion demands, on the basis established in the previous topic, a thorough analysis of the allowed supersymmetric parameter space, taking into account the stringent constraints resulting from the recent WMAP data. The main ideas are presented in § 3.6.2.
  However, there are additional constraints, related to particle-physics phenomenology, that must also be taken into account, further limiting the preferred parameter space. Such constraints are the following:
  1. *BR($b \to s\gamma$).* The 2-σ interval is constructed including an intrinsic $0.15 \cdot 10^{-4}$ MSSM correction as in Ref. [Ellis et al., 2007b], leading to
  $$2.15 \cdot 10^{-4} < BR(b \to s\gamma) < 4.9 \cdot 10^{-4}\,; \tag{3.23}$$
  without including the MSSM intrinsic error, the above range becomes
  $$2.8 \cdot 10^{-4} < BR(b \to s\gamma) < 4.4 \cdot 10^{-4}\,. \tag{3.24}$$
  2. *Mass of the lightest CP-even Higgs.* We take the conservative bound of $m_h =$ 114 GeV and also consider an uncertainty of $\sim$ 3 GeV [Heinemeyer et al., 2006] to its theoretical computation.
- *Lepton Flavour Violation.* We shall be briefly addressing the question as to how flavour violation arises in supersymmetric models in § 3.6.3.

3.6.1 Massive Neutrinos and Gauge and Yukawa Unification

The most straightforward extension of the Standard Model that can accommodate neutrino masses is to include three very heavy right-handed neutrino states and assume that the smallness of masses arises from the see-saw mechanism. In this case, the predictions for m_b and unification clearly get modified. In particular, radiative corrections from the neutrino Yukawa couplings have to be included for renormalisation group runs from M_{GUT} to M_N (scale of the heavy right-handed neutrinos) [Barger et al., 2008]. Below M_N, right-handed neutrinos decouple from the spectrum and an effective see-saw mechanism is operative (with the neutrino mass operator running down to low energies). The relevant equations are given in [Grzadkowski and Lindner, 1987, Pirogov and Zenin, 1999, Haba et al., 2000, Haba et al., 1999].

In addition, if the GUT scale lies significantly below a scale M_X, at which gravitational effects can no longer be neglected, the renormalisation of couplings at scales between M_X and M_{GUT} may induce additional effects to the running. The simplest such example is provided within the framework of the minimal supersymmetric SU(5) GUT, and the renormalisation group equations (RGEs) in this case are given in [Hisano and Nomura, 1999]. These runs affect Yukawa unification via the supersymmetric corrections to the bottom quark mass; however, since in the leading-logarithmic approximation the induced modifications to soft masses are proportional to the V_{CKM} mixing [Hisano and Nomura, 1999], they are significantly suppressed.

Nevertheless, it has been realized that the influence of the runs above the GUT scale on the Dark Matter abundance can be very sizeable [Calibbi et al., 2007], due to changes in the relation between $m_{\tilde{\tau}}$ and m_χ, which is crucial in the coannihilation area. In dark matter calculations, therefore, the impact of such effects has to be also analysed, and compared to the more standard scenarios.

Our starting point will be the $SU(5)$ superpotential at M_X which (omitting generation indices) is given by

$$\mathcal{W}_X = T^T\,\lambda_u^\delta\,T\,H + T^T\,\lambda_d\,\bar{F}\,\bar{H} + \bar{F}^T\,\lambda_N^\delta\,S\,H + S^T\,M_N\,S\,. \tag{3.25}$$

Here, $T, \bar{F}$ and S are the **10, 5*** and **1** $SU(5)$ superfields, respectively, and H and $\bar{H}$ are the **5** and **5*** Higgs superfields. The couplings $\lambda_{u,d,N}$ stand for the up-type quark, down-type quarks/charged lepton and Dirac neutrino Yukawa matrices. The symbol δ stands for *diagonal* (the up- and down-type quark Yukawa matrices cannot be diagonalised simultaneously). Finally, M_N is the right-handed neutrino mass matrix. In addition, we work with the soft SUSY-breaking Lagrangian

$$\begin{aligned}\mathcal{L}_{soft} \;=\;& \tilde{T}^\dagger\,m_{\mathbf{10}}^2\,\tilde{T} + \tilde{\bar{F}}^\dagger\,m_{\mathbf{5}}^2\,\tilde{\bar{F}} + \tilde{S}^\dagger\,m_{\mathbf{1}}^2\,\tilde{S} + \\ & m_h^2\,h^\dagger\,h + m_{\bar{h}}^2\,\bar{h}^\dagger\,\bar{h} + M_5\,\lambda_{5L}\,\lambda_{5L} + \text{h.c.} + \\ & \{\tilde{T}^T\,A_u\,\tilde{T}\,h + \tilde{T}^T\,A_d\,\tilde{\bar{F}}\,\bar{h} + \tilde{\bar{F}}^T\,A_N\,\tilde{S}\,h + \text{h.c.}\}\,,\end{aligned} \tag{3.26}$$

where $m_{\mathbf{10},\mathbf{5},\mathbf{1}}$ are the **10, 5*** and **1** $SU(5)$ superfields masses, respectively, and $m_{h,\bar{h}}$ are the **5** and **5*** Higgs superfields masses. $A_{u,d,N}$ are the up-type quark, down-type quark/charged lepton and Dirac neutrino trilinear terms. Finally, λ_{5L} is the $SU(5)$ gaugino and M_5 its soft Majorana mass. In the mSUGRA scenario, universal boundary conditions are assumed at the gravitational scale

$$m_{\mathbf{10}}^2 = m_{\mathbf{5}}^2 = m_{\mathbf{1}}^2 = m_h = m_{\bar{h}} = m_0^2\,, \tag{3.27}$$

$$A_f = a_0\,\lambda_f, \quad f = \{u, d, \nu\}\,. \tag{3.28}$$

The physical values of the masses will then be obtained by integrating the renormalisation group equations from M_X down to low energies. In the first part of the runs we work with the RGEs of SU(5) from the high energy scale M_X down to M_{GUT}. Subsequently, we run the MSSM RGEs, supplemented with right-handed neutrinos, from M_{GUT} to the right-handed neutrino scale, M_N. At this scale, right-handed neutrinos decouple from the spectrum, leaving us with the MSSM RGEs and the see-saw operator (which is also renormalised down to the low energy scale). The effects of neutrinos on Yukawa unification crucially depend on the magnitude of the dominant neutrino Dirac Yukawa coupling, λ_N, and consequently the magnitude of M_N (which determines λ_N through the See-Saw conditions). Obviously, for low M_N and λ_N, the effects are less significant.
Large neutrino Yukawa couplings affect unification by increasing the predicted value of $m_b(M_Z)$. This can be understood for small $\tan\beta$ by simple, semi-analytic expressions [Leontaris et al., 1995, Carena et al., 2000] since only the top and the Dirac-type neutrino Yukawa couplings (λ_t and λ_N) may be large at the GUT scale. In this case, the RGEs take simple form

$$\begin{aligned} 16\pi^2\frac{d}{dt}\lambda_t &= \left(6\lambda_t^2+\lambda_N^2-G_U\right)\lambda_t, \qquad 16\pi^2\frac{d}{dt}\lambda_N = \left(4\lambda_N^2+3\lambda_t^2-G_N\right)\lambda_N \\ 16\pi^2\frac{d}{dt}\lambda_b &= \left(\lambda_t^2-G_D\right)\lambda_b, \qquad 16\pi^2\frac{d}{dt}\lambda_\tau = \left(\lambda_N^2-G_E\right)\lambda_\tau\,. \end{aligned} \tag{3.29}$$

Here, λ_α, $\alpha = U, D, E, N$, represent the 3×3 Yukawa matrices for the up and down quarks, charged lepton and Dirac neutrinos, and $G_\alpha = \sum_{i=1}^{3} c_\alpha^i g_i(t)^2$ are functions of the gauge couplings with the coefficients c_α^i's as in [Vissani and Smirnov, 1994, Brignole et al., 1994]. Denoting by $\lambda_{b_0}, \lambda_{\tau_0}$ the b and τ couplings at the unification scale, it is found that

$$\lambda_b(t_N) = \rho\xi_t\frac{\gamma_D}{\gamma_E}\lambda_\tau(t_N),\ \ \rho = \frac{\lambda_{b_0}}{\lambda_{\tau_0}\xi_N}\,, \tag{3.30}$$

where $\gamma_\alpha(t)$ and ξ_i depend only on gauge and Yukawa couplings. Let us then assume successful $b-\tau$ unification at M_{GUT}, with $\lambda_{\tau_0} = \lambda_{b_0}$. In the absence of right-handed neutrinos $\xi_N \equiv 1$, thus $\rho = 1$. In the presence of them, however, $\lambda_{\tau_0} = \lambda_{b_0}$ at the GUT scale implies that $\rho \neq 1$ (since $\xi_N < 1$). To restore ρ to unity, a deviation from $b-\tau$ unification would seem to be required. However, large lepton mixing is reconciled with Yukawa unification, by making the simple observation that the $b-\tau$ equality at the GUT scale refers to the $(3,3)$ entries of the charged lepton and down quark mass matrices. It is then possible to assume mass textures, such that, after the diagonalisation at the GUT scale, the $(m_E^{diag})_{33}$ and $(m_D^{diag})_{33}$ entries are no-longer equal [Leontaris et al., 1995]. The simplest example is the one with symmetric mass matrices:

$$\mathcal{M}_d^0 \propto A\begin{pmatrix} y & 0 \\ 0 & 1\end{pmatrix},\ \ \mathcal{M}_\ell^0 \propto A\begin{pmatrix} x^2 & x \\ x & 1\end{pmatrix}. \tag{3.31}$$

Here, A may be identified with $m_b(M_{GUT})$. While the textures ensure equality of the (3,3) elements of the down and charged lepton mass matrices, the eigenvalue of the charged lepton mass matrix is not 1, but $1+x^2$, thus implying that $\lambda_b \neq \lambda_\tau$ after diagonalisation. Within this framework, the issue of Yukawa Unification in the presence of large lepton mixing has been analysed in [Leontaris et al., 1995, Carena et al., 2000] for small and moderate values of $\tan\beta$.

In § 4.1, we will extend these results to all values of $\tan\beta$ and both signs of μ, taking appropriately into account the large supersymmetric corrections to m_b [Hall et al., 1994, Carena et al., 1994, Gomez et al., 2004], and using up-to-date experimental bounds. In practice, potentially large lepton mixing effects will be taken into account by imposing the GUT condition $\lambda_\tau = \lambda_b(1+\delta)$ where δ is determined by requiring that $m_b(M_Z)$ is in the allowed experimental range. The third generation Dirac neutrino Yukawa coupling λ_N is determined through the see-saw mechanism by assuming $m_{\nu_3} = 0.05$ eV and a heavy Majorana neutrino scale $M_N = 3\times10^{14}$ GeV (a value that ensures that λ_N stays within the perturbative regime). On the other hand, the behaviour associated with the absence of a Dirac neutrino coupling should be recovered by appropriately lowering M_N and thus λ_N.

The resulting effects to the allowed parameter space for unification are significant, and give interesting information on the magnitude and origin of lepton mixing. We then proceed to discuss in detail the cosmological implications in the framework of the WMAP data.

3.6.2 Yukawa Unification in Supersymmetric Models

While gauge unification is considered as one of the most attractive features of Supersymmetry, the relations among Yukawa couplings derived by embedding $SU(3)\otimes SU(2)\otimes U(1)$ in a larger gauge group are less clear. The charged fermion mass hierarchies indicate that any mass correlations induced by a unified gauge symmetry will be more explicitly manifest in the relation between the Yukawa couplings of the third generation, λ_t, λ_b, λ_τ. How this works depends on the theoretical framework that defines the initial conditions. In $SO(10)$, for instance, all fermions are in the same representation, thus, in the simplest realizations, neutrino couplings will be unified with the rest. In $SU(5)$, the field structure is $(Q,u^c,e^c)_i \in \mathtt{10}$ of $SU(5)$ and $(L,d^c)_i \in \overline{\mathtt{5}}$, only implying $b-\tau$ unification; neutrinos are singlets and thus there is a freedom of choice (although the fact that they couple to the same Higgs as the up-type quarks could be a motivation for some link with the top coupling). Clearly, the more restrictive the unification schemes, the stronger the correlations between quarks and leptons.

In supersymmetric models, unification turns out to be very sensitive to the model parameters. Among others, the compatibility of $b-\tau$ unification with the observed b and τ masses has a sensitive dependence on the sign of the Higgs mixing parameter, μ, and on the details of the superparticle spectrum [Komine and Yamaguchi, 2002]. Moreover, the universality condition on the soft terms can be removed as in [Gomez et al., 2000a, Gomez et al., 2000b, Baer et al., 2008], resulting in an enhancement of the allowed parameter space. In fact, it turns out that full third generation Yukawa Unification $\lambda_t = \lambda_b = \lambda_\tau$ in the CMSSM fails to accurately predict the experimental values of the third generation fermion masses, unless one goes to a large $\tan\beta$ regime with a heavy spectrum and a dark matter abundance that would tend to overclose the universe.

To correctly obtain pole masses within this framework, the standard model and supersymmetric threshold corrections have to be included; for the bottom quark, these corrections result to a Δm_b that can be very large, particularly for large values of $\tan\beta$. In mSUGRA, in the absence of phases, Δm_b has the sign of μ [Hall et al., 1994, Carena et al., 1994, Gomez et al., 2004], which is required in order to obtain a b-quark mass in the allowed range in models with $b-\tau$ unification. However, this also results to a positive supersymmetric contribution to BR($b \to s\gamma$), which can be compatible with the bounds only for a heavy sparticle spectrum. The allowed parameter space is nevertheless extremely constrained from the bounds on Flavour Changing Neutral Currents, and the new bounds on $b \to s\gamma$ [Barberio et al., 2007] are crucial for the whole discussion and comparisons with the SM prediction [Misiak et al., 2007, Misiak and Steinhauser, 2007]. The $g_\mu - 2$ constraints from Brookhaven National Laboratory (BbNL) [Bennett et al., 2004] are also relevant since the supersymmetric contribution to the muon anomalous magnetic moment takes the sign of μ in mSUGRA [Lopez et al., 1994, Chattopadhyay and Nath, 1996]. There are also some uncertainties $\mathcal{O}(\alpha^2)$ in the Standard Model prediction, due to the hadronic vacuum polarisation correction [Miller et al., 2007]. For a heavy sparticle spectrum the SUSY contribution to Δa_μ is small with respect to the Standard Model one. However, we do not take into account that constraint derived from $g_\mu - 2$, since there is no significant deviation from the SM predictions if the hadronic contribution is calculated using τ-decay data (the current experimental value deviates by 3.4 σ from the SM prediction if the hadronic vacuum polarisation is determined using e^+e^- annihilation data).

Let us also summarise a few facts on the possible range of the mass of the bottom quark: The 2-σ range for the $\overline{MS}$ bottom running mass, $m_b(m_b)$, is from 4.1-4.4 GeV (with corresponding pole masses from 4.7 to 5 GeV). We also know that $\alpha_s(M_Z) = 0.1172 \pm 0.002$, and the central value of α_s corresponds to $m_b(M_Z)$ from 2.82 to 3.06 GeV, while the value $m_b(m_b) = 4.25$ GeV is mapped to $m_b(M_Z) = 2.92$ GeV. The allowed strip for $m_b(M_Z)$ moves to 2.74 GeV $< m_b(M_Z) <$ 3.014 GeV for α_s^{max} and to 2.862 GeV $< m_b(M_Z) <$ 3.114 GeV for α_s^{min}.

In the left panel of Fig. 3.3, we summarise the predictions for m_b in mSUGRA, where all CP phases are either zero or π (defining the sign of μ). In order to discuss the dependence of $m_b(M_Z)$ on $\tan\beta$, we consider the following set of soft parameters: $M_{1/2} = 800$ GeV, $A_0 = 0$, $m_0 = 600$ GeV. We study this set for both $\mu > 0$ and $\mu < 0$, setting $\alpha_s(M_z) = 0.1172$. We also include a reference line without the SUSY corrections to the bottom mass (double-dot-dash line, for $\Delta m_b = 0$). The figure exhibits the known fact that in the absence of phases or large trilinear terms, Δm_b is positive for μ positive, and therefore the theoretical prediction for the b quark pole mass is too high to be reconciled with $b-\tau$ unification. On the other hand, for $\mu < 0$, Δm_b is negative and the theoretical prediction for the b-quark mass can lie within the experimental range for values of $\tan\beta$ between roughly 30 and 40; clearly, for a large $\tan\beta$ it is mandatory to take into account the large supersymmetric corrections to m_b [Hall et al., 1994, Carena et al., 1994, Gomez et al., 2004]. The figure also illustrates that, for $\mu > 0$ and after taken properly into account supersymmetric corrections, m_b scales very slowly with $\tan\beta$. In other words, the renormalisation flow for the bottom mass is attracted by some fixed value regardless of the initial conditions [Floratos et al., 1996, Carena et al., 1994] (which as we see in the figure turns out to be too large). This *quasi-fixed* point, however, is not generated purely by the renormalisation flow given by the RGE, and only appears after including supersymmetric corrections (the curve for $\Delta m_b = 0$ is not flat in Fig. 3.3). Furthermore, for $\mu < 0$ there is no fixed point at all.

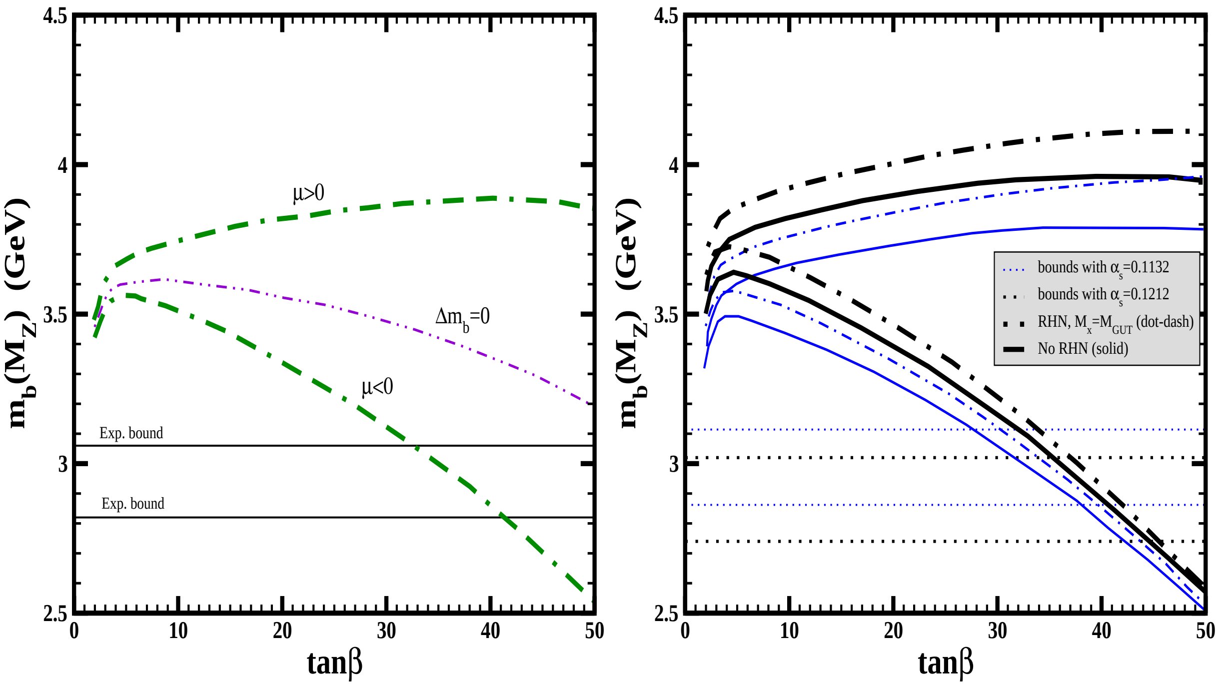

Figure 3.3: *The value of* $m_b(M_Z)$ *versus* $\tan\beta$ *assuming* $\lambda_b = \lambda_\tau$ *at the high scale in the absence (left plot) and presence (right plot) of massive neutrinos and for both signs of* μ *for the following set of parameters:* $M_{1/2} = 800$ *GeV ,* $A_0 = 0$ *GeV,* $m_0 = 600$ *GeV. The experimental range of* m_b *(horizontal lines) shown in the left panel is computed for the central value of* $\alpha_s(M_Z)$*. In the right panel, the solid lines are obtained within the MSSM, while the dot-dash lines include Dirac Yukawa couplings up to the scale* $M_N = 3 \times 10^{14}$ *GeV. Here we take the lowest (blue thin) and highest (black thick) experimental values of* $\alpha_s(M_Z)$*.*

In the right panel of Fig. 3.3 we repeat the analysis in the presence of massive neutrinos, keeping only the third generation couplings (and ignoring lepton mixing effects) from the M_{GUT} to the scale of the right-handed neutrino masses, M_N, and evolve the light neutrino mass operator from this scale down to M_Z. A large value of the Dirac-type neutrino Yukawa coupling, λ_N, at the GUT scale may arise naturally within the framework of Grand Unification, and its value is determined, through the see-saw mechanism, by demanding a third generation low energy neutrino mass of $m_{\nu_3} = 0.05$ eV and evaluating the respective heavy neutrino Majorana mass from the see-saw conditions. The predictions for $m_b(M_Z)$ using the lower and upper bounds of the 2-σ experimental range of α_s and the corresponding range for $m_b(M_Z)$ after the evolution of the bounds on $m_b(m_b)$ are shown for $M_N = 3 \times 10^{14}$ GeV. We use $m_t = 172.6$ GeV [Lancaster, 2008] in all computations.
We observe that for $\mu > 0$ the prediction for $m_b(M_Z)$ is always very large, despite its dependence on the soft terms through Δm_b. For $\mu < 0$, there is a window of values of $\tan\beta$ compatible with asymptotic $b-\tau$ Yukawa unification. For the values of the soft terms considered in Fig. 3.3, the allowed range of $\tan\beta$ moves from $27 - 44$ to $30 - 45$ when we introduce the effect of see-saw neutrinos. Let us stress that $b-\tau$ Yukawa unification is (is not) compatible with m_b, for $\mu < 0\,(> 0)$, regardless on whether or not massive neutrinos are included,

when lepton mixing effects are ignored.

3.6.3 Charged Lepton Flavour Violation in Supersymmetric Models

In SUSY theories, charged lepton flavour violation may be generated at the loop-level, even in models with universal soft terms at a high scale. This is also true for the MSSM, when extended with a see-saw mechanism to generate small neutrino masses [Hisano et al., 1996, Hisano and Nomura, 1999]. In this case, LFV terms are generated radiatively, since it is not possible to simultaneously diagonalise neutrino, charged lepton and slepton mass matrices [Borzumati and Masiero, 1986].

In order to explain the observed lepton hierarchies by suitable Yukawa textures, we introduce flavour symmetries, which may also predict the structure of the soft mass terms. LFV processes like $l_i \to l_j \gamma$ impose severe constraints on the allowed patterns [Leontaris and Tracas, 1998, Gomez et al., 1999, Chankowski et al., 2005]. In this work we focus on the implications of massive neutrinos in SU(5) Yukawa unification with an additional $U(1)_F$ family symmetry, including RGE effects not only below, but also above M_{GUT}.

We start by considering the following SUSY $SU(5)$ superpotential:

$$\mathcal{W}_X = T_1^T\, \mathcal{Y}_u^\delta\, T_1\, H + T_1^T\, \bar{\mathcal{Y}}_d\, \bar{F}_1\, \bar{H} + \bar{F}_1{}^T\, \mathcal{Y}_\nu^\delta\, S_1\, H + S_1^T\, \bar{M}_R\, S_1\,, \tag{3.32}$$

where $\mathcal{Y}_\alpha$ ($\alpha = u, d, \nu$) are the Yukawa matrices for the up-type quarks, down-quarks/charged-leptons and Dirac neutrinos, respectively. M_R is the heavy Majorana mass matrix. The symbol δ stands for *diagonal*, the original fields rotated as

$$T = U_{10} T_1, \qquad \bar{F} = U_{\nu L} \bar{F}_1, \qquad S = U_{\nu R} S_1\,, \tag{3.33}$$

with the rotation matrices defined as

$$\mathcal{Y}_u = U_{10} \mathcal{Y}_u^\delta U_{10}^T, \qquad \mathcal{Y}_d = U_{5L}^* \mathcal{Y}_d^\delta U_{5R}^\dagger, \qquad \mathcal{Y}_\nu = U_{\nu L}^* \mathcal{Y}_\nu^\delta U_{\nu R}^\dagger\,, \tag{3.34}$$

and

$$\bar{\mathcal{Y}}_d = V_{CKM}^* \mathcal{Y}_d^\delta V_E^\dagger\,. \tag{3.35}$$

Here, $V_{CKM} = U_{10}^\dagger U_{5L}$ and $V_E = U_{\nu L}^\dagger U_{5R}$ denote the mixings in the quark and lepton sectors, while $\bar{M}_R = U_{\nu R}^T M_R U_{\nu R}$.

The off-diagonal contributions to slepton mass matrices, when the superfields are rotated so that charged leptons become diagonal, can be understood through three rotations at different energy scales:

- M_X. The rotations in the superpotential fields lead to the following transformation of the soft terms:

$$\bar{m}_{10}^2 = U_{10}^\dagger m_{10}^2 U_{10}, \qquad \bar{m}_5^2 = U_{\nu L}^\dagger m_5^2 U_{\nu L}\,. \tag{3.36}$$

- M_{GUT}. Assuming that $SU(5)$ is broken down to the MSSM gauge group, the superpotential becomes

$$\mathcal{W}_{MSSM} = Q^T \mathcal{Y}_u^\delta U H_2 + Q^T (V_{CKM}^* \mathcal{Y}_d^\delta) D H_2 + L^T (V_E^* \mathcal{Y}_d^\delta) E H_2 + L^T \mathcal{Y}_\nu^\delta S H_2 + S^T \bar{M}_R S \,, \tag{3.37}$$

where we have absorbed the matrices V_E^* and V_{CKM}^* in the definitions of the superfields E and D respectively. The scalar soft masses then become:

$$\begin{aligned} m_E^2 &= V_{CKM}^\dagger \bar{m}_{10}^2 V_{CKM}, \qquad m_L^2 = \bar{m}_5^2 \,, \\ m_Q^2 &= m_U^2 = \bar{m}_{10}^2, \qquad m_D^2 = V_E^\dagger \bar{m}_5^2 V_E \,. \end{aligned} \tag{3.38}$$

We write m_D^2 and m_E^2 in the following way:

$$\begin{aligned} m_D^2 &= V_E^\dagger \bar{m}_5^2 V_E \\ &= U_{5R}^\dagger U_{\nu L} U_{\nu L}^\dagger m_5^2 U_{\nu L} U_{\nu L}^\dagger U_{5R} \\ &\simeq U_{5R}^\dagger m_5^2 U_{5R}, \\ m_E^2 &= V_{CKM}^\dagger \bar{m}_{10}^2 V_{CKM} \\ &= U_{5L}^\dagger U_{10} U_{10}^\dagger m_{10}^2 U_{10} U_{10}^\dagger U_{5L} \\ &\simeq U_{5L}^\dagger m_{10}^2 U_{5L} \,, \end{aligned} \tag{3.39}$$

where the last step holds if radiative corrections to the rotation matrices are neglected.

- M_N. M_N is the scale at which the heavy right handed neutrinos decouple. Below this scale, the particle content is just the one of the MSSM complemented with the neutrino mass operator resulting from the see-saw mechanism. Consequently, the superpotential can be written in a basis where the charged-lepton mass matrix becomes diagonal and the left slepton mass matrix becomes

$$\bar{m}_L^2 = V_E^\dagger m_L^2 V_E \simeq U_{5R}^\dagger m_5^2 U_{5R} \,. \tag{3.40}$$

RGE effects play a significant role in the calculation of flavour-violating processes. Even in the case of universal soft terms at M_X, RGE runs between M_X and M_{GUT} (arising mainly through superpotential terms of the form $\bar{E}\bar{U}\bar{H}$ where $\bar{H}$ is a colour-triplet Higgs field) give rise to one-loop diagrams that also renormalise the right-handed slepton masses (which in the CMSSM would remain to a large extent diagonal). In the leading-logarithmic approximation these corrections are given by [Hisano and Nomura, 1999]

$$(m_E^2)_{ij} \simeq -\frac{3}{8\pi^2} \mathcal{Y}_{u_3}^2 V_{CKM}^{3i} V_{CKM}^{*3j} (3m_0^2 + a_0^2) \log \frac{M_{\rm X}}{M_{\rm GUT}} \tag{3.41}$$

for $i \neq j$, and, as we mentioned, are suppressed due to the smallness of V_{CKM}; this holds in the minimal supersymmetric $SU(5)$, since in extensions of the theory this mixing may be further amplified [Ellis et al., 2009, Carquin et al., 2009]. On the contrary, runs from $M_{GUT} \to M_N$ are crucial. In the leading-logarithmic approximation, the non-universal renormalisation of the soft supersymmetry-breaking scalar masses is given by

$$(m_L^2)_{ij} \simeq -\frac{1}{8\pi^2} \left(\mathcal{Y}_{\nu_3}^2 V_E^{*3i} V_E^{3j} \log \frac{M_{\rm X}}{M_{\nu_3}} + \mathcal{Y}_{\nu_2}^2 V_E^{*2i} V_E^{2j} \log \frac{M_{\rm X}}{M_{\nu_2}} \right) (3m_0^2 + a_0^2) \,, \tag{3.42}$$

implying that the corresponding corrections to left-handed slepton masses are proportional to V_E (the Dirac neutrino mixing matrix in the basis where the d-quark and charged-lepton masses are diagonal). In this approach, non-universality in the soft supersymmetry-breaking left-slepton masses is much larger than the one in the right-slepton masses.

Chapter 4

Yukawa Unification & Dark Matter

A brief general overview on the necessary background required to the topic at hand has hitherto been presented. In the following, we will address the original work that has been developed aiming at shedding some light on the difficulties discussed, which constitutes the main body of our research. The work presented in this chapter has been published in [Gomez et al., 2009b, Gomez et al., 2009a, Gomez et al., 2009c] (Chapter 5 will discuss the work on LFV). As stated in the Preface, our work belongs in the area of supersymmetric extensions of the SM of particle physics, with emphasis on the origin of fermion masses including massive neutrinos. The investigation of these issues is hereafter discussed.
We shall first be addressing the implications of lepton mixing on the b-quark mass prediction within Yukawa-unified models. Next, we will use this framework to investigate the compatibility of a neutralino LSP as a Dark Matter candidate.

4.1 b-quark mass and Yukawa Unification

In § 3.6.2, we showed the predictions generally expected for the b-quark mass in a supersymmetric Yukawa-unified model in the presence of massive neutrinos, but *neglecting* the lepton mixing induced as a consequence of the impossibility of simultaneously diagonalise both the charged-lepton and neutrino mass textures. Our first main result concerns the compatibility of both signs of the μ parameter with the predicted bottom mass, once the mentioned lepton mixing is included.
The results are significantly modified once we consider the effects of lepton mixing in the diagonalisation and running of couplings from high to low energies. In order to show this, we focus on $b-\tau$ unification within the framework of $SU(5)$ gauge unification and flavour symmetries that provide consistent patterns for mass and mixing hierarchies, and naturally reconcile a small V_{CKM} mixing with a large charged lepton one. Taking into account the particle content of $SU(5)$ representations (with symmetric up-type mass matrices, and down-type mass matrices that are transpose to the ones for charged leptons), one finds that

$$\mathcal{M}_u \propto \begin{pmatrix} \bar{\varepsilon}^4 & \bar{\varepsilon}^2 \\ \bar{\varepsilon}^2 & 1 \end{pmatrix}, \quad \mathcal{M}_d^0 \propto A \begin{pmatrix} 0 & 0 \\ x & 1 \end{pmatrix}, \quad \mathcal{M}_\ell^0 \propto A \begin{pmatrix} 0 & x \\ 0 & 1 \end{pmatrix}, \tag{4.1}$$

which, after diagonalisation, lead to the relation

$$\frac{m_b^0}{1+x^2} = \frac{m_\tau^0}{1-x^2} \implies m_b^0 = m_\tau^0 \left(1 - \underbrace{2x^2}_{\delta} + \mathcal{O}\left(\delta^2\right)\right), \tag{4.2}$$

where δ parameterises the flavour mixing in the (2,3) charged lepton sector (any additional mixing required to match the data would then arise from the neutrino sector).
In the left panel of Fig. 4.1 we show the change of m_b as a function of $\tan\beta$, when the effects from large lepton mixing are correctly considered. Comparing with the previous plots, we see how solutions with positive μ are now viable, for the whole range of $\tan\beta$. The appropriate size of the parameter δ in each case can be determined by imposing the relation $\lambda_\tau = \lambda_b(1+\delta)$ at M_{GUT} and investigating the values that are required in order to obtain a correct prediction for $m_b(M_Z)$. This is shown in the right panel of Fig. 4.1, where we demand a value of $m_b(M_Z)$ at the centre of its experimental range, for the central value of α_s. We checked (although not shown) that the effect from runs above M_{GUT} is very small, as expected. We then observe the following:

- Solutions with a positive μ are now enabled, for the whole range of $\tan\beta$.

- Negative μ is also allowed in the whole range of $\tan\beta$ and requires a smaller mixing parameter δ than the $\mu > 0$ case.

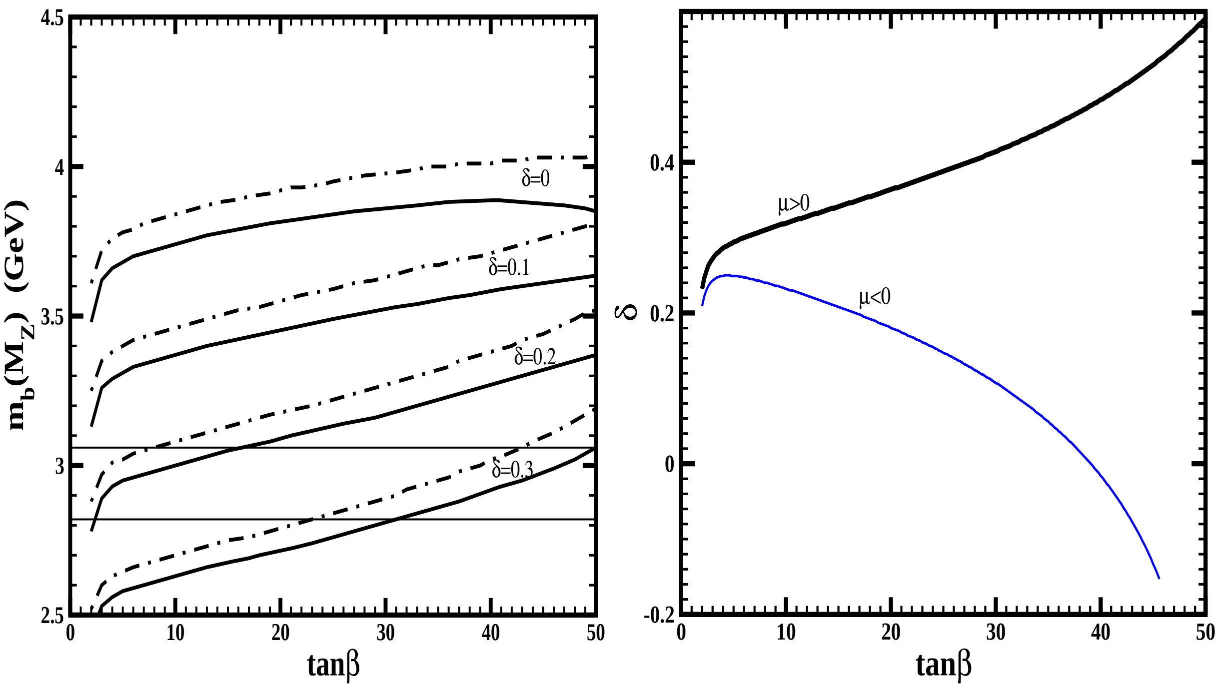

Figure 4.1: *In the left panel, we show m_b as a function of $\tan\beta$ for $\mu > 0$, without (solid lines) and with (dot dashed) massive neutrinos, in the latter case assuming $m_{\nu_3} = 0.05$ eV and $M_N = 3 \times 10^{14}$ GeV. In the right panel, we show the required values of δ compatible with $b - \tau$ unification at the central experimental value of m_b for both signs of μ.*

4.2 Yukawa Unification and Dark Matter

Having shown that lepton mixing helps $b - \tau$ Yukawa-unified models successfully predict the b-quark mass, we will next discuss the implications of these models for Dark Matter abundances. Since we are considering R-parity conserved models, we will be focusing on the lightest neutralino as a Dark Matter candidate.
In mSUGRA (or the CMSSM), for choices of the soft terms below the TeV scale, the LSP is mainly Bino-like and the prediction for $\Omega_\chi h^2$ is typically too large for models that satisfy the experimental constraints on SUSY parameters. These constraints exclude models with relatively small values of m_0 and $M_{1/2}$ in which neutralino annihilates mainly through sfermion exchange. The values of WMAP can be obtained mainly in two regions:

- $\chi - \tilde{\tau}$ coannihilation region, which occurs for $m_\chi \sim m_{\tilde{\tau}}$.
- Resonances in the $\chi - \chi$ annihilation channel, which occur for $m_A \sim 2m_\chi$.

Nonetheless, since the above regions are finely-tuned [Lahanas et al., 2000, Ellis et al., 2001, Lahanas and Spanos, 2002], they will inevitably be sensitive to the changes induced by GUT unification and sizeable mixing in the charged lepton sector. In this respect, it is illustrative to first investigate the impact of the parameter δ on the value of m_A. We show this in Fig. 4.2, where we present the variation of m_A with $\tan\beta$ for $m_0 = 600$ GeV, $M_{1/2} = 800$ GeV and $A_0 = 0$. In the upper lines the value of $\delta(\neq 0)$ is fixed by demanding $m_b(M_Z) = 2.921$ GeV while in the lower ones $\delta = 0$ with no restrictions on the $m_b(M_Z)$ prediction. We observe that the upper lines do not meet the resonance condition, while for the lower lines this condition is achieved for values of $\tan\beta$ in the range $40 - 50$. We also see that, assuming soft term universality at a scale $M_X > M_{GUT}$, the runs beyond M_{GUT} change only moderately the prediction for m_A.

Our next step is a global study of the supersymmetric parameter space by assuming universal soft terms m_0, $M_{1/2}$ and A_0 at some scale $M_X \geq M_{GUT}$, in order to obtain the mass spectrum by integrating the appropriate RGEs at each energy range, as described above. The physical observables are computed using the code provided by *micromegas* [Belanger et al., 2007, Belanger et al., 2009]. The 2-σ range for BR($b \to s\gamma$) is constructed including an intrinsic $0.15 \cdot 10^{-4}$ MSSM correction as in Ref. [Ellis et al., 2007b], leading to (as previously shown in section 3.6, Eqs. (3.23, 3.24))

$$2.15 \cdot 10^{-4} < BR(b \to s\gamma) < 4.9 \cdot 10^{-4}; \tag{4.3}$$

without including the MSSM intrinsic error, the above range becomes

$$2.8 \cdot 10^{-4} < BR(b \to s\gamma) < 4.4 \cdot 10^{-4}. \tag{4.4}$$

We take a conservative bound for the mass of the lightest CP-even Higgs of $m_h > 114$ GeV and also consider an uncertainty of ~ 3 GeV [Heinemeyer et al., 2006] to its theoretical computation.

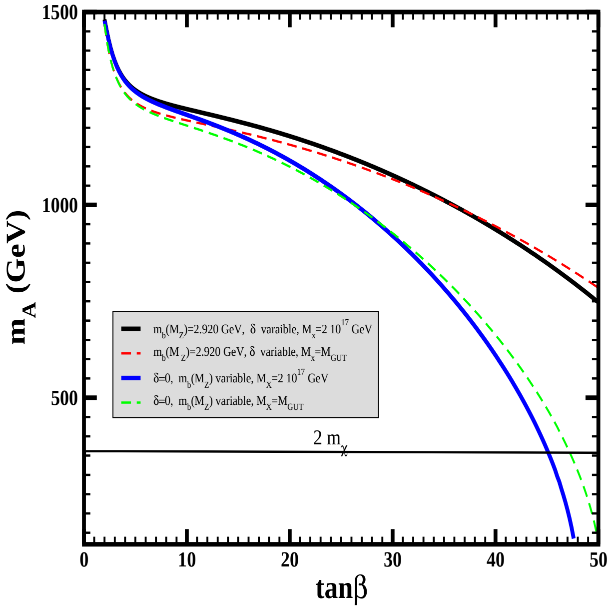

Figure 4.2: *Values of the pseudoscalar higgs mass* m_A *versus* $\tan\beta$ *for the same setting of parameters as in Fig. 3.3. In the upper lines the parameter* δ *varies so that* $m_b(M_Z) = 2.921$ *GeV, while in the lower lines* $\delta = 0$*. The solid lines correspond to* $M_X = 2 \cdot 10^{17}$ *GeV and the dashed lines to* $M_X = M_{GUT}$*.*

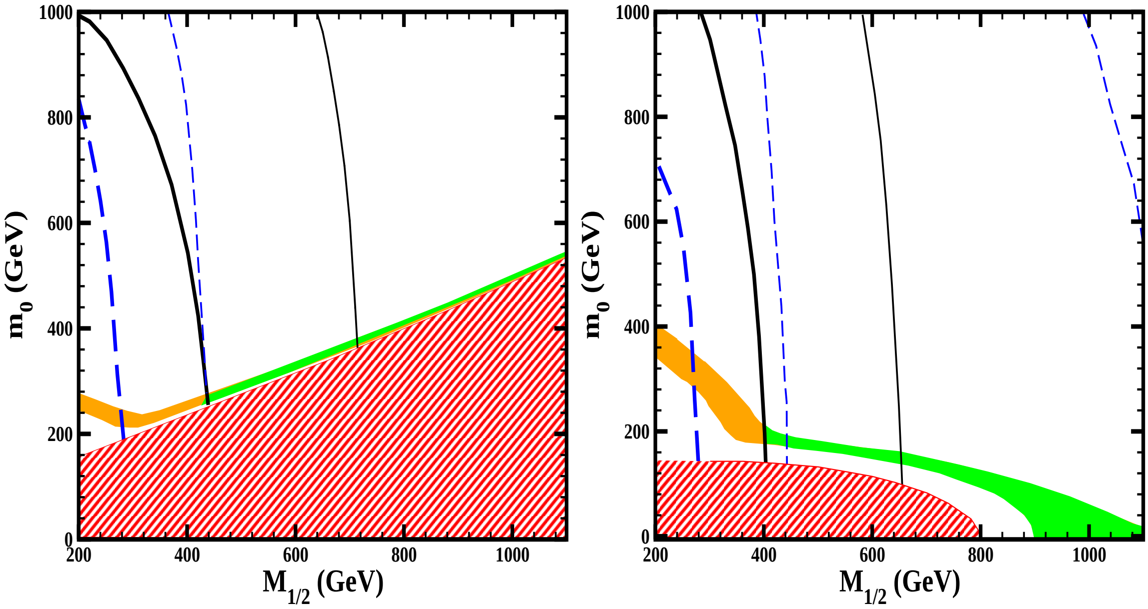

Figure 4.3: *WMAP allowed area (green-shaded) for the case of* $\tan\beta = 45$, $\mu > 0$, $A_0 = 0$, *when* $\alpha_s(M_Z)$ *and* $m_b(M_Z)$ *are set at their central values. The solid black lines indicates the* $b \to s\gamma$ *constraints [thick* $BR(b \to s\gamma) = 2.15 \cdot 10^{-4}$, *thin* $BR(b \to s\gamma) = 2.8 \cdot 10^{-4}$*], and the dash blue line the Higgs mass bound [thin* $m_h = 114$ *GeV, thick* $m_h = 111$ *GeV]. On the left graph,* $M_X = M_{GUT}$ *while on the right panel,* $M_X = 2 \cdot 10^{17}$ *GeV.*

In Fig. 4.3 we present the WMAP favoured area for $\tan\beta = 45$, including lepton mixing effects and considering the most tolerant bounds on BR($b \to s\gamma$) and m_h (stricter bounds on these constraints are also displayed). In this example, we keep $m_b(M_Z) = 2.920$ GeV which corresponds to the evolution of the experimental value of m_b obtained for the central value $\alpha_s(M_Z) = 0.1172$. When $M_X = M_{GUT}$ we find the WMAP favoured region to be on the familiar CMSSM $\chi-\tau$ coannihilation area. However, as in [Calibbi et al., 2007], we find that the runs corresponding to $M_X > M_{GUT}$ have a big impact on the neutralino relic density and Fig. 4.3 shows clearly this effect. The large values of the gauge unified coupling $\alpha_{SU(5)}$ tend to increase the values of $m_{\tilde{\tau}}$ in a way that, even if we start with $m_0 = 0$ at M_X the model predicts $m_{\tilde{\tau}} > m_\chi$. This implies that the allowed parameter space with a neutralino LSP is significantly enhanced (green area), while the coannihilation condition becomes harder to achieve; this effect is more visible for large $\tan\beta$ and is understood since the lightest stau has a mass $m_{\tilde{\tau}_1} \simeq m_{\tilde{\tau}_{RR}} + m_{\tilde{\tau}_{LR}}$, where

$$m_{\tilde{\tau}_{RR}} \simeq (1-\rho_\beta)\, m_0^2 + 0.3 M_{1/2}^2,\;\; m_{\tilde{\tau}_{LR}} \simeq -m_\tau \mu \tan\beta\,, \tag{4.5}$$

ρ_β being a positive coefficient dependent on $\tan\beta$ (in our case $\rho_\beta < 1$) [Calibbi et al., 2007]. The increase of $m_{\tilde{\tau}_1}$ is due to the GUT runnings ($\sim M_{1/2}^2$). The picture for $tan\beta = 35$ and $A_0 = m_0$ has been discussed in [Carquin et al., 2009], where it was shown that the WMAP allowed region can be compatible with observable flavour violation at the LHC.
We stress again that $b-\tau$ Yukawa unification in the context of the CMSSM requires $\delta > 0$. An exhaustive study of the parameter space for the case of $A_0 = 0$ and keeping the prediction of $m_b(M_Z) = 2.92$ GeV is presented in Figs. 4.4 and 4.5. The left panel of Fig. 4.4, shows that the phenomenological and cosmological constraints are fulfilled for $\tan\beta \geq 31$ and $M_{1/2} \geq 330$ GeV respectively. On the other hand, the right panel of Fig. 4.4 indicates that sizeable values of the mixing parameter δ are needed in order to maintain $m_b(M_Z)$ in the experimental range and that the required mixing increases with $\tan\beta$. Fig. 4.5 shows the $\left(m_0, M_{1/2}\right)$ plane for different values of $\tan\beta$, taking all constraints into account; it can be seen that the resonant effects start becoming important at $\tan\beta \geq 45$.

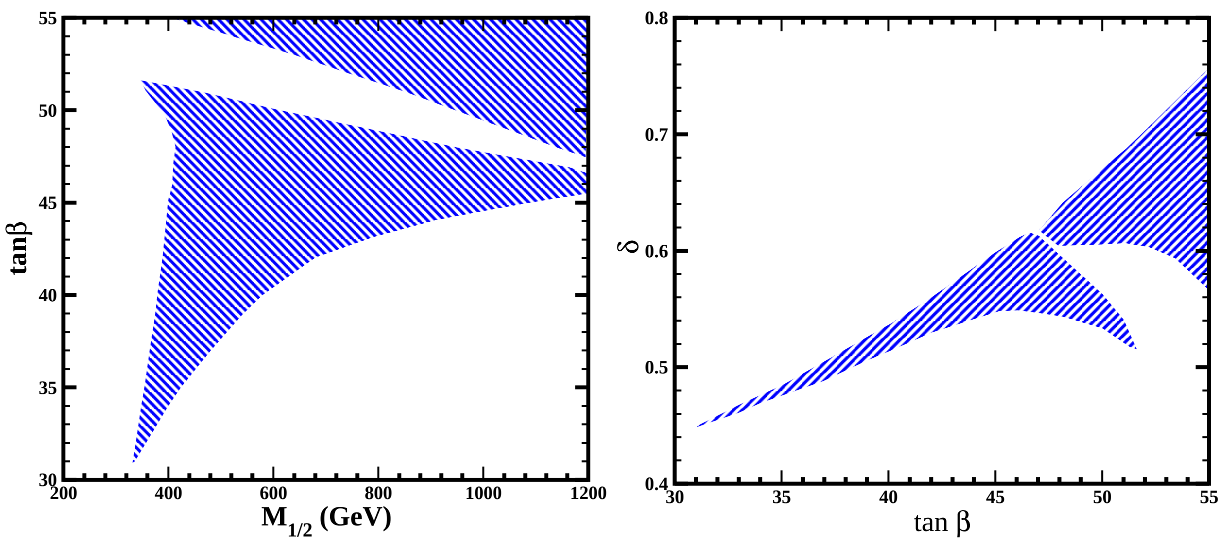

Figure 4.4: *The left panel provides a plot equivalent to Fig. 4 of [Calibbi et al., 2007] for the purpose of comparisons; the upper part corresponds to areas with resonant Higgs channels. In the right panel, we show the values of δ corresponding to the previous plot; these inevitably implies $\delta \neq 0$.*

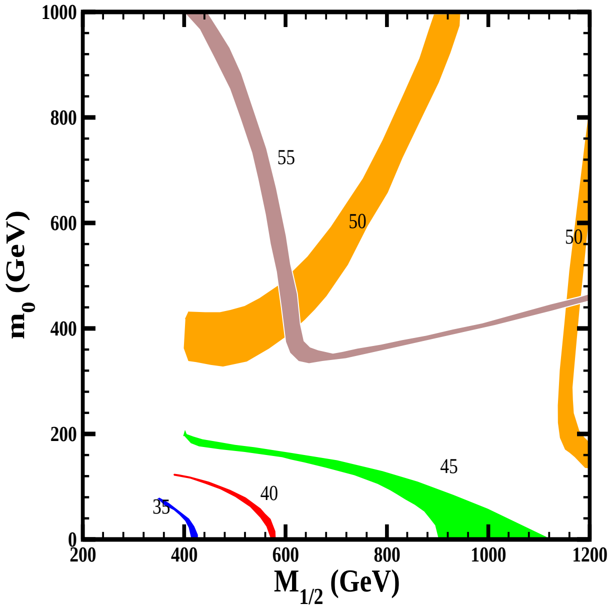

Figure 4.5: *WMAP allowed areas for several values of* $\tan\beta$. *We consider* $m_b(M_Z) = 2.92$ *GeV,* $A_0 = 0$ *GeV and* $\mu > 0$.

The impact of varying m_b and α_s within their experimental range can be seen in Fig. 4.6 which indicates that, for $\tan\beta$ below 40 there are no significant changes with the variation of m_b on the WMAP area that lies on the coannihilation region. For larger values of $\tan\beta$, however, small changes of the bottom Yukawa coupling due to the modification of m_b have a significant impact on resonant annihilation, as is manifest by the locations of the bands for $\tan\beta = 45$ and 50 in the two lower plots.

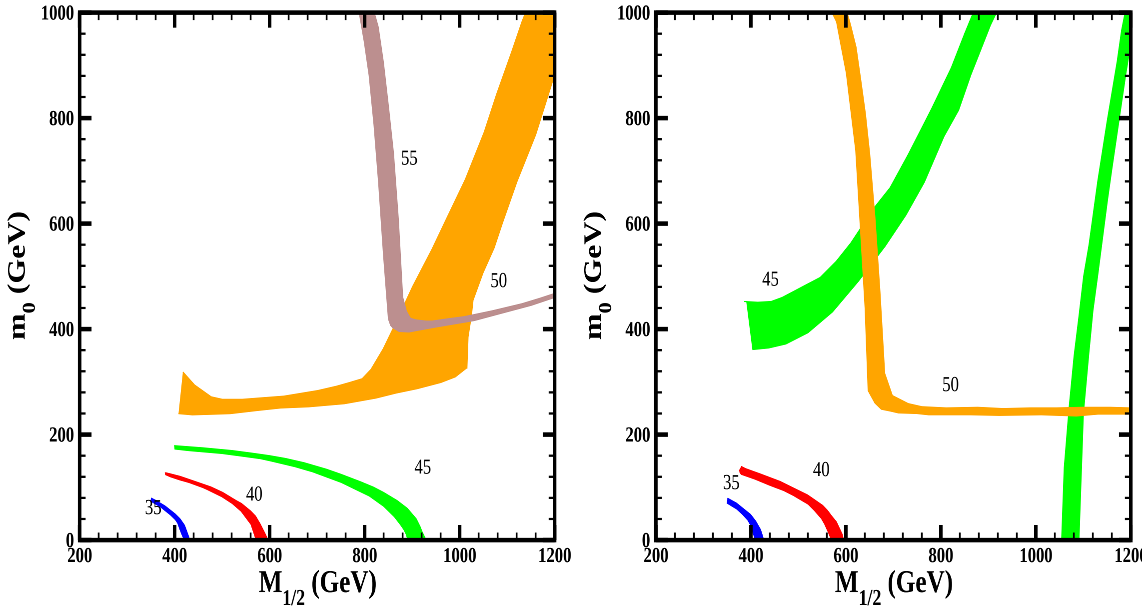

Figure 4.6: *Same plot as Fig. 4.5, but taking the extreme values of both* $\alpha_s(M_Z)$ *and* $m_b(M_Z)$. *On the left panel,* $\alpha_s(M_Z) = 0.121$ *and* $m_b(M_Z) = 2.74$ *GeV while on the right panel* $\alpha_s(M_Z) = 0.113$ *and* $m_b\,(M_Z) = 3.114$ *GeV. In both cases* $M_X = 2 \cdot 10^{17}$ *GeV.*

The case $\mu < 0$ is not yet ruled out by the $(g-2)_\mu$ data, since as we mentioned before by using τ data, a small negative discrepancy of the experimental measurement as compared to the SM prediction can be accommodated. However, the upper limit on the BR$(b \to s\gamma)$ imposes a severe constraint on the SUSY parameter space since the supersymmetric contribution adds to the SM prediction.

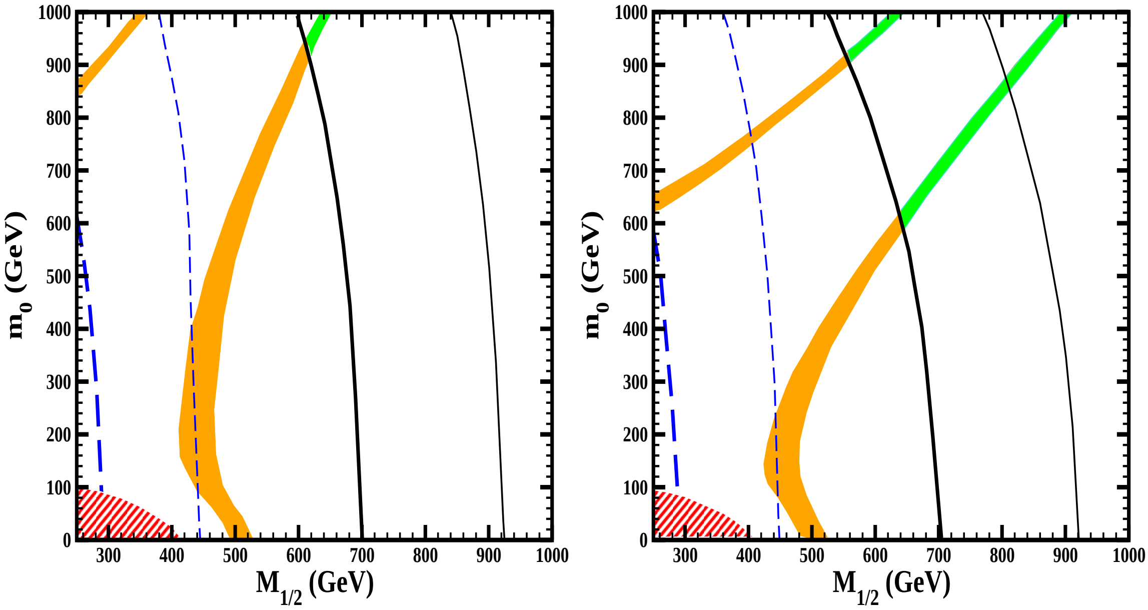

Figure 4.7: $(m_0, M_{1/2})$ *plane for tan*$\beta = 40$*,* $A_0 = 0$*, and* $\mu < 0$ *. On the left panel we take the right-handed neutrino scale* $M_N = 6 \times 10^{14}$ *GeV, while on the right panel* $M_N = 10^{13}$ *GeV. Here,* $m_b(M_Z)$ *and* $\alpha_s(M_Z)$ *are set to their central values. The lines follow the same notation as Fig. 4.3.*

For $\mu < 0$ and $M_X > M_{GUT}$, we find that the WMAP areas allowed due to $\chi - \tilde{\tau}$ coannihilations at low values of $\tan\beta$ are excluded by the $b \to s\gamma$ higher bound. However, at larger values of $\tan\beta$, the areas with resonant annihilations are marginally allowed, as we can see in Fig. 4.7.

As already underlined, these results are very sensitive to λ_N. Once we decrease it by lowering M_N (so that, from the see-saw condition, m_{ν_3} remains 0.05 eV) we allow larger regions of the parameter space for negative μ as well. In both cases, however, the values of the parameter δ have to be very small ($\delta < 0.05$); therefore, negative μ is not compatible with large charged lepton Yukawa mixing.

Chapter 5

Charged-Lepton Flavour Violation and $U(1)_F$ Symmetry

As a final analysis, we will address the issue of LFV processes within the context of Yukawa-unified models, taking into account the cosmologically preferred parameter space obtained in the previous section for $SU(5)$ runs. As a matter of tentative comparison (as this is work in progress), we will end this chapter by briefly commenting on how left-right symmetric models may enhance the allowed parameter space even if pre-GUT runs are not considered. The work presented in this chapter refers to [Gomez et al., 2010b, Gomez et al., 2010a]. It should be mentioned that, unlike the two first analysis, LFV involves a significant model-dependence, as it depends on the mass spectrum, which substantially differ for different models. For this reason, we shall first consider the specific textures to be used for the subsequent analysis carried out.

5.1 Yukawa Interactions from $U(1)_F$ in $SU(5)$ SUSY-GUT Models

Having defined the general framework, the next step consists of summarising $SU(5)$ Yukawa textures that match the fermion data and may also predict the pattern of soft terms to be expected. The mass matrices are constructed by looking at the field content of SU(5) representations, namely: three families of $(Q, u^c, e^c)_i \in \mathtt{10}$, three families of $(L, d^c)_i \in \overline{\mathtt{5}}$ representations, and heavy right-handed neutrinos in singlet representations. This model has therefore the following properties: (i) the up-quark mass matrix is symmetric, and (ii) the charged-lepton mass matrix is the transpose of the down-quark mass matrix, which relates the mixing of the left-handed leptons to that of the right-handed down-type quarks. Since the CKM mixing in the quark sector is due to a mismatch between the mixing of the left-handed up- and down-type quarks, it is independent of mixing in the lepton sector, easily reconciling the large atmospheric neutrino mixing angle with the observed small V_{CKM} mixing. Following the $U(1)_F$ charge assignment in [Ellis et al., 2007a], the Yukawa matrices have the form

$$\mathcal{Y}_u \propto \begin{pmatrix} \varepsilon^6 & \varepsilon^5 & \varepsilon^3 \\ \varepsilon^5 & \varepsilon^4 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix}, \quad \mathcal{Y}_\ell^T \propto \mathcal{Y}_d \propto \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon & 1 & 1 \end{pmatrix}, \quad \mathcal{Y}_\nu \propto \begin{pmatrix} \varepsilon^{|1\pm n_1|} & \varepsilon^{|1\pm n_2|} & \varepsilon^{|1\pm n_3|} \\ \varepsilon^{|n_1|} & \varepsilon^{|n_2|} & \varepsilon^{|n_3|} \\ \varepsilon^{|n_1|} & \varepsilon^{|n_2|} & \varepsilon^{|n_3|} \end{pmatrix}, \tag{5.1}$$

where n_i stand for the heavy Majorana neutrino charges. As discussed in the Preface, we will assume that the entire lepton mixing is arising from the charged-lepton sector, which is potentially the most dangerous case as far as LFV is concerned.

The rotation matrices that diagonalise $\mathcal{Y}_u$ and $\mathcal{Y}_\ell^T \propto \mathcal{Y}_d$ are

$$U_{10} = \begin{pmatrix} -1+\frac{\varepsilon^2}{2} & \varepsilon & 0 \\ -\varepsilon & -1+\frac{\varepsilon^2}{2} & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix}, \tag{5.2}$$

$$U_{5L} = \begin{pmatrix} -1+\frac{\varepsilon^2}{2} & \varepsilon & 0 \\ -\varepsilon & -1+\frac{\varepsilon^2}{2} & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix}, \quad U_{5R} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{2}-\frac{\varepsilon}{2\sqrt{2}} & \frac{1}{2}-\frac{\varepsilon}{2\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2}+\frac{\varepsilon}{2\sqrt{2}} & \frac{1}{2}+\frac{\varepsilon}{2\sqrt{2}} \\ \frac{\varepsilon}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{5.3}$$

While there is no unique choice of the right handed neutrino charges n_1, n_2, n_3 (several choices may lead to correct low energy neutrino data) representative choices can be made, and among the simplest patterns is the one provided by the assignment $\{n_1, n_2, n_3\} = \{1, 1, 1\}$. In this case,

$$V_E = U_{\nu L}^\dagger U_{5R} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{2}+\frac{\varepsilon}{2\sqrt{2}} & \frac{1}{2}+\frac{\varepsilon}{2\sqrt{2}} \\ -\frac{1}{2}+\frac{\varepsilon}{2} & \frac{1}{2}\left(1+\frac{\sqrt{2}}{2}\right)+\frac{\varepsilon}{4} & \frac{1}{2}\left(1-\frac{\sqrt{2}}{2}\right)+\frac{\varepsilon}{4} \\ \frac{1}{2}+\frac{\varepsilon}{2} & \frac{1}{2}\left(1-\frac{\sqrt{2}}{2}\right)-\frac{\varepsilon}{4} & \frac{1}{2}\left(1+\frac{\sqrt{2}}{2}\right)-\frac{\varepsilon}{4} \end{pmatrix}, \tag{5.4}$$

where $U_{\nu L}$ is the left rotation matrix for the Dirac neutrino sector.

In addition, flavour symmetries generally imply non-universal soft terms [Raidal et al., 2008], since the structure of the soft terms is linked to the family charges. For the Yukawa textures in Eq. (5.1) the soft mass matrices m_{10}^2 and m_5^2 become

$$m_{10}^2 \propto \begin{pmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix} m_0^2, \quad m_5^2 \propto \begin{pmatrix} 1 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix} m_0^2. \tag{5.5}$$

The diagonalisations performed on the superfields also influence the soft mass terms, thus we must rotate the textures accordingly:

$$\bar{m}_{10}^2 = \begin{pmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix} m_0^2, \quad \bar{m}_5^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & \varepsilon \\ 0 & \varepsilon & 1 \end{pmatrix} m_0^2. \tag{5.6}$$

In what follows, we will analyse the predictions of the above textures for on LFV processes of the type $l_i \to l_j + \gamma$, considering that all textures initially arise at a scale $M_X > M_{GUT}$.

5.2 RGE runs and results

Let us briefly discuss the running procedure. We use a top-down approach, the scale M_X being the starting point. At this scale, both the Yukawa textures and the soft mass matrices are determined by the family symmetry charges. We evolve the *3rd generation* parameters down to M_{GUT} using the $SU(5)$ RGEs. In this way, both the V_{CKM} and the V_E mixing matrices are *predicted*. Then, we further evolve the corresponding RGEs (including the right-handed neutrino mass scale M_N and the SUSY threshold corrections) down to the electro-weak scale. At M_Z we *impose* the experimental constraints on the gauge couplings, as well as acceptable fermion masses and mixings. From this point, we employ a bottom-up approach to evolve the RGEs, using the experimental constraints, up to the GUT scale (properly re-obtaining the SUSY scale and introducing the right-handed neutrino modes). At the GUT scale we end up with V_{CKM} and V_E that are to be compared with the ones computed in the top-down method. Such a comparison allows us to extract information about the off-diagonal soft terms at the high scale M_X.

As in Ref. [Gomez et al., 2009b] the values we use are $M_X = 2 \cdot 10^{17}$ GeV, $M_N = 3 \cdot 10^{14}$ GeV. The coupling λ_{ν_3} is determined such that $m_{\nu_3} \sim 0.05$ eV; $m_b(M_Z) = 2.92$ GeV and $\alpha_s = 0.1172$. The evaluation of the LFV observables is done by performing a full diagonalisation of the slepton mass matrices (for instance, see [Hisano et al., 1996]), inserting the full rotation matrices in the lepton-slepton-gaugino vertices and summing over all the mass eigenstates of the exchanged particles. Soft terms are computed in the basis where the charged leptons are diagonal. Our results agree with other updated estimates of the branching ratios, such as those given in Ref. [Paradisi, 2005].

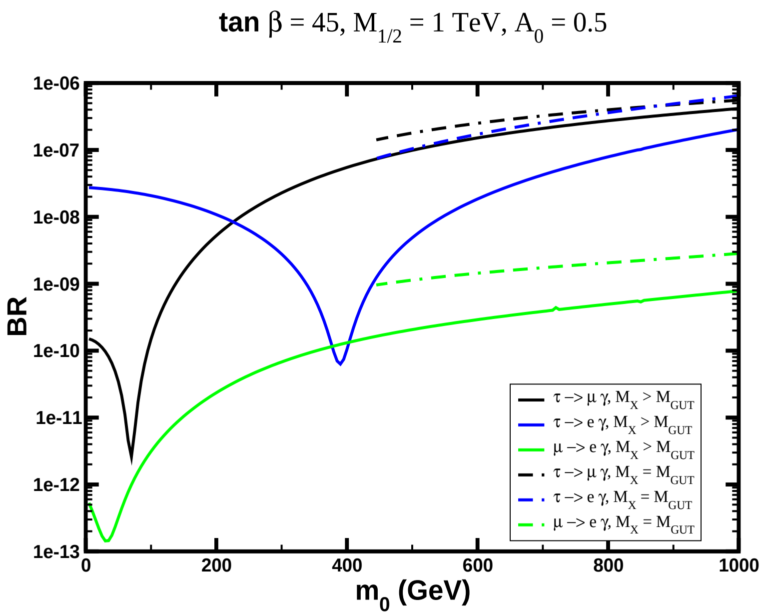

Figure 5.1: *Prediction for the charged-lepton flavour violating branching ratios showing the difference of taking either M_X or M_{GUT} as the starting point of the runs.*

5.2.1 Runs above M_{GUT}

The introduction of a non-trivial flavour structure for the slepton soft terms at M_{GUT}, as predicted by the family symmetry that also generates Yukawa couplings, typically results to a large violation of the bounds on $\ell_j \to \ell_i \gamma$ [Leontaris and Tracas, 1998, Gomez et al., 1999, Chankowski et al., 2005]. This picture may be remedied by taking into account RGE effects from a scale $M_X > M_{GUT}$. In this case, the cosmological requirement of having a neutral particle as the LSP imposes low values on m_0, such that $m_{\tilde{\tau}} > m_\chi$ [Calibbi et al., 2007, Gomez et al., 2009b, Ellis et al., 2010] (diagonal terms in the soft mass matrices have a large RGE growth, while non-diagonal elements remain almost unaffected by the runs). Thus, even assuming non-diagonal soft terms with matrix elements of the same order of magnitude at M_X, the corresponding matrix at M_{GUT} exhibits dominant diagonal elements. To some extent, the RGE effect is similar to the action of closing an umbrella: the general non-universal soft terms at M_X resemble an open umbrella that approaches a diagonal matrix at the GUT scale.

In Fig. 5.1, we show the differences between the following: i) SU(5) RGE evolution of the soft terms from a high scale M_X down to M_{GUT} and then to the MSSM with see-saw neutrinos (solid lines), and ii) Soft SUSY breaking terms given at M_{GUT} and then the MSSM with see-saw neutrinos (dash-lines). In case ii) we stop the lines at the value of m_0 below which $m_{\tilde{\tau}}$ becomes the LSP. In contrast, m_0 can even vanish at M_X in case i). The textures and soft terms we use are similar to Ref. [Chankowski et al., 2005]. However, unlike these authors, we decouple the right-handed neutrinos below M_{GUT}. As a result, the predicted BR's do not vanish in the limit $m_0 = 0$. We also observe the presence of one peak for each decay; the origin of such peaks can be traced back to the cancellations coming from the RR sector, in agreement with [Paradisi, 2005].

The advantageous feature of runs above the GUT scale relies on the increase of the mass of the lightest stau, such that the condition $m_\chi < m_{\tilde{\tau}}$ is achieved even at low values of m_0. These values are sufficiently low to predict rates for charged lepton violation within the current experimental bounds and a relic density on the WMAP range[Calibbi et al., 2007, Gomez et al., 2009b].

We can provide an explicit example of the growth of the diagonal terms of the slepton mass matrix in models with interesting predictions for both LFV and $\Omega_\chi h^2$. Let us consider the $0 < m_0 < 100$ GeV region. In the area of the parameter space where the WMAP bounds are satisfied due to $\tau - \chi$ co-annihilations, we find that $m_{1/2}$ is essentially a linear function of m_0, $m_{1/2} \sim a_1^i + a_2^i m_0$, where i runs over the multiplets. Taking into account that the radiative corrections to the off-diagonal entries of the soft mass matrices are subdominant as compared with those of the diagonal ones, these diagonal elements can be expressed as follows:

$$m_{S_i}^2 \simeq C_i^2\,(m_0)\, m_0^2\,, \tag{5.7}$$

where we have defined

$$C_i^2\,(m_0) \equiv \frac{144}{20\pi}\alpha_5 \left(\left(\frac{a_1^i}{m_0}\right)^2 + \frac{2a_1^i a_2^i}{m_0} + a_2^2 \right) \ln\left(\frac{M_X}{M_{GUT}}\right)\,, \tag{5.8}$$

and S_i stands for the supermultiplets **10** and $\bar{\mathbf{5}}$. As stated, Eq. (5.7) implies a large enhancement only for the diagonal entries of the soft matrices, further suppressing the off-diagonal elements. It turns out indeed that for values of $m_0 \simeq 60 - 80$ GeV at M_X such an enhancement at the GUT scale is as large as $\simeq 100$. As a consequence, the soft mass matrices $\bar{m}^2_{10}$ and $\bar{m}^2_{\bar{5}}$ at GUT scale read as

$$\bar{m}^2_{10} = \begin{pmatrix} 1 & \varepsilon^3 & \varepsilon^5 \\ \varepsilon^3 & 1 & \varepsilon^4 \\ \varepsilon^5 & \varepsilon^4 & 1 \end{pmatrix} C^2\left(m_0\right) m_0^2, \quad \bar{m}^2_{\bar{5}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & \varepsilon^3 \\ 0 & \varepsilon^3 & 1 \end{pmatrix} C^2\left(m_0\right) m_0^2\,, \tag{5.9}$$

clearly exhibiting the suppression on the off-diagonal terms (as compared with textures (5.6)).

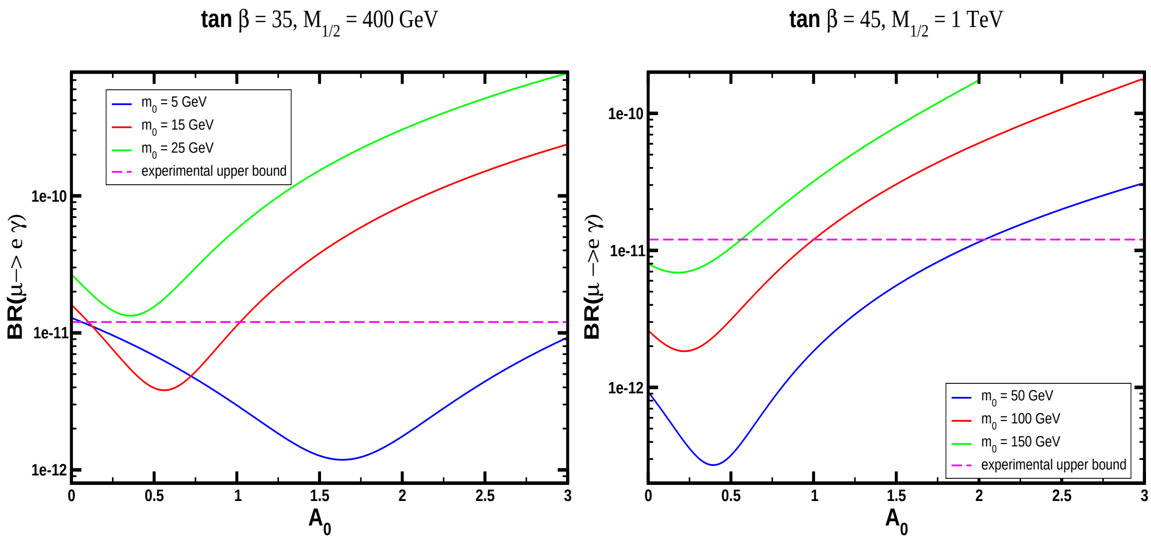

Figure 5.2: *Variation of the charged-lepton flavour violating branching ratio for $\mu \to e\gamma$ with A_0.*

Before going through the main results, it will be instructive to analyse the dependence of the LFV rates on the universal soft parameter A_0. In Fig. 5.2, we used again the textures and soft terms of Ref. [Chankowski et al., 2005] with the first set of right-handed neutrino charges discussed above. We observe that, for fixed m_0 and $M_{1/2}$, the LFV rates yield unacceptable predictions beyond some A_0, hence further constraining the allowed parameter space (note that the allowed range for m_0 increases with $\tan\beta$). Recall that the upper limit on A_0 arises from the appearance of tachyonic soft masses.

5.2.2 Runs below M_{GUT} and Charged-Lepton Flavour Violation rates

An immediate question is how sensitive the results are upon variations of m_0. As shown in Fig. 5.3, the applied constraints imply that solutions only exist between $\tan\beta \simeq 35$ and $\tan\beta \simeq 45$. Naturally, smaller values of m_0 lead to an enhancement of the allowed parameter space (while Eq. 5.8 indicates how a higher M_X enhances the allowed values of m_0). Furthermore, small values of m_0 become favoured once LFV processes rates are taking under consideration and the “umbrella effect” emerges. In particular, it can be seen that no reliable parameter combination survives above $m_0 \simeq 150$ GeV.

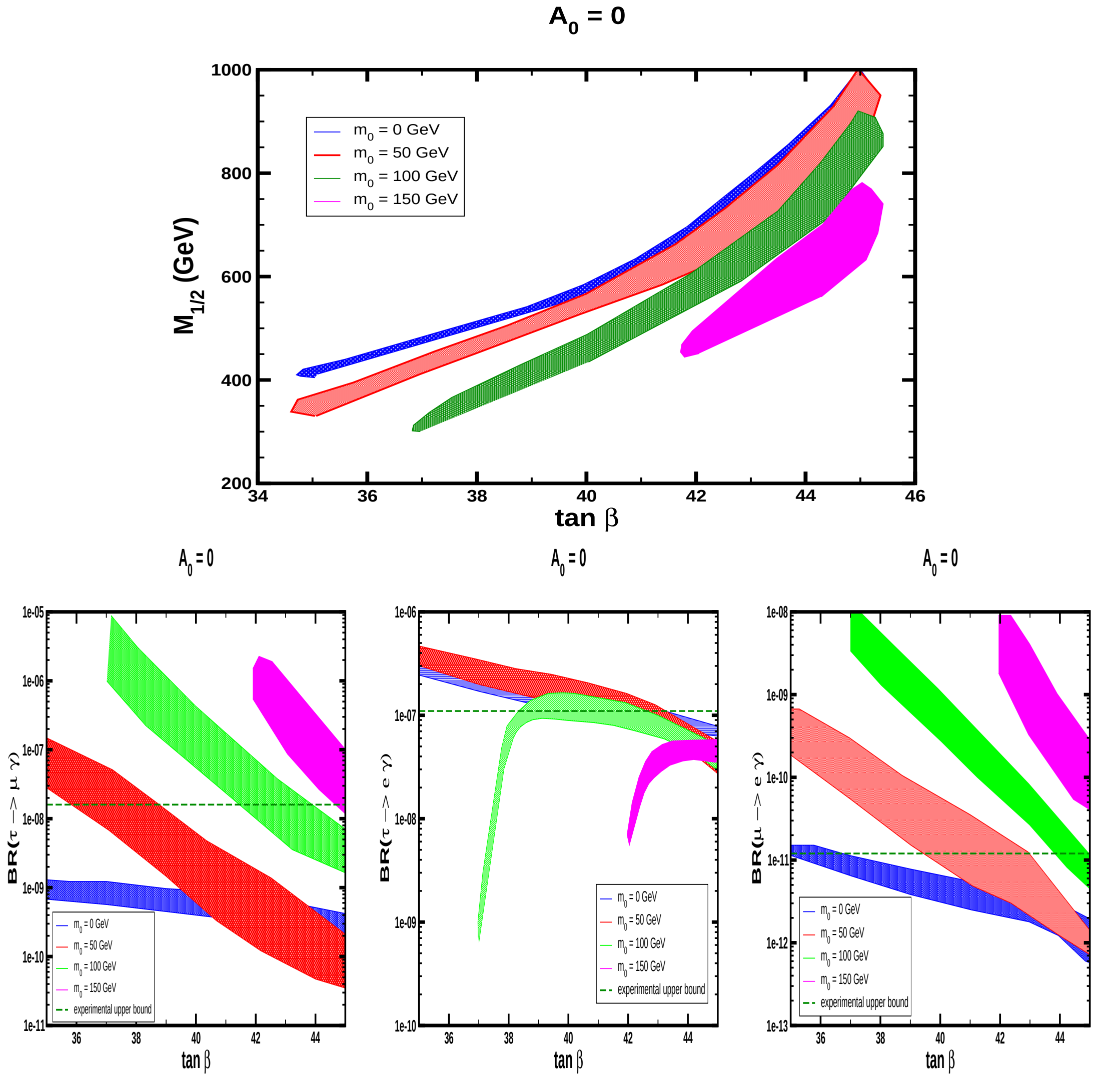

Figure 5.3: *We plot the allowed parameter space resulting from applying the constraints, as explained in the text, for* $m_0 = 0, 50, 100, 150$ *(upper plot). The BR's in terms of* $\tan\beta$ *computed all along the allowed parameter space is also plotted (lower three plots).*

In the following, we shall show how the textures defined in the previous section yield acceptable charged-LFV rates once the umbrella effect is considered. As observed from Figs. 5.4 the predictions for the branching ratios for $\ell_i \to \ell_j + \gamma$ decays lie within the current experimental bounds for properly chosen parameters.

We should stress that the depicted ranges for both m_0 and $M_{1/2}$ are the cosmologically preferred parameter space, as found in [Gomez et al., 2009b]. We can see that the case $\tan\beta$ = 35 is ruled out, as there is no overlapping region for the three decays. This is because of the RR sector-induced cancellations mentioned above, which arises for much larger values of m_0 for $\tau \to e\gamma$ than for the other two processes. The case $\tan\beta = 40$ does possess a common area for $20 < m_0 < 50$ GeV. However, such an area lies outside the cosmologically preferred region, as shown on the left panel of Fig. 5.5. Thus, we conclude that the case $\tan\beta = 40$ is only marginally allowed. Finally, for $\tan\beta = 45$ the whole range for $m_0 < 100$

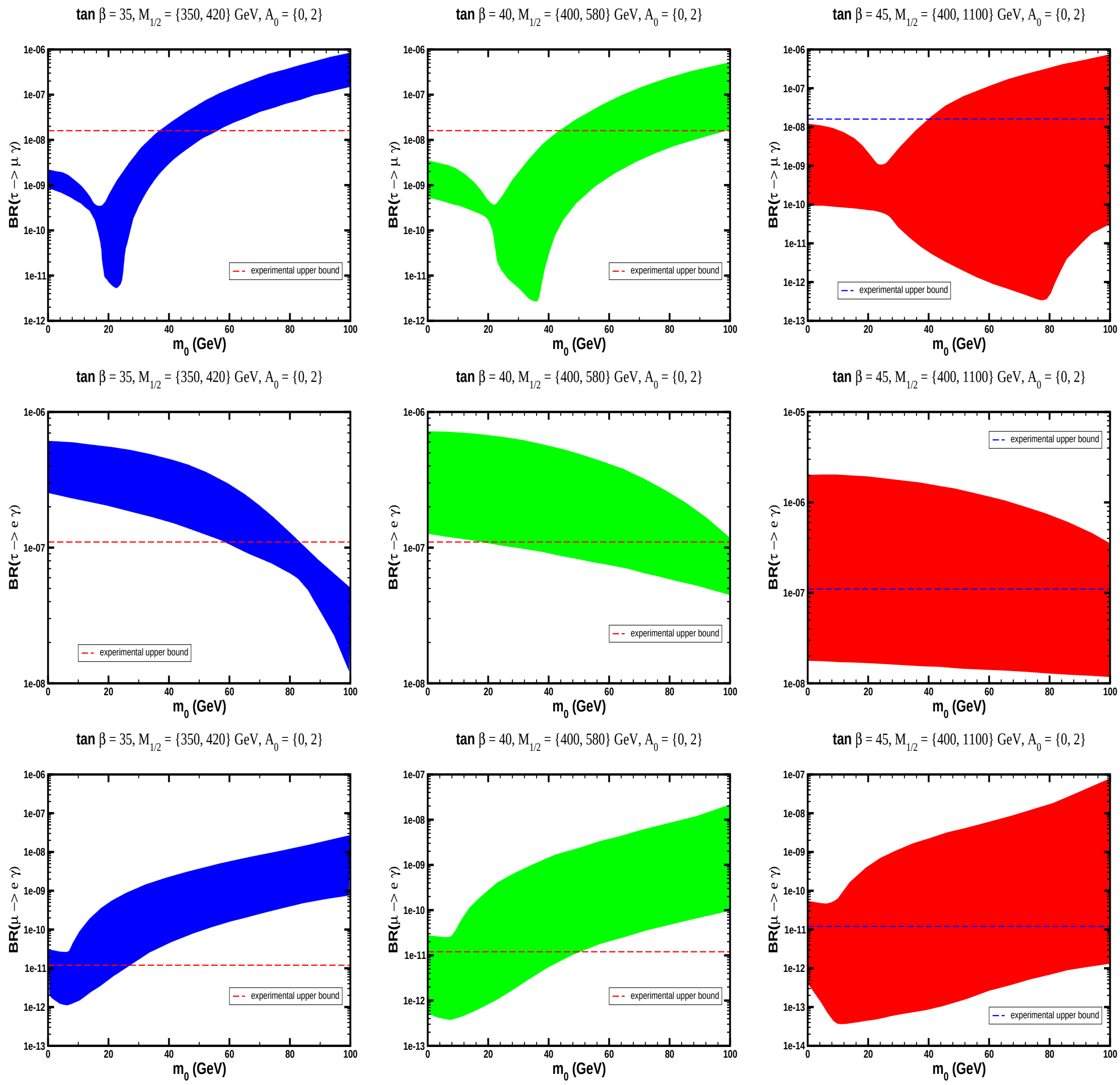

Figure 5.4: *Prediction for the* $\tau \to \mu\gamma$ *(top),* $\tau \to e\gamma$ *(middle) and* $\mu \to e\gamma$ *branching ratios for the cosmologically preferred area of values of* $M_{1/2}$, A_0, *for three different values of* $\tan\beta$.

GeV is allowed, when suitable values for $M_{1/2}$, A_0 are chosen. Moreover, as shown on the right panel of Fig. 5.5, there exists an overlapping region when considering the cosmologically favoured parameter space (values of any parameters involved in these plots (m_0, $M_{1/2}$, A_0) beyond the ranges shown lead to tachyonic soft masses (see also comment before Fig. 5.2)). Thus, GUT runs efficiently suppress the off-diagonal entries, yielding charged-LFV rates that render the $SU(5)$ model *compatible* with current experimental bounds.

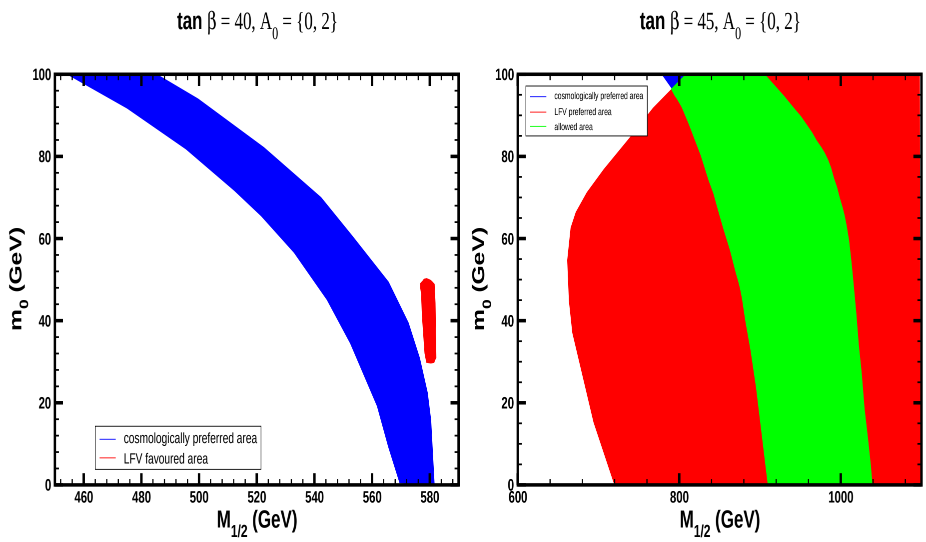

Figure 5.5: *Allowed parameter space when both LFV and cosmological constraints are taken into account for* $\tan\beta = 40$ *and* 45*.*

5.3 Charged-Lepton Flavour Violation within left-right models

Finally, let us make a few remarks on the predictions for LFV within left-right symmetric models. As mentioned above, the following are meant to be just some preliminary results, whose final findings will be reported in the near future.
The specific structure of left-right symmetric models enables one to expect *lower* LFV rates than those of non-symmetric scenarios, as the $SU(5)$ discussed so far. We will consider the Yukawa textures defined in [King and Ross, 2001, King and Ross, 2003], where the Georgi-Jarlskog texture (see section 3.5) is implemented so as to account for the different masses of the down-type quarks and charged-leptons, which would otherwise be the same in left-right symmetric scenarios (provided the same expansion parameter is taken for both those mass matrices).
Recall that the very purpose of taking pre-GUT runs into account was to make the $SU(5)$ model compatible with LFV experimental bounds thanks to what has in this work been called the umbrella effect. The aim now is understanding to what extent such pre-GUT runs are really mandatory. As shown in Fig. 5.6, one achieves two major results within left-right symmetric models, namely:

- Remarkable enhancement regarding the allowed values for m_0, which now can be as large as ~ 500 GeV.
- Decrease in the allowed values of $\tan\beta$; intermediate values such as $\tan\beta \sim 35$ are now permitted.

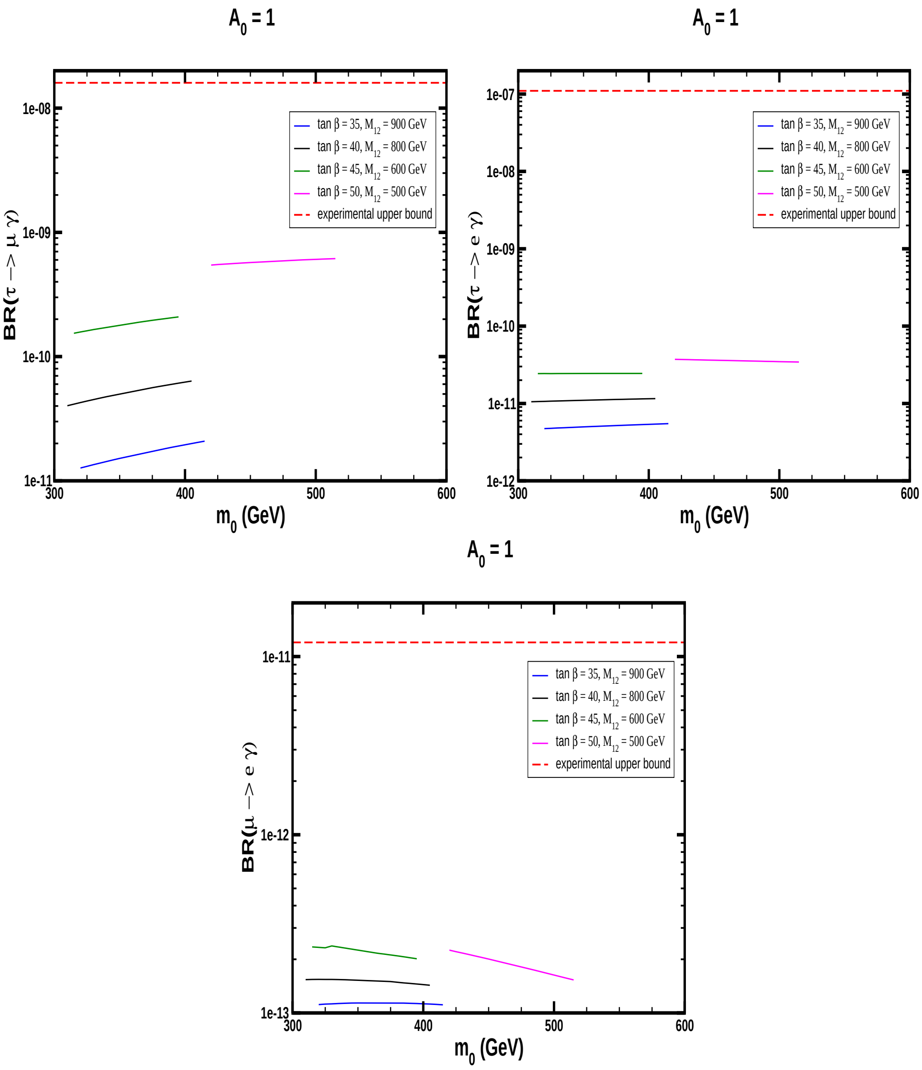

Figure 5.6: *Predictions for LFV rates for* $\ell_i \to \ell_j \gamma$ *for different values of* $\tan\beta$ *and* $M_{1/2}$.

Part III

Conclusions

Chapter 6

Conclusions and Outlook

The purpose of this work has been the systematic analysis of some of the currently open issues in supersymmetric scenarios of particle physics with massive neutrinos, mostly focusing on issues of Dark Matter and charged-lepton flavour violation.

Before establishing the main results, we have given in Chapter 1 a detailed description regarding the current status of particle physics and cosmology, as well as the main directions along which the problems of these two branches of physics may be solved. Concerning particle physics, special emphasis was put on the gauge coupling unification, which is easily achieved when supersymmetry is brought about. However, as stressed, rendering a theory supersymmetry amounts to introducing a fairly large number of free parameters, thereby decreasing predictability. Here is where GUT models show their beauty, since the cherished idea of describing the fundamental interactions within a single framework, thus largely enhancing predictability, is nicely obtained when GUT models are enhanced with supersymmetry.

Regarding Standard Cosmology, since our research is linked to cosmology through the Dark Matter problem, the discussion in Chapter 1 was intended to address the particle physics-cosmology interface.

Chapter 2 analysed the experimental evidence for new physics as well as the ideas put forward for physics beyond the Standard Model, focusing on those directly related to the present work. Thus, supersymmetric gauge field theories were discussed in some detail, as was the idea of unification.

Having introduced the background required to this work, Chapter 3 addressed specific topics within the context of SUSY-GUT models that have been the focus of our research, namely phenomenological issues within Yukawa-unified models enhanced with massive neutrinos. Firstly, we discussed one of the most popular mechanisms to account for the smallness of neutrino masses, known as the *see-saw mechanism*. Next, we commented on the embedding of neutrino masses within a unified framework taking all bounds on fermion masses into account. The direction taken in this work has been based on *family symmetries*, which assume that the extremely hierarchical structure of fermion masses arises from the breaking of a hidden symmetry.

We then moved on to discuss the implications of Yukawa-unified scenarios for Dark Matter. To this end, we analysed the main candidates within SUSY-GUT models in order to explain the dominance of Dark Matter over ordinary matter. This has been done in the framework of models that conserve R-parity, whose most natural Dark Matter candidate is the lightest *neutralino*.

Finally, we also studied Charged-Lepton Flavour Violation, focusing on $\ell_i \to \ell_j \gamma$, which is suppressed in the SM. Any evidence in this respect would be a major step towards the understanding of physics at the fundamental level.

Having described the theoretical topics directly related to the original research put forward in this work, Chapters 4 and 5 have presented the main results. We first argued that lepton mixing arising in models with massive neutrinos enhances the allowed parameter space in $b-\tau$ Yukawa-unified scenarios, making it possible to consider a broader range for $\tan\beta$ as well as both signs for the μ parameter. Along these lines, we discussed the prediction for Dark Matter abundance assuming a neutralino LSP and showed that the above scenario, compatible with low-energy bounds, allows a larger parameter space satisfying the striking WMAP Dark Matter constraints.

Equipped with these results, we have ended our work by showing that the $b-\tau$ Yukawa-unified model with massive neutrinos may shed some light on the pattern of fermion masses once a family symmetry is considered. For definiteness, we have invoked an $U(1)_F$ symmetry that reproduces all low-energy data. In addition, it has been argued that the predictions for LFV rates may lie within current experimental bounds if the quantum corrections above the unification scale are properly taken into account, hence rendering the $SU(5)$ $b-\tau$ Yukawa-unified model feasible.

More precisely, the first part of the original work of this dissertation (Chapter 4) discusses the WMAP Dark Matter constrains on $b-\tau$ Yukawa Unification in the presence of massive neutrinos. Large lepton mixing, as indicated by the data, modifies the predictions for the bottom quark mass and enables Yukawa Unification also for large $\tan\beta$ and for positive values of the μ parameter, which were previously disfavoured. The larger the Dirac neutrino Yukawa coupling, the larger the effects. A direct outcome is that the allowed parameter space for neutralino dark matter also increases, particularly when the effects of large lepton mixing are combined with runs above the GUT scale.

In the second part of this work (Chapter 5), we have shown that $SU(5)$ runs above the GUT scale, naturally suppress the off-diagonal entries of the soft matrices, through what has been called the *umbrella effect*, leading to a nearly flavour-independent contribution for $m_0 < 100$ GeV. Such low values of m_0 are discarded if $SU(5)$ runs are not taken into account, since in this case the lightest stau becomes the LSP. Within this framework, $SU(5)$ runs lead to acceptable LFV rates for the cosmologically preferred parameter space found in [Gomez et al., 2009b], favouring values of $\tan\beta$ around 45. Significant deviations from this value, in either way, are harder to reconcile with cosmological data.

In order to illustrate the above, we have studied LFV predictions in the case that conventional $SU(5)$ is enhanced by an Abelian family symmetry. This is in fact one of the potentially most dangerous scenarios as far as charged-lepton flavour violation is concerned, particularly in the simple realisations where lepton mixing (at the GUT multiplet basis) is dominated by the charged lepton sector. Our results indicate that even in this case, the umbrella effect leads to suppressions to LFV rates, leading to a very significant enhancement of the available parameter space, as compared to the conventional schemes, with runs below M_{GUT}. Summarising, we find the following:

- $b-\tau$ unification is only allowed for $\mu > 0$ in the presence of large charged lepton mixing, which is motivated by the experimental data of the recent years. In particular, large lepton mixing enables Yukawa unification also for large $\tan\beta$, which was previously disfavoured. The larger the Dirac neutrino Yukawa couplings, the larger the effects.

- For $\mu < 0$ we still find values compatible with an exact $b-\tau$ unification at M_X. However, for large Dirac neutrino Yukawa couplings and mixing arising dominantly from the charged lepton sector in the basis where the down-quark mass matrix is diagonal, the space of parameters compatible with WMAP is i) only marginally compatible with the upper bound on $b \to s\gamma$ and ii) entirely excluded on the grounds of the $(g-2)_\mu$ observations, if we use e^-e^+ data to estimate the SM vacuum polarisation contribution.

- The allowed parameter space for neutralino Dark Matter also increases, mostly when the effects of large lepton mixing are combined with runs above M_{GUT}. We have shown that $\tan\beta$ must be bounded from below, $35 < \tan\beta$, to effectively account for LSP co-annihilation.

- Natural suppression of dangerous non-diagonal entries in the slepton mass matrix is achieved by considering the runs above the GUT scale for $0 < m_0 < 100$ GeV, thus yielding acceptable LFV rates. Family symmetries, such as the simple $U(1)_F$ considered in this work, might provide some clues on the underlying fermion textures. In particular, a large set of neutrino family charges is compatible with low energy data.

Since the topics discussed in this work include somewhat speculative research, one should bear in mind that the particular analyses carried out are simply exploring some of the potential possibilities. Even restricting ourselves to the general framework of SUSY GUT models, one enjoys much freedom in identifying the preferred parameter space. In this respect, there remain some open issues to be dealt with.

Firstly (what is the subject of current work and has very briefly commented upon in § 5.3 in relation to LFV), the very use of $SU(5)$ as the GUT group may be abandoned, since the right-handed neutrinos are singlets in this model, which might be unnatural. More complex GUT groups are able to naturally accommodate all fermions within the fundamental representation. Such an example is given by $SO(10)$, which turns out to be the most natural framework to embed the see-saw mechanism.

Along these lines, within a left-right symmetric model such as $SO(10)$, Yukawa unification should be extended to $t-b-\tau$ unified models, as now they all live in the same **16**-plet representation. One might even consider the whole $t-b-\tau-\nu_\tau$ Yukawa unification, although the differences in hierarchies when introducing massive neutrinos might require fine-tuning. This is something that might be worth exploring.

Likewise, the model-dependent analysis discussed for the LFV processes (which would be worth extending to also account for squark mixing) might appear to find a better understanding within left-right models, which would impose further correlations among the family symmetry charges. In particular, even within the framework of family symmetries, one is left with a large set of *a priori* suitable values for the family charges. Further constraining these charges is quite model-dependent and might be related to the details of the heavy Majorana neutrino sector. In this respect, flavour violating decays may shed some light to the mass patterns of right-handed neutrinos, which are not easily constrained by the fermion data alone.

Finally, it may be instructive to make some remarks on the codes implemented to compute the results shown in this work. For the sake of simplicity, we have restricted ourselves to the third-generation approximation, something reasonable taking into account the large hierarchy of the mass matrices. However, there exists a number of drawbacks associated with this approach. Not been able to simultaneously diagonalising the whole set of mass matrices, implies that a careful analysis in a third-generation approach is essential. A natural extension of the present work would include the full 3×3 mass matrices in the numerical analysis.

As an epilogue, it is worth saying a few words regarding the current situation in high-energy physics and cosmology. From the particle-physics side, the LHC has finally started running. It will be exciting to see what it may tell us about the tiniest distance scales, as it is intended to discriminate among the large number of models put forward for physics beyond the standard model. Particularly, one *light* Higgs boson should be discovered, otherwise the whole supersymmetric program would be greatly questioned.

Nevertheless, it seems clear that accelerator physics may soon come to an end, since the required structures to carry these experiments have reached colossal dimensions. One feasible alternative could well be astrophysical and cosmological probes. In this respect, huge efforts have been invested from both the economical and human side, in order to penetrate the deepest structure of the Universe. As mentioned, WMAP has accomplished a major task, and ongoing satellite experiments, such as PLANCK and LIGO, will further shed light on the Universe infancy.

Appendix A

Renormalisation Group Equations

In this appendix we summarise the RGEs that are most relevant for the purposes of the work addressed in this dissertation.
For runs above the GUT scale the equations involving the Yukawa couplings and the soft mass terms corresponding to the **10** and $\bar{\mathbf{5}}$ representations of $SU(5)$, for the 3^{rd} generation, take the form [Hisano and Nomura, 1999]:

$$16\pi^2 \frac{d\lambda_N}{dt} = \left[-\frac{48}{5}g_5^2 + 7\lambda_N^2 + 3\lambda_t^2 + 4\lambda_b^2\right]\lambda_N \ , \tag{A.1}$$

$$16\pi^2 \frac{d\lambda_d}{dt} = \left[-\frac{84}{5}g_5^2 + 10\lambda_d^2 + 3\lambda_t^2 + \lambda_N^2\right]\lambda_d \ , \tag{A.2}$$

$$16\pi^2 \frac{d\lambda_t}{dt} = \left[-\frac{96}{5}g_5^2 + 9\lambda_t^2 + 4\lambda_d^2 + \lambda_N^2\right]\lambda_t \ , \tag{A.3}$$

$$\begin{aligned} 16\pi^2 \frac{dm^2_{\mathbf{10}}}{dt} &= -\frac{144}{5}g_5^2\, M_5^2 + \left(12\lambda_t^2 + 4\lambda_d^2\right) m^2_{\mathbf{10}} \\ &\quad +4\left[\left(m^2_{\mathbf{5}} + m^2_{\bar{h}}\right)\lambda_d^2 + A_d^2\right] + 6\left(\lambda_t^2\, m_h^2 + A_t^2\right) \ , \end{aligned} \tag{A.4}$$

$$\begin{aligned} 16\pi^2 \frac{dm^2_{\mathbf{5}}}{dt} &= -\frac{96}{5}g_5^2\, M_5^2 + 2\left(4\lambda_d^2 + \lambda_N^2\right) m^2_{\mathbf{5}} \\ &\quad +8\left[\left(m^2_{\mathbf{10}} + m^2_{\bar{h}}\right)\lambda_d^2 + A_d^2\right] + 2\left(\lambda_N^2\, m_h^2 + \lambda_N^2\, m^2_{\mathbf{1}} + A_N^2\right) \ . \end{aligned} \tag{A.5}$$

For runs from M_{GUT} down to M_N, the equations for the 3^{rd} generation Yukawa matrices become [Casas and Ibarra, 2001]:

$$16\pi^2 \frac{d\lambda_N}{dt} = -\left[\left(\frac{3}{5}g_1^2 + 3g_2^2\right) I_3 - \left(4\lambda_N^2 + 3\lambda_t^2 + \lambda_\tau^2\right)\right]\lambda_N \ , \tag{A.6}$$

$$16\pi^2 \frac{d\lambda_\tau}{dt} = -\left[\left(\frac{9}{5}g_1^2 + 3g_2^2\right) I_3 - \left(4\lambda_\tau^2 + 3\lambda_b^2\lambda_N^2\right)\right]\lambda_\tau \ , \tag{A.7}$$

$$16\pi^2 \frac{d\lambda_t}{dt} = -\left[\left(\frac{13}{5}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right) I_3 - \left(6\lambda_t^2 + \lambda_b^2\right) + \lambda_N^2\right] \lambda_t \,. \tag{A.8}$$

Let us next summarise the procedure employed in the runs in Chapter 5.

- Firstly, the RGEs for the two lightest generations were implemented, because they enter the calculation of the spectrum needed for obtaining the branching ratios. However, only the Yukawas for the 3^{rd} generation were kept. Thus, we used a first correction to the 3^{rd} generation approximation (no lightest generations at all), but without using the machinery of the full 3-generations RGEs.

- Secondly, the treatment of the A-terms was slightly different. While the usual factorisation $A_{ij} = A_0\, Y_{ij}$ was taken throughout the runs in the analysis of Yukawa Unification and Dark Matter, for the study of LFV we decided to relax such a condition, which only held as a initial condition constraint. Thus, we implemented the RGEs taking the A-terms as independent parameters. The motivation was to properly evaluate the A-term contributions to the LFV rates.

- Finally, we would like to point out a technical detail regarding sign conventions used in the RGEs. The authors of [Hisano and Nomura, 1999, Casas and Ibarra, 2001] use a different sign convention for the A-terms than the authors of [Martin and Vaughn, 1994]. We should stress that such a different convention only holds for the A-terms; the RGEs for the remaining soft terms are the same.

Appendix B

Branching Ratios: Numerical Code

In this Appendix we will collect the specific expressions for the calculation of the branching ratios that have been used in this work. However, instead of explicitly quoting the expressions, we shall rather cite the papers that exhibit such formulae and attach the piece of the numerical code where the computation of the branching ratios is implemented.
The derivation of the explicit expressions for the interaction *gaugino-sfermion-fermion* can be found in Appendix B of Ref. [Hisano et al., 1996]. There is also a pedagogical discussion in Ref. [Romao, 2005], which explicitly shows both the soft mass matrices and all couplings within the MSSM. For the implementation of the final formulae for the branching ratios used in this work, both references were combined, as Ref. [Romao, 2005] does not include the final form for the branching ratios.
In the following piece of code, therefore, we have implemented the relevant couplings as stated in Ref. [Romao, 2005], and then added the relevant neutralino/chargino contributions to the LFV processes.

```
(*You need to run the file tools.m which includes some diagonalizations***)

(****FILE TO WRITE RESULTS****)

xfile = OpenWrite["tb50.dat"]

<<tools.m


(**********Reading Files *****)





f1=ReadList["couplings.out",Number,RecordLists->True];
f2=ReadList["results.out",Number,RecordLists->True];
fml=ReadList["sml.out",Number,RecordLists->True];
fme=ReadList["sme.out",Number,RecordLists->True];
fAl=ReadList["sAl.out",Number,RecordLists->True];
fmic=ReadList["observables.mic",Number,RecordLists->True];
fmasmic=ReadList["masses.mic",Number,RecordLists->True];

(***********DICTIONARY*************************************
files

f1=couplings.out
f2=results.out

         write(23,113) dfloat(ij),m0,m12,a_0,
     ,         MG3,mu,MH3,tb,Mq1,
     ,         Md1,Mu1,Ml1,Mr1,Mq2,
     ,         Md2,Mu2,Ml2,Mr2,Mq3,
     ,    Md3,Mu3,Ml3,Mr3,Atop,
     ,    Ab,Atau,MG1,MG2,Am,b_susy,botz

 *)

(*
1)order
2)m0
3)m12
4)a0
*)


pm0=Interpolation[Transpose[f2][[2]]];
pm12=Interpolation[Transpose[f2][[3]]];
pa0=Interpolation[Transpose[f2][[4]]];



(*
5)MG3
6)mu
7)Mh3
8)tb
*)
fMG3=Interpolation[Transpose[f2][[5]]];

umup=Interpolation[Transpose[f2][[6]]];
```



```
umu[xx_]:=umup[xx];
mA=Interpolation[Transpose[f2][[7]]];
tbf=Interpolation[Transpose[f2][[8]]];
(*
24)Atop
25)Ab
26)Atau
27)MG1
28)MG2
29)Am
*)

fatop=Interpolation[Transpose[f2][[24]]];
fab=Interpolation[Transpose[f2][[25]]];
fatau=Interpolation[Transpose[f2][[26]]];
fam=Interpolation[Transpose[f2][[29]]];

fMG1=Interpolation[Transpose[f2][[27]]];
fMG2=Interpolation[Transpose[f2][[28]]];




(****FILE couplings.out************)
     (*

         write(24,114) dfloat(ij),y25(1),y25(2),y25(3),
     ,            y25(4),y25(5),y25(6),xpar,smass

  *)
     (*
1)Order
2)y(1)
3)y(2)
4)y(3)

5)ytop
6)ybot
7)ytau

8)xpar
9)msusy

*)

s1=Interpolation[Transpose[f1][[2]]];
s2=Interpolation[Transpose[f1][[3]]];
s3=Interpolation[Transpose[f1][[4]]];
s4=Interpolation[Transpose[f1][[5]]];
s5=Interpolation[Transpose[f1][[6]]];
s6=Interpolation[Transpose[f1][[7]]];
xpar=Interpolation[Transpose[f1][[8]]];
fmsusy=Interpolation[Transpose[f1][[9]]];



(****SLEPTON MASS MATRIX***)

ssml11=Interpolation[Transpose[fml][[1]]];
ssml12=Interpolation[Transpose[fml][[2]]];
ssml13=Interpolation[Transpose[fml][[3]]];
ssml21=Interpolation[Transpose[fml][[4]]];
```

```
ssml22=Interpolation[Transpose[fml][[5]]];
ssml23=Interpolation[Transpose[fml][[6]]];
ssml31=Interpolation[Transpose[fml][[7]]];
ssml32=Interpolation[Transpose[fml][[8]]];
ssml33=Interpolation[Transpose[fml][[9]]];

ssme11=Interpolation[Transpose[fme][[1]]];
ssme12=Interpolation[Transpose[fme][[2]]];
ssme13=Interpolation[Transpose[fme][[3]]];
ssme21=Interpolation[Transpose[fme][[4]]];
ssme22=Interpolation[Transpose[fme][[5]]];
ssme23=Interpolation[Transpose[fme][[6]]];
ssme31=Interpolation[Transpose[fme][[7]]];
ssme32=Interpolation[Transpose[fme][[8]]];
ssme33=Interpolation[Transpose[fme][[9]]];

ssAl11=Interpolation[Transpose[fAl][[1]]];
ssAl12=Interpolation[Transpose[fAl][[2]]];
ssAl13=Interpolation[Transpose[fAl][[3]]];
ssAl21=Interpolation[Transpose[fAl][[4]]];
ssAl22=Interpolation[Transpose[fAl][[5]]];
ssAl23=Interpolation[Transpose[fAl][[6]]];
ssAl31=Interpolation[Transpose[fAl][[7]]];
ssAl32=Interpolation[Transpose[fAl][[8]]];
ssAl33=Interpolation[Transpose[fAl][[9]]];

(**********************Micromegas Observables****)


fOh2=Interpolation[Transpose[fmic][[8]]];
fbsg=Interpolation[Transpose[fmic][[9]]];
fwNE2=Interpolation[Transpose[fmic][[12]]];
fwSl1=Interpolation[Transpose[fmic][[14]]];
fwSl2=Interpolation[Transpose[fmic][[16]]];
fwSmR=Interpolation[Transpose[fmic][[18]]];
fwSmL=Interpolation[Transpose[fmic][[20]]];
fwSeR=Interpolation[Transpose[fmic][[22]]];
fwSeL=Interpolation[Transpose[fmic][[24]]];

(**********************Micromegas Masses****)

fMNE1=Interpolation[Transpose[fmasmic][[13]]];
fMNE2=Interpolation[Transpose[fmasmic][[14]]];
fMSl1=Interpolation[Transpose[fmasmic][[27]]];
fMSl2=Interpolation[Transpose[fmasmic][[28]]];
fMSmR=Interpolation[Transpose[fmasmic][[26]]];
fMSmL=Interpolation[Transpose[fmasmic][[25]]];
fMSeR=Interpolation[Transpose[fmasmic][[24]]];
fMSeL=Interpolation[Transpose[fmasmic][[23]]];
fMSnl=Interpolation[Transpose[fmasmic][[22]]];
fMSt1=Interpolation[Transpose[fmasmic][[33]]];
fMSt2=Interpolation[Transpose[fmasmic][[34]]];
fMSb1=Interpolation[Transpose[fmasmic][[39]]];
fMSb2=Interpolation[Transpose[fmasmic][[40]]];
fMh=Interpolation[Transpose[fmasmic][[5]]];
fMH3=Interpolation[Transpose[fmasmic][[6]]];


(****beta***)

beta:=ArcTan[tb];
cbet:=Cos[beta];
```



```
sbet:=Sin[beta];
c2bet:=Cos[2 beta];
s2bet:=Sin[2 beta];




computation[ii_]:=Module[
    {mz=91.19,
  GF = 1.166*10^(-5),
     (****PLANCK VALUES****)
tb=tbf[ii],
m0=pm0[ii],
m12=pm12[ii],
a0=pa0[ii],




     (****Values at msusy********)

gms=s2[ii],
gp=s1[ii]*Sqrt[3/5]//N,
hht=s4[ii],
hhb=s5[ii],
hhe=s6[ii],
     mu=-umup[ii],
     MG1=fMG1[ii],
     MG2=fMG2[ii],
     MG3=fMG3[ii],

Al=fatau[ii],
Am=fam[ii],

     (***slepton mass matrix*********)
     sml={{ssml11[ii],ssml12[ii],ssml13[ii]},
          {ssml21[ii],ssml22[ii],ssml23[ii]},
          {ssml31[ii],ssml32[ii],ssml33[ii]}},

     sme={{ssme11[ii],ssme12[ii],ssme13[ii]},
          {ssme21[ii],ssme22[ii],ssme23[ii]},
          {ssme31[ii],ssme32[ii],ssme33[ii]}},

     sAl={{ssAl11[ii],ssAl12[ii],ssAl13[ii]},
          {ssAl21[ii],ssAl22[ii],ssAl23[ii]},
          {ssAl31[ii],ssAl32[ii],ssAl33[ii]}},



  (****from micromegas file***)

Oh2=fOh2[ii],
bsg=fbsg[ii],
mMNE1=fMNE1[ii],
mMNE2=fMNE2[ii],
mMSl1=fMSl1[ii],
mMSl2=fMSl2[ii],
mMSmR=fMSmR[ii],
mMSmL=fMSmL[ii],
mMSeR=fMSeR[ii],
```

```
mMSeL=fMSeL[ii],
Mh=fMh[ii],
MH3=fMH3[ii],
wNE2=fwNE2[ii],
wSl1=fwSl1[ii],
wSl2=fwSl2[ii],
wSmR=fwSmR[ii],
wSmL=fwSmL[ii],
wSeR=fwSeR[ii],
wSeL=fwSeL[ii]
},

(****beta***)

beta=ArcTan[tb];
cbet=Cos[beta];
sbet=Sin[beta];
c2bet=Cos[2 beta];
s2bet=Sin[2 beta];

twms=gp/gms;
wms=ArcTan[twms];
swms=Sin[wms];
cwms=Cos[wms];

mws=mz*cwms;
bqm=174*cbet*hhb;
tqm=174*sbet*hht;
taum=174*cbet*hhe;

(*We will assume a diagonal charged lepton Yukawa in all couplings*)

dlm=DiagonalMatrix[{0.000511, 0.105, taum}];

(*****NEUTRALINO MASS MATRIX****)
      (****Diagonalization

       Nn^*.neum.Nn^dag=dia(neu)

       ***)

neum={{MG1,0,−mz*Cos[beta]*swms,mz*Sin[beta]*swms },
{0,MG2,mz*Cos[beta]*cwms,−mz*Sin[beta]*cwms},
{−mz*Cos[beta]*swms,mz*Cos[beta]*cwms,0,−mu},
{mz*Sin[beta]*swms,−mz*Sin[beta]*cwms,−mu,0 }};
(*Print[neum//MatrixForm];*)


      neusv= sing4[neum];
      Nnd= neusv[[1]];
      Nn=Transpose[Conjugate[Nnd]];
      Nnc=Conjugate[Nn];
      neuva=neusv[[2]];
      (**Test Nnc.neum.Nnd***)
(*      Print[Nnc.neum.Nnd//Chop//MatrixForm];
      Print[ neuva];
              *)



            (*******Chargino******)
```



```
            (**USE Romao, observe that Mc(Romao)=Mc^T(CPsuperH)***)
            (***diagonalization***)
            (**UC^*.Mc.Vc^+***)

Mc={{MG2,Sqrt[2]*mws*Sin[beta]},{Sqrt[2]*mws*Cos[beta],mu}};
Mcsv= sing2[Mc];
            Uc= Transpose[Mcsv[[1]]];
            Vc=Transpose[Conjugate[Mcsv[[3]]]];
      Ucd=Transpose[Conjugate[Uc]];
      Uct=Transpose[Uc];
      Ucc=Conjugate[Uc];

      Vcd=Transpose[Conjugate[Vc]];
      Vct=Transpose[Vc];
      Vcc=Conjugate[Vc];

      Mcva=Mcsv[[2]];
 (*
Print[Ucc.Mc.Vcd//MatrixForm//Chop];
Print[Mcva];
      *)


              (***CHARGED SLEPTON 6X6 MASS MATRIX***)
              (**ROTATED AS Rsl.Msl.Rsl^+=Diag***)
              (**Eigenvectors are m1>m2....**)



dlm=DiagonalMatrix[{0.000511, 0.105, taum}];


sldtermLL=−1/2(2 mws^2−mz^2)*Cos[2*beta];
sldtermRR=−(mz^2−mws^2)*Cos[2*beta];
sndterm=1/2 mz^2*Cos[2*beta];


(***non diagonal terms***)

MsRR=sme+IdentityMatrix[3]*sldtermRR+dlm.dlm;
MsLL=sml+IdentityMatrix[3]*sldtermLL+dlm.dlm;


MsLR=sAl−mu*tb*IdentityMatrix[3].dlm;
MsRL=Transpose[Conjugate[MsLR]];

Msl={
  Flatten[{Flatten[MsLL[[1]]], Flatten[MsLR[[1]]]}],
  Flatten[{Flatten[MsLL[[2]]], Flatten[MsLR[[2]]]}],
  Flatten[{Flatten[MsLL[[3]]], Flatten[MsLR[[3]]]}],
  Flatten[{Flatten[MsRL[[1]]], Flatten[MsRR[[1]]]}],
  Flatten[{Flatten[MsRL[[2]]], Flatten[MsRR[[2]]]}],
  Flatten[{Flatten[MsRL[[3]]], Flatten[MsRR[[3]]]}]};


      Mslsv=sing6[Msl];
      Rsl=Mslsv[[1]];
      Rsld=Transpose[Conjugate[Rsl]];
      Rslc=Conjugate[Rsl];

      Mslva=Mslsv[[2]]^(1/2);
```

```
(*      Print["Msl="];
Print[Msl//MatrixForm];
      Print["Mslva="];
Print[Rsl.Msl.Rsld//MatrixForm//Chop];
      Print[Mslva];*)
      (****SNEUTRINO 3x3 mass matrix***)
      (*rotarion Rsn.Msn*Rsn^+***)

      (***eigenvectors m1>m2>m3***)

Msn=sml+IdentityMatrix[3]*sndterm;

Msnsv=sing3[Msn];
Rsn=Msnsv[[1]];
Rsnd=Transpose[Conjugate[Rsn]];
Rsnc=Conjugate[Rsn];
Msnva=Msnsv[[2]]^(1/2);


(*      Print["Msn="];
Print[Msn//MatrixForm];
      Print["Msnva="];
Print[Rsn.Msn.Rsnd//MatrixForm//Chop];

      Print[Msnva];
      *)

      (************ neutralino−slep−lep ****vertices***)
      (***Tau−lepton***)

NLt[aa_,xx_]:=
−gms*Sqrt[2]*twms*Nnc[[aa,1]]*Rslc[[xx,6]]−hhe*Nnc[[aa,3]]*Rslc[[xx,3]];

NRt[aa_,xx_]:=
gms/Sqrt[2](Nn[[aa,2]]+twms*Nn[[aa,1]])*Rslc[[xx,3]]−
hhe*Nn[[aa,3]]*Rslc[[xx,6]];

       (****Muon−lepton***)
NLm[aa_,xx_]:=−gms*Sqrt[2]*twms*Nnc[[aa,1]]*Rslc[[xx,5]];
NRm[aa_,xx_]:=gms/Sqrt[2](Nn[[aa,2]]+twms*Nn[[aa,1]])*Rslc[[xx,2]];



       (***Electron−lepton***)
NLe[aa_,xx_]:=−gms*Sqrt[2]*twms*Nnc[[aa,1]]*Rslc[[xx,4]];
NRe[aa_,xx_]:=gms/Sqrt[2]*(Nn[[aa,2]]+twms*Nn[[aa,1]])*Rslc[[xx,1]];

      (***Tau−neutrino***)

NLnt[aa_,xx_]:=0;
NRnt[aa_,xx_]:=−gms/Sqrt[2]*(Nn[[aa,2]]−twms*Nn[[aa,1]])*Rsnc[[xx,3]];

      (***Muon−neutrino***)

NLnm[aa_,xx_]:=0;
NRnm[aa_,xx_]:=−gms/Sqrt[2]*(Nn[[aa,2]]−twms*Nn[[aa,1]])*Rsnc[[xx,2]];

      (***Electron−neutrino***)
NLne[aa_,xx_]:=0;
NRne[aa_,xx_]:=−gms/Sqrt[2]*(Nn[[aa,2]]−twms*Nn[[aa,1]])*Rsnc[[xx,1]];

      (***chargino−slep−lep***vertices***)
```



```
      (***Tau−lepton***)

CLt[aa_,xx_]:=hhe*Ucc[[aa,2]]*Rsnc[[xx,3]];
CRt[aa_,xx_]:=−gms*Vc[[aa,1]]*Rsnc[[xx,3]];

      (***Muon−lepton***)

      CLm[aa_,xx_]:=0;
CRm[aa_,xx_]:=−gms*Vc[[aa,1]]*Rsnc[[xx,2]];


      (***Electron−lepton***)

      CLe[aa_,xx_]:=0;
CRe[aa_,xx_]:=−gms*Vc[[aa,1]]*Rsnc[[xx,1]];

      (***Tau−neutrino***)

CLnt[aa_,xx_]:=−gms*Ucc[[aa,1]]*Rsl[[xx,3]] + hhe*Ucc[[aa,2]]*Rsl[[xx,6]];
CRnt[aa_,xx_]:=0;


      (***Muon−neutrino***)

      CLnm[aa_,xx_]:=−gms*Ucc[[aa,1]]*Rsl[[xx,2]];
CRnm[aa_,xx_]:=0;

      (***Electron−neutrino***)

      CLne[aa_,xx_]:=−gms*Ucc[[aa,1]]*Rsl[[xx,1]];
CRne[aa_,xx_]:=0;



      (****Matrix Elements****)
      (***1 Ligtest Neutralino***)
      (***IN GBet m1>m2***)

Gbet[mm1_,mm2_]:=If[
mm1<mm2,0,mm1*(1−mm2^2/mm1^2)^2/16/Pi];



      (*******BR(li−>lj+g) ******)


      (***Tau−Mu gamma***)


      (***Amplitudes, Neutralino−mediated diagram***)

      ZZ[aa_, xx_] := (neuva[[aa]]/Mslva[[xx]])^2;

      ffN1[aa_,xx_]:=1/(32*Pi^2)*1/(6*(1 − ZZ[aa, xx])^4)*(1 − 6*ZZ[aa, xx] +
      3*ZZ[aa, xx]^2 + 2*ZZ[aa, xx]^3 − 6*ZZ[aa, xx]^2*Log[ZZ[aa, xx]]);

      ffN2[aa_,xx_]:=1/(32*Pi^2)*1/(1 − ZZ[aa, xx])^3*(1 − ZZ[aa, xx]^2 +
                 2*ZZ[aa, xx]*Log[ZZ[aa, xx]]);

AA2NL1 =
 Sum[1/(Mslva[[xx]]^2)*NLm[aa, xx] Conjugate[NLt[aa, xx]]*ffN1[aa,xx],
```

```
{aa, 1, 4}, {xx, 1, 6}];

AA2NL2 =
 Sum[1/(Mslva[[xx]]^2)*NLm[aa, xx] Conjugate[NRt[aa, xx]]*neuva[[aa]]/taum*
ffN2[aa,xx], {aa, 1, 4}, {xx, 1, 6}];


AA2NR1 =
 Sum[1/(Mslva[[xx]]^2)*NRm[aa, xx] Conjugate[NRt[aa, xx]]*ffN1[aa,xx],
{aa, 1, 4}, {xx, 1, 6}];

AA2NR2 =
 Sum[1/(Mslva[[xx]]^2)*NRm[aa, xx] Conjugate[NLt[aa, xx]]*neuva[[aa]]/taum*
ffN2[aa,xx], {aa, 1, 4}, {xx, 1, 6}];



     (***Amplitudes, chargino-mediated diagram***)

      TT[aa_, xx_] := (Mcva[[aa]]/Msnva[[xx]])^2;

  ffC1[aa_,xx_]:=-1/(32*Pi^2)*1/(6*(1 - TT[aa, xx])^4)*(2 + 3*TT[aa, xx] -
     6*TT[aa, xx]^2 + TT[aa, xx]^3 +
     6*TT[aa, xx]*Log[TT[aa, xx]]);

ffC2[aa_,xx_]:=-1/(32*Pi^2)*1/(1 - TT[aa, xx])^3*(-3 + 4*TT[aa, xx] -
      TT[aa, xx]^2 - 2*Log[TT[aa, xx]]);

 AA2CL1 =
 Sum[1/(Msnva[[xx]]^2)*CLm[aa, xx] Conjugate[CLt[aa, xx]]*ffC1[aa,xx],
{aa, 1, 2}, {xx, 1, 3}];

AA2CL2 =
 Sum[1/(Msnva[[xx]]^2)*CLm[aa, xx] Conjugate[CRt[aa, xx]]*Mcva[[aa]]/taum*
ffC2[aa,xx], {aa, 1, 2}, {xx, 1, 3}];

AA2CR1 = Sum[1/(Msnva[[xx]]^2)*CRm[aa, xx] Conjugate[CRt[aa, xx]]*ffC1[aa,xx],
{aa, 1, 2}, {xx, 1, 3}];

AA2CR2 =
 Sum[1/(Msnva[[xx]]^2)*CRm[aa, xx] Conjugate[CLt[aa, xx]]*Mcva[[aa]]/taum*
ffC2[aa,xx], {aa, 1, 2}, {xx, 1, 3}];




      AA2NL = AA2NL1 + AA2NL2;
      AA2NR = AA2NR1 + AA2NR2;


      AA2CL = AA2CL1 + AA2CL2;
      AA2CR = AA2CR1 + AA2CR2;

      AA2L = AA2NL + AA2CL;
      AA2R = AA2NR + AA2CR;


      (*****Width tau->mu + g****)


Widthtm =
 gms^2*1/(1 + twms^(-2))*1/(16*Pi)*
```



```
      taum^5*(Conjugate[AA2L]*AA2L + Conjugate[AA2R]*AA2R);
(***Normalization***)
      Widthtm2nu = taum^5*GF^2/(192*Pi^3);
      (***Branching ratio tau->mu+g***)
      BRtm = (Widthtm/Widthtm2nu)*0.17;

      (***tau->e+gamma***)

      (***Amplitudes for the neutralino-mediated diagram***)


BB2NL1 = Sum[1/(Mslva[[xx]]^2)*NLe[aa, xx] Conjugate[NLt[aa, xx]]*ffN1[aa,xx],
{aa, 1, 4}, {xx, 1, 6}];

BB2NL2 = Sum[1/(Mslva[[xx]]^2)*NLe[aa, xx] Conjugate[NRt[aa, xx]]*
      neuva[[aa]]/taum*ffN2[aa,xx], {aa, 1, 4}, {xx, 1, 6}];


BB2NR1 = Sum[1/(Mslva[[xx]]^2)*NRe[aa, xx] Conjugate[NRt[aa, xx]]*ffN1[aa,xx],
{aa, 1, 4}, {xx, 1, 6}];


BB2NR2 = Sum[1/(Mslva[[xx]]^2)*NRe[aa, xx] Conjugate[NLt[aa, xx]]*
      neuva[[aa]]/taum*ffN2[aa,xx], {aa, 1, 4}, {xx, 1, 6}];


      (***Amplitudes for the chargino-mediated diagram***)



 BB2CL1 =
  Sum[1/(Msnva[[xx]]^2)*CLe[aa, xx] Conjugate[CLt[aa, xx]]*ffC1[aa,xx],
{aa, 1, 2}, {xx, 1, 3}];


BB2CL2 = Sum[1/(Msnva[[xx]]^2)*CLe[aa, xx] Conjugate[CRt[aa, xx]]*
      Mcva[[aa]]/taum*ffC2[aa,xx], {aa, 1, 2}, {xx, 1, 3}];

BB2CR1 = Sum[1/(Msnva[[xx]]^2)*CRe[aa, xx] Conjugate[CRt[aa, xx]]*ffC1[aa,xx],
{aa, 1, 2}, {xx, 1, 3}];

BB2CR2 = Sum[1/(Msnva[[xx]]^2)*CRe[aa, xx] Conjugate[CLt[aa, xx]]*
      Mcva[[aa]]/taum*ffC2[aa,xx], {aa, 1, 2}, {xx, 1, 3}];




      BB2NL = BB2NL1 + BB2NL2;
      BB2NR = BB2NR1 + BB2NR2;


      BB2CL = BB2CL1 + BB2CL2;
      BB2CR = BB2CR1 + BB2CR2;

      BB2L = BB2NL + BB2CL;
      BB2R = BB2NR + BB2CR;


(***Width(tau->e+gamma)***)

Widthte =
  gms^2*1/(1 + twms^(-2))*1/(16*Pi)*
      taum^5*(Conjugate[BB2L]*BB2L + Conjugate[BB2R]*BB2R);
```

```
(***Branching ratio***)

BRte = (Widthte/Widthtm2nu)*0.17;


(***mu->e+gamma***)

     (***Amplitudes for the neutralino-mediated diagram***)

CC2NL1 = Sum[
    1/(Mslva[[xx]]^2)*NLe[aa, xx] Conjugate[NLm[aa, xx]]*ffN1[aa,xx],
{aa, 1, 4}, {xx, 1, 6}];

CC2NL2 = Sum[
    1/(Mslva[[xx]]^2)*NLe[aa, xx] Conjugate[NRm[aa, xx]]*
      neuva[[aa]]/taum*ffN2[aa,xx], {aa, 1, 4}, {xx, 1, 6}];

CC2NR1 = Sum[
    1/(Mslva[[xx]]^2)*NRe[aa, xx] Conjugate[NRm[aa, xx]]*ffN1[aa,xx],
{aa, 1, 4}, {xx, 1, 6}];

CC2NR2 = Sum[
    1/(Mslva[[xx]]^2)*NRe[aa, xx] Conjugate[NLm[aa, xx]]*
      neuva[[aa]]/taum*ffN2[aa,xx], {aa, 1, 4}, {xx, 1, 6}];

(***Amplitudes for the chargino-mediated diagram***)


CC2CL1 = Sum[1/(Msnva[[xx]]^2)*CLe[aa, xx] Conjugate[CLm[aa, xx]]*ffC1[aa,xx],
{aa, 1, 2}, {xx, 1, 3}];

CC2CL2 = Sum[1/(Msnva[[xx]]^2)*CLe[aa, xx] Conjugate[CRm[aa, xx]]*
      Mcva[[aa]]/taum*ffC2[aa,xx], {aa, 1, 2}, {xx, 1, 3}];

CC2CR1 = Sum[1/(Msnva[[xx]]^2)*CRe[aa, xx] Conjugate[CRm[aa, xx]]*ffC1[aa,xx],
{aa, 1, 2}, {xx, 1, 3}];


CC2CR2 = Sum[1/(Msnva[[xx]]^2)*CRe[aa, xx] Conjugate[CLm[aa, xx]]*
      Mcva[[aa]]/taum*ffC2[aa,xx], {aa, 1, 2}, {xx, 1, 3}];


CC2NL = CC2NL1 + CC2NL2;
CC2NR = CC2NR1 + CC2NR2;

CC2CR = CC2CR1 + CC2CR2;
CC2CL = CC2CL1 + CC2CL2;

CC2L = CC2NL + CC2CL;
CC2R = CC2NR + CC2CR;


(***Width(mu->e+gamma)***)

Widthme =
  gms^2*1/(1 + twms^(-2))*1/(16*Pi)*
mmu^5*(Conjugate[CC2L]*CC2L + Conjugate[CC2R]*CC2R);
(***Normalization***)
Widthme2nu = mmu^5*GF^2/(192*Pi^3);
(***Branching ratio***)

BRme = Widthme/Widthme2nu;
```



```
(*Print[{tb,m0,m12,a0,mMNE1,mMNE2,mMSl1,mMSl2,mMSmR,mMSmL,Oh2,bsg,
         brNmmN,brNttN, brNtmN,Mh,MH3, Abs[Re[BRtm]],Abs[Re[BRte]],
Abs[Re[BRme]],AA2NL1,AA2NL2,AA2NR1,AA2NR2,AA2CL1,AA2CL2,AA2CR1,AA2CR2,
BB2NL1,BB2NL2,BB2NR1,BB2NR2,BB2CL1,BB2CL2,BB2CR1,BB2CR2,CC2NL1,CC2NL2,
CC2NR1,CC2NR2,CC2CL1,CC2CL2,CC2CR1,CC2CR2}];
*)

WriteString[xfile, FortranForm[tb], "\t",FortranForm[m0],"\t",
FortranForm[m12],"\t",FortranForm[a0], "\t",FortranForm[Abs[Re[BRtm]]],"\t",
FortranForm[Abs[Re[BRte]]],"\t",FortranForm[Abs[Re[BRme]]],"\t",
FortranForm[Oh2], "\t",FortranForm[bsg], "\t",FortranForm[Mh],"\n"];


Print[ii]


]


Do[computation[ii],{ii,1,Length[f1]}]
 (*Do[computation[ii],{ii,300,600,100}]*)
```

Appendix C

Contributions

Here we summarise the list of original publications from the work presented in this thesis.

Journals

- *WMAP Dark Matter Constraints on Yukawa Unification with Massive Neutrinos*, Gomez, M. E., Lola, S., Naranjo, P. & Rodriguez-Quintero, J., JHEP **04**, 043 (2009).
- *Suppression of Lepton Flavour Violation from Quantum Corrections above M_{GUT}*, Gomez, M. E., Lola, S., Naranjo, P. & Rodriguez-Quintero, J., JHEP **06**, 053 (2010).

Conference Proceedings

- *τ-Flavour Violation at the LHC*, Carquin, E., Gomez, M. E., Naranjo, P. & Rodriguez-Quintero, J.
  To appear in the proceedings of LC09: e^+e^- Physics at the TeV Scale and the Dark Matter Connection, Perugia, Italy, 21^{st}-24^{th} September, 2009.
- *Lepton Flavour Violation in $b-\tau$ Yukawa Unification with Massive Neutrinos*, Carquin, E., Gomez, M. E., Lola, S., Naranjo, P. & Rodriguez-Quintero, J.
  To appear in the proceedings of SUSY 2009: 17^{th} International Conference on Supersymmetry and Unification of Fundamental Interactions, Boston, USA, 5^{th}-10^{th} June, 2009.
- *Dark Matter and Lepton Flavour Violation in Yukawa Unification with Massive Neutrinos*, Carquin, E., Gomez, M. E., Lola, S., Naranjo, P. & Rodriguez-Quintero, J., PoS **IDM2008**, 090 (2009).
- *Dark Matter and Yukawa Unification with Massive Neutrinos*, Gomez, M. E., Lola, S., Naranjo, P. & Rodriguez-Quintero, J., AIP Conf. Proc. **1115**, 273-278 (2009).

Additional Conference Presentations by P. Naranjo

- *WMAP Dark Matter Constraints on Yukawa Unification with Massive Neutrinos*, Gomez, M. E., Lola, S., Naranjo, P. & Rodriguez-Quintero, J.
  Talk given at the ENTApP Workshop on Dark Matter, CERN, February, 2^{nd}- 6^{th} 2009.

- *WMAP Constraints on Dark Matter and Yukawa unification with Massive Neutrinos*, Gomez, M. E., Lola, S., Naranjo, P. & Rodriguez-Quintero, J.
  Talk given at the 2^{nd} International Meeting of UniverseNet: Seeking for links between Fundamental Physics and Cosmology, Oxford (UK), September, 22^{nd}-26^{th} 2008.

Bibliography

[Abazajian et al., 2009] Abazajian, K. N. et al. (2009). The Seventh Data Release of the Sloan Digital Sky Survey. *Astrophys. J. Suppl.*, 182:543–558.

[Ahmad et al., 2001] Ahmad, Q. R. et al. (2001). Measurement of the charged current interactions produced by B-8 solar neutrinos at the Sudbury Neutrino Observatory. *Phys. Rev. Lett.*, 87:071301.

[Ahmad et al., 2002] Ahmad, Q. R. et al. (2002). Direct evidence for neutrino flavour transformation from neutral-current interactions in the Sudbury Neutrino Observatory. *Phys. Rev. Lett.*, 89:011301.

[Ahn et al., 2003] Ahn, M. H. et al. (2003). Indications of Neutrino Oscillation in a 250 km Long-baseline Experiment. *Phys. Rev. Lett.*, 90:041801.

[Altarelli and Feruglio, 2004] Altarelli, G. and Feruglio, F. (2004). Models of neutrino masses and mixings. *New J. Phys.*, 6:106.

[Amsler et al., 2008] Amsler, C. et al. (2008). Review of particle physics. *Phys. Lett.*, B667:1.

[Antoniadis, 1990] Antoniadis, I. (1990). A possible new dimension at a few TeV. *Phys. Lett.*, B246:377–384.

[Antoniadis et al., 1998] Antoniadis, I., Arkani-Hamed, N., Dimopoulos, S., and Dvali, G. R. (1998). New dimensions at a millimeter to a Fermi and superstrings at a TeV. *Phys. Lett.*, B436:257–263.

[Antusch et al., 2008] Antusch, S., King, S. F., and Malinsky, M. (2008). Solving the SUSY Flavour and CP Problems with $SU(3)$ Family Symmetry. *JHEP*, 06:068.

[Antusch et al., 2009] Antusch, S., King, S. F., Malinsky, M., and Ross, G. G. (2009). Solving the SUSY Flavour and CP Problems with Non-Abelian Family Symmetry and Supergravity. *Phys. Lett.*, B670:383–389.

[Araki et al., 2005] Araki, T. et al. (2005). Measurement of neutrino oscillation with KamLAND: Evidence of spectral distortion. *Phys. Rev. Lett.*, 94:081801.

[Arkani-Hamed et al., 2001a] Arkani-Hamed, N., Cohen, A. G., and Georgi, H. (2001a). (De)constructing dimensions. *Phys. Rev. Lett.*, 86:4757–4761.

[Arkani-Hamed et al., 2001b] Arkani-Hamed, N., Cohen, A. G., and Georgi, H. (2001b). Electroweak symmetry breaking from dimensional deconstruction. *Phys. Lett.*, B513:232–240.

[Arkani-Hamed et al., 2002] Arkani-Hamed, N., Cohen, A. G., Gregoire, T., and Wacker, J. G. (2002). Phenomenology of electroweak symmetry breaking from theory space. *JHEP*, 08:020.

[Arkani-Hamed et al., 1998] Arkani-Hamed, N., Dimopoulos, S., and Dvali, G. R. (1998). The hierarchy problem and new dimensions at a millimeter. *Phys. Lett.*, B429:263–272.

[Arkani-Hamed et al., 1999] Arkani-Hamed, N., Dimopoulos, S., and Dvali, G. R. (1999). Phenomenology, astrophysics and cosmology of theories with sub-millimeter dimensions and TeV scale quantum gravity. *Phys. Rev.*, D59:086004.

[Babu and Mohapatra, 1995] Babu, K. S. and Mohapatra, R. N. (1995). Mass matrix textures from superstrings inspired $SO10$ models. *Phys. Rev. Lett.*, 74:2418–2421.

[Baer et al., 2003] Baer, H., Balazs, C., Belyaev, A., Krupovnickas, T., and Tata, X. (2003). Updated reach of the CERN LHC and constraints from relic density, $b \to s\gamma$ and $a(\mu)$ in the mSUGRA model. *JHEP*, 06:054.

[Baer et al., 2004] Baer, H., Belyaev, A., Krupovnickas, T., and O'Farrill, J. (2004). Indirect, direct and collider detection of neutralino dark matter. *JCAP*, 0408:005.

[Baer et al., 2008] Baer, H., Kraml, S., Sekmen, S., and Summy, H. (2008). Dark matter allowed scenarios for Yukawa-unified $SO(10)$ SUSY GUTs. *JHEP*, 03:056.

[Barberio et al., 2007] Barberio, E. et al. (2007). Averages of $b-$hadron properties at the end of 2006.

[Barbieri et al., 1982] Barbieri, R., Ferrara, S., and Savoy, C. A. (1982). Gauge Models with Spontaneously Broken Local Supersymmetry. *Phys. Lett.*, B119:343.

[Barbieri et al., 1997] Barbieri, R., Hall, L. J., Raby, S., and Romanino, A. (1997). Unified theories with $U(2)$ flavour symmetry. *Nucl. Phys.*, B493:3–26.

[Barger et al., 2008] Barger, V., Marfatia, D., and Mustafayev, A. (2008). Neutrino sector impacts SUSY dark matter. *Phys. Lett.*, B665:242–251.

[Belanger et al., 2007] Belanger, G., Boudjema, F., Pukhov, A., and Semenov, A. (2007). micrOMEGAs2.0: A program to calculate the relic density of dark matter in a generic model. *Comput. Phys. Commun.*, 176:367–382.

[Belanger et al., 2009] Belanger, G., Boudjema, F., Pukhov, A., and Semenov, A. (2009). Dark matter direct detection rate in a generic model with micrOMEGAs2.1. *Comput. Phys. Commun.*, 180:747–767.

[Bennett et al., 1996] Bennett, C. L. et al. (1996). 4-Year COBE DMR Cosmic Microwave Background Observations: Maps and Basic Results. *Astrophys. J.*, 464:L1–L4.

[Bennett et al., 2004] Bennett, G. W. et al. (2004). Measurement of the negative muon anomalous magnetic moment to 0.7-ppm. *Phys. Rev. Lett.*, 92:161802.

[Binetruy et al., 1996] Binetruy, P., Lavignac, S., and Ramond, P. (1996). Yukawa textures with an anomalous horizontal Abelian symmetry. *Nucl. Phys.*, B477:353–377.

[Bomark et al., 2009] Bomark, N. E., Lola, S., Osland, P., and Raklev, A. R. (2009). Gravitino Dark Matter and the Flavour Structure of R- violating Operators. *Phys. Lett.*, B677:62–70.

[Borzumati and Masiero, 1986] Borzumati, F. and Masiero, A. (1986). Large Muon and electron Number Violations in Supergravity Theories. *Phys. Rev. Lett.*, 57:961.

[Brignole et al., 1994] Brignole, A., Murayama, H., and Rattazzi, R. (1994). Upper bound on hot dark matter density from $SO(10)$ Yukawa unification. *Phys. Lett.*, B335:345–354.

[Buchmuller et al., 2007] Buchmuller, W., Covi, L., Hamaguchi, K., Ibarra, A., and Yanagida, T. (2007). Gravitino dark matter in R-parity breaking vacua. *JHEP*, 03:037.

[Caldwell, 2002] Caldwell, R. R. (2002). A Phantom Menace? *Phys. Lett.*, B545:23–29.

[Calibbi et al., 2007] Calibbi, L., Mambrini, Y., and Vempati, S. K. (2007). SUSY-GUTs, SUSY-Seesaw and the Neutralino Dark Matter. *JHEP*, 09:081.

[Carena et al., 2000] Carena, M. S., Ellis, J. R., Lola, S., and Wagner, C. E. M. (2000). Neutrino masses, mixing angles and the unification of couplings in the MSSM. *Eur. Phys. J.*, C12:507–520.

[Carena et al., 1994] Carena, M. S., Olechowski, M., Pokorski, S., and Wagner, C. E. M. (1994). Electroweak symmetry breaking and $b-t$ Yukawa unification. *Nucl. Phys.*, B426:269–300.

[Carquin et al., 2009] Carquin, E., Ellis, J., Gomez, M. E., Lola, S., and Rodriguez-Quintero, J. (2009). Search for Tau Flavour Violation at the LHC. *JHEP*, 05:026.

[Casas and Ibarra, 2001] Casas, J. A. and Ibarra, A. (2001). Oscillating neutrinos and $\mu \to e\gamma$. *Nucl. Phys.*, B618:171–204.

[Chamseddine et al., 1982] Chamseddine, A. H., Arnowitt, R. L., and Nath, P. (1982). Locally Supersymmetric Grand Unification. *Phys. Rev. Lett.*, 49:970.

[Chankowski et al., 2005] Chankowski, P. H., Kowalska, K., Lavignac, S., and Pokorski, S. (2005). Flavour changing neutral currents and inverted sfermion mass hierarchy.

[Chattopadhyay and Nath, 1996] Chattopadhyay, U. and Nath, P. (1996). Probing supergravity grand unification in the Brookhaven $(g-2)_\mu$ experiment. *Phys. Rev.*, D53:1648–1657.

[Chiang et al., 1993] Chiang, H. C., Oset, E., Kosmas, T. S., Faessler, A., and Vergados, J. D. (1993). Coherent and incoherent $\mu^- - e^-$ conversion in nuclei. *Nucl. Phys.*, A559:526–542.

[Cole et al., 2005] Cole, S. et al. (2005). The 2dF Galaxy Redshift Survey: Power-spectrum analysis of the final dataset and cosmological implications. *Mon. Not. Roy. Astron. Soc.*, 362:505–534.

[Deppisch et al., 2006] Deppisch, F., Kosmas, T. S., and Valle, J. W. F. (2006). Enhanced $\mu^- - e^-$ conversion in nuclei in the inverse seesaw model. *Nucl. Phys.*, B752:80–92.

[Deppisch and Valle, 2005] Deppisch, F. and Valle, J. W. F. (2005). Enhanced lepton flavour violation in the supersymmetric inverse seesaw model. *Phys. Rev.*, D72:036001.

[Dudas et al., 1995] Dudas, E., Pokorski, S., and Savoy, C. A. (1995). Yukawa matrices from a spontaneously broken Abelian symmetry. *Phys. Lett.*, B356:45–55.

[Dunkley et al., 2009] Dunkley, J. et al. (2009). Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Likelihoods and Parameters from the WMAP data. *Astrophys. J. Suppl.*, 180:306–329.

[Eguchi et al., 2003] Eguchi, K. et al. (2003). First results from KamLAND: Evidence for reactor anti- neutrino disappearance. *Phys. Rev. Lett.*, 90:021802.

[Ellis et al., 2009] Ellis, J., Lola, S., and Raidal, M. (2009). Supersymmetric Grand Unification and Lepton Universality in $K \to l\,\nu$ Decays. *Nucl. Phys.*, B812:128–143.

[Ellis et al., 2010] Ellis, J., Mustafayev, A., and Olive, K. A. (2010). What if Supersymmetry Breaking Unifies Beyond the GUT Scale?

[Ellis et al., 2001] Ellis, J. R., Falk, T., Ganis, G., Olive, K. A., and Srednicki, M. (2001). The CMSSM Parameter Space at Large $\tan\beta$. *Phys. Lett.*, B510:236–246.

[Ellis et al., 2007a] Ellis, J. R., Gomez, M. E., and Lola, S. (2007a). CP and Lepton-Number Violation in GUT Neutrino Models with Abelian Flavour Symmetries. *JHEP*, 07:052.

[Ellis et al., 1984] Ellis, J. R., Hagelin, J. S., Nanopoulos, D. V., Olive, K. A., and Srednicki, M. (1984). Supersymmetric relics from the big bang. *Nucl. Phys.*, B238:453–476.

[Ellis et al., 2007b] Ellis, J. R., Heinemeyer, S., Olive, K. A., Weber, A. M., and Weiglein, G. (2007b). The Supersymmetric Parameter Space in Light of B^- physics Observables and Electroweak Precision Data. *JHEP*, 08:083.

[Ellis and Nanopoulos, 1982] Ellis, J. R. and Nanopoulos, D. V. (1982). Flavour Changing Neutral Interactions in Broken Supersymmetric Theories. *Phys. Lett.*, B110:44.

[Ellis et al., 2004] Ellis, J. R., Olive, K. A., Santoso, Y., and Spanos, V. C. (2004). Gravitino dark matter in the CMSSM. *Phys. Lett.*, B588:7–16.

[Fayet and Ferrara, 1977] Fayet, P. and Ferrara, S. (1977). Supersymmetry. *Phys. Rept.*, 32:249–334.

[Floratos et al., 1996] Floratos, E. G., Leontaris, G. K., and Lola, S. (1996). MSSM phenomenology in the large $\tan\beta$ regime. *Phys. Lett.*, B365:149–156.

[Froggatt and Nielsen, 1979] Froggatt, C. D. and Nielsen, H. B. (1979). Hierarchy of Quark Masses, Cabibbo Angles and CP Violation. *Nucl. Phys.*, B147:277.

[Fukuda et al., 1998] Fukuda, Y. et al. (1998). Evidence for oscillation of atmospheric neutrinos. *Phys. Rev. Lett.*, 81:1562–1567.

[Fukuda et al., 1999a] Fukuda, Y. et al. (1999a). Constraints on neutrino oscillation parameters from the measurement of day-night solar neutrino fluxes at Super- Kamiokande. *Phys. Rev. Lett.*, 82:1810–1814.

[Fukuda et al., 1999b] Fukuda, Y. et al. (1999b). Measurement of the solar neutrino energy spectrum using neutrino electron scattering. *Phys. Rev. Lett.*, 82:2430–2434.

[Gates et al., 1983] Gates, S. J., Grisaru, M. T., Rocek, M., and Siegel, W. (1983). Superspace, or one thousand and one lessons in supersymmetry. *Front. Phys.*, 58:1–548.

[Gell-Mann et al., 1979] Gell-Mann, M., Ramond, P., and Slansky, R. (1979). Complex Spinors and Unified Theories. Print-80-0576 (CERN).

[Glashow, 1961] Glashow, S. L. (1961). Partial Symmetries of Weak Interactions. *Nucl. Phys.*, 22:579–588.

[Goldberg, 1983] Goldberg, H. (1983). Constraint on the photino mass from cosmology. *Phys. Rev. Lett.*, 50:1419.

[Gomez et al., 2010a] Gomez, M. E., Carquin, E., Naranjo, P., and Rodriguez-Quintero, J. (2010a). τ-Flavour Violation at the LHC. *To appear in the proceedings of LC09: e^-e^+ Physics at the TeV Scale and the Dark Matter Connection, Perugia, Italy, 21-24 Sep 2009.*

[Gomez et al., 2004] Gomez, M. E., Ibrahim, T., Nath, P., and Skadhauge, S. (2004). Sensitivity of supersymmetric dark matter to the b-quark mass. *Phys. Rev.*, D70:035014.

[Gomez et al., 2000a] Gomez, M. E., Lazarides, G., and Pallis, C. (2000a). Supersymmetric cold dark matter with Yukawa unification. *Phys. Rev.*, D61:123512.

[Gomez et al., 2000b] Gomez, M. E., Lazarides, G., and Pallis, C. (2000b). Yukawa unification, $b \to s\,\gamma$ and bino stau coannihilation. *Phys. Lett.*, B487:313–320.

[Gomez et al., 1999] Gomez, M. E., Leontaris, G. K., Lola, S., and Vergados, J. D. (1999). $U(1)$-textures and lepton flavor violation. *Phys. Rev.*, D59:116009.

[Gomez et al., 2009a] Gomez, M. E., Lola, S., Naranjo, P., and Rodriguez-Quintero, J. (2009a). Dark Matter and Yukawa Unification with Massive Neutrinos. *AIP Conf. Proc.*, 1115:273–278.

[Gomez et al., 2009b] Gomez, M. E., Lola, S., Naranjo, P., and Rodriguez-Quintero, J. (2009b). WMAP Dark Matter Constraints on Yukawa Unification with Massive Neutrinos. *JHEP*, 04:043.

[Gomez et al., 2010b] Gomez, M. E., Lola, S., Naranjo, P., and Rodriguez-Quintero, J. (2010b). Suppression of Lepton Flavour Violation from Quantum Corrections above M_{GUT}. *JHEP*, 06:053.

[Gomez et al., 2009c] Gomez, M. E., Rodriguez-Quintero, J., Lola, S., Naranjo, P., and Carquin, E. (2009c). Dark Matter and Lepton Flavour Violation in Yukawa Unification with Massive Neutrinos. *PoS*, IDM2008:090.

[Gonzalez-Garcia, 2005] Gonzalez-Garcia, M. C. (2005). Global analysis of neutrino data. *Phys. Scripta*, T121:72–77.

[Green et al., 1988] Green, M. B., Schwarz, J. H., and Witten, E. (1988). *Superstring Theory, Vol. 1: Introduction; Vol. 2: Loop Amplitudes, Anomalies and Phenomenology.* Cambridge Monographs on Mathematical Physics. Cambridge University Press, Cambridge, UK.

[Gross and Wilczek, 1973a] Gross, D. J. and Wilczek, F. (1973a). Asymptotically Free Gauge Theories. 1. *Phys. Rev.*, D8:3633–3652.

[Gross and Wilczek, 1973b] Gross, D. J. and Wilczek, F. (1973b). Ultraviolet Behaviour of non-Abelian Gauge Theories. *Phys. Rev. Lett.*, 30:1343–1346.

[Grzadkowski and Lindner, 1987] Grzadkowski, B. and Lindner, M. (1987). Non-linear Evolution of Yukawa Couplings. *Phys. Lett.*, B193:71.

[Guth, 1981] Guth, A. H. (1981). The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems. *Phys. Rev.*, D23:347–356.

[Haba et al., 1999] Haba, N., Matsui, Y., Okamura, N., and Sugiura, M. (1999). Energy-scale dependence of the lepton-flavor-mixing matrix. *Eur. Phys. J.*, C10:677–680.

[Haba et al., 2000] Haba, N., Okamura, N., and Sugiura, M. (2000). The renormalization group analysis of the large lepton flavor mixing and the neutrino mass. *Prog. Theor. Phys.*, 103:367–377.

[Haber and Kane, 1985] Haber, H. E. and Kane, G. L. (1985). The Search for Supersymmetry: Probing Physics Beyond the Standard Model. *Phys. Rept.*, 117:75–263.

[Hall et al., 1983] Hall, L. J., Lykken, J. D., and Weinberg, S. (1983). Supergravity as the Messenger of Supersymmetry Breaking. *Phys. Rev.*, D27:2359–2378.

[Hall et al., 1994] Hall, L. J., Rattazzi, R., and Sarid, U. (1994). The Top quark mass in supersymmetric $SO(10)$ unification. *Phys. Rev.*, D50:7048–7065.

[Heinemeyer et al., 2006] Heinemeyer, S., Hollik, W., and Weiglein, G. (2006). Electroweak precision observables in the minimal supersymmetric standard model. *Phys. Rept.*, 425:265–368.

[Hinshaw et al., 2009] Hinshaw, G. et al. (2009). Five-Year Wilkinson Microwave Anisotropy Probe (WMAP 1) Observations: Data Processing, Sky Maps & Basic Results. *Astrophys. J. Suppl.*, 180:225–245.

[Hisano et al., 1996] Hisano, J., Moroi, T., Tobe, K., and Yamaguchi, M. (1996). Lepton-Flavour Violation via Right-Handed Neutrino Yukawa Couplings in Supersymmetric Standard Model. *Phys. Rev.*, D53:2442–2459.

[Hisano et al., 1995] Hisano, J., Moroi, T., Tobe, K., Yamaguchi, M., and Yanagida, T. (1995). Lepton flavour violation in the supersymmetric standard model with seesaw induced neutrino masses. *Phys. Lett.*, B357:579–587.

[Hisano and Nomura, 1999] Hisano, J. and Nomura, D. (1999). Solar and atmospheric neutrino oscillations and lepton flavor violation in supersymmetric models with the right-handed neutrinos. *Phys. Rev.*, D59:116005.

[Hubble, 1929] Hubble, E. (1929). A relation between distance and radial velocity among extra–galactic nebulae. *Proc. Nat. Acad. Sci.*, 15:168–173.

[Irges et al., 1998] Irges, N., Lavignac, S., and Ramond, P. (1998). Predictions from an anomalous $U(1)$ model of Yukawa hierarchies. *Phys. Rev.*, D58:035003.

[Itzykson and Zuber, 2006] Itzykson, C. and Zuber, J.-B. (2006). *Quantum Field Theory.* Dover, New York.

[King and Ross, 2001] King, S. F. and Ross, G. G. (2001). Fermion masses and mixing angles from $SU(3)$ family symmetry. *Phys. Lett.*, B520:243–253.

[King and Ross, 2003] King, S. F. and Ross, G. G. (2003). Fermion masses and mixing angles from $SU(3)$ family symmetry and unification. *Phys. Lett.*, B574:239–252.

[Kolb and Turner, 1994] Kolb, E. and Turner, M. (1994). *The Early Universe.* Frontiers in Physics. Westview Press, Boulder, Colo.

[Komatsu et al., 2009] Komatsu, E. et al. (2009). Five-Year Wilkinson Microwave Anisotropy Probe (WMAP 1) Observations: Cosmological Interpretation. *Astrophys. J. Suppl.*, 180:330–376.

[Komine and Yamaguchi, 2002] Komine, S. and Yamaguchi, M. (2002). Bottom-tau unification in SUSY $SU(5)$ GUT and constraints from $b \to s\gamma$ and $(g-2)_\mu$. *Phys. Rev.*, D65:075013.

[Lahanas et al., 2000] Lahanas, A. B., Nanopoulos, D. V., and Spanos, V. C. (2000). Neutralino relic density in a Universe with a non- vanishing cosmological constant. *Phys. Rev.*, D62:023515.

[Lahanas and Spanos, 2002] Lahanas, A. B. and Spanos, V. C. (2002). Implications of the Pseudo-Scalar Higgs Boson in determining the Neutralino Dark Matter. *Eur. Phys. J.*, C23:185–190.

[Lancaster, 2008] Lancaster, M. (2008). A Combination of CDF and D0 Results on the Mass of the Top Quark.

[Langacker, 1981] Langacker, P. (1981). Grand Unified Theories and Proton Decay. *Phys. Rept.*, 72:185.

[Leontaris et al., 1995] Leontaris, G. K., Lola, S., and Ross, G. G. (1995). Heavy neutrino threshold effects in low-energy phenomenology. *Nucl. Phys.*, B454:25–44.

[Leontaris and Tracas, 1998] Leontaris, G. K. and Tracas, N. D. (1998). Lepton flavour violation in unified models with $U(1)$- family symmetries. *Phys. Lett.*, B431:90–97.

[Linde, 1983] Linde, A. D. (1983). Chaotic Inflation. *Phys. Lett.*, B129:177–181.

[Linde, 1990] Linde, A. D. (1990). *Particle Physics and Inflationary Cosmology*, volume 5 of *Contemporary Concepts in Physics*. CRC Press, UK, 1^{st} edition.

[Lola et al., 2007] Lola, S., Osland, P., and Raklev, A. R. (2007). Radiative gravitino decays from R-parity violation. *Phys. Lett.*, B656:83–90.

[Lopez et al., 1994] Lopez, J. L., Nanopoulos, D. V., and Wang, X. (1994). Large $(g-2)_\mu$ in $SU(5) \otimes U(1)$ supergravity models. *Phys. Rev.*, D49:366–372.

[Lyth and Liddle, 2009] Lyth, D. H. and Liddle, A. R. (2009). *The Primordial Density Perturbation: Cosmology, Inflation and the Origin of Cosmic Structure*. Cambridge University Press, Cambridge, UK, revised edition edition.

[Maki et al., 1962] Maki, Z., Nakagawa, M., and Sakata, S. (1962). Remarks on the unified model of elementary particles. *Prog. Theor. Phys.*, 28:870–880.

[Maltoni et al., 2004] Maltoni, M., Schwetz, T., Tortola, M. A., and Valle, J. W. F. (2004). Status of global fits to neutrino oscillations. *New J. Phys.*, 6:122.

[Martin, 1997] Martin, S. P. (1997). A Supersymmetry Primer.

[Martin and Vaughn, 1994] Martin, S. P. and Vaughn, M. T. (1994). Two loop renormalization group equations for soft supersymmetry breaking couplings. *Phys. Rev.*, D50:2282.

[Michael et al., 2006] Michael, D. G. et al. (2006). Observation of muon neutrino disappearance with the MINOS detectors and the NuMI neutrino beam. *Phys. Rev. Lett.*, 97:191801.

[Mikheev and Smirnov, 1985] Mikheev, S. P. and Smirnov, A. Y. (1985). Resonance enhancement of oscillations in matter and solar neutrino spectroscopy. *Sov. J. Nucl. Phys.*, 42:913–917.

[Mikheev and Smirnov, 1986] Mikheev, S. P. and Smirnov, A. Y. (1986). Resonant amplification of neutrino oscillations in matter and solar neutrino spectroscopy. *Nuovo Cim.*, C9:17–26.

[Miller et al., 2007] Miller, J. P., de Rafael, E., and Roberts, B. L. (2007). Muon g-2: Review of Theory and Experiment. *Rept. Prog. Phys.*, 70:795.

[Misiak et al., 2007] Misiak, M. et al. (2007). The first estimate of $B(\bar{B} \to X_s \gamma)$ at $\mathcal{O}(\alpha_s^2)$. *Phys. Rev. Lett.*, 98:022002.

[Misiak and Steinhauser, 2007] Misiak, M. and Steinhauser, M. (2007). NNLO QCD corrections to the $B \to X_s \gamma$ matrix elements using interpolation in m_c. *Nucl. Phys.*, B764:62–82.

[Montvay and Munster, 1997] Montvay, I. and Munster, G. (1997). *Quantum Fields on a Lattice*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Cambridge, UK.

[Nath, 2003] Nath, P. (2003). Twenty years of SUGRA.

[Nath et al., 1983] Nath, P., Arnowitt, R. L., and Chamseddine, A. H. (1983). Gauge Hierarchy in Supergravity Guts. *Nucl. Phys.*, B227:121.

[Nilles, 1984] Nilles, H. P. (1984). Supersymmetry, Supergravity and Particle Physics. *Phys. Rept.*, 110:1–162.

[Nir and Seiberg, 1993] Nir, Y. and Seiberg, N. (1993). Should squarks be degenerate? *Phys. Lett.*, B309:337–343.

[Padmanabhan, 2003] Padmanabhan, T. (2003). Cosmological constant: the weight of the vacuum. *Phys. Rept.*, 380:235–320.

[Paradisi, 2005] Paradisi, P. (2005). Constraints on SUSY lepton flavour violation by rare processes. *JHEP*, 10:006.

[Peebles and Ratra, 2003] Peebles, P. J. E. and Ratra, B. (2003). The cosmological constant and dark energy. *Rev. Mod. Phys.*, 75:559–606.

[Penzias and Wilson, 1965] Penzias, A. A. and Wilson, R. W. (1965). A Measurement of excess antenna temperature at 4080-Mc/s. *Astrophys. J.*, 142:419–421.

[Perlmutter et al., 1999] Perlmutter, S. et al. (1999). Measurements of Omega and Lambda from 42 High-Redshift Supernovae. *Astrophys. J.*, 517:565–586.

[Peskin and Schroeder, 1995] Peskin, M. E. and Schroeder, D. V. (1995). *An Introduction To Quantum Field Theory*. Frontiers in Physics. Westview Press, Boulder, Colo.

[Pirogov and Zenin, 1999] Pirogov, Y. F. and Zenin, O. V. (1999). Two-loop renormalization group restrictions on the standard model and the fourth chiral family. *Eur. Phys. J.*, C10:629–638.

[Polchinski, 2005] Polchinski, J. (2005). *String Theory, Vol. 1; Vol. 2: Superstring Theory and Beyond*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Cambridge, UK.

[Politzer, 1973] Politzer, H. D. (1973). Reliable Perturbative Results for Strong Interactions? *Phys. Rev. Lett.*, 30:1346–1349.

[Pontecorvo, 1968] Pontecorvo, B. (1968). Neutrino experiments and the question of leptonic-charge conservation. *Sov. Phys. JETP*, 26:984–988.

[Raidal et al., 2008] Raidal, M. et al. (2008). Flavour physics of leptons and dipole moments. *Eur. Phys. J.*, C57:13–182.

[Randall and Sundrum, 1999a] Randall, L. and Sundrum, R. (1999a). A large mass hierarchy from a small extra dimension. *Phys. Rev. Lett.*, 83:3370–3373.

[Randall and Sundrum, 1999b] Randall, L. and Sundrum, R. (1999b). An alternative to compactification. *Phys. Rev. Lett.*, 83:4690–4693.

[Riess et al., 1998] Riess, A. G. et al. (1998). Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *Astron. J.*, 116:1009–1038.

[Romao, 2005] Romao, J. C. (2005). The Minimal Supersymmetric Standard Model. *Private communication (unpublished).*

[Sakharov, 1967] Sakharov, A. D. (1967). Violation of CP Invariance, c Asymmetry, and Baryon Asymmetry of the Universe. *Pisma Zh. Eksp. Teor. Fiz.*, 5:32–35.

[Shaposhnikov, 1986] Shaposhnikov, M. E. (1986). Possible Appearance of the Baryon Asymmetry of the Universe in an Electroweak Theory. *JETP Lett.*, 44:465–468.

[Shaposhnikov, 1987] Shaposhnikov, M. E. (1987). Baryon Asymmetry of the Universe in Standard Electroweak Theory. *Nucl. Phys.*, B287:757–775.

[Sohnius, 1985] Sohnius, M. F. (1985). Introducing Supersymmetry. *Phys. Rept.*, 128:39–204.

[Spergel et al., 2007] Spergel, D. N. et al. (2007). Wilkinson Microwave Anisotropy Probe (WMAP) three year results: Implications for cosmology. *Astrophys. J. Suppl.*, 170:377.

[Starobinsky, 1980] Starobinsky, A. A. (1980). A new type of isotropic cosmological models without singularity. *Phys. Lett.*, B91:99–102.

['t Hooft, 1971] 't Hooft, G. (1971). Renormalisable Lagrangians for Massive Yang-Mills Fields. *Nucl. Phys.*, B35:167–188.

['t Hooft and Veltman, 1972] 't Hooft, G. and Veltman, M. J. G. (1972). Regularization and Renormalization of Gauge Fields. *Nucl. Phys.*, B44:189–213.

[Takayama and Yamaguchi, 2000] Takayama, F. and Yamaguchi, M. (2000). Gravitino dark matter without R-parity. *Phys. Lett.*, B485:388–392.

[Trodden and Carroll, 2004] Trodden, M. and Carroll, S. M. (2004). TASI lectures: Introduction to cosmology.

[Van Nieuwenhuizen, 1981] Van Nieuwenhuizen, P. (1981). Supergravity. *Phys. Rept.*, 68:189–398.

[Vissani and Smirnov, 1994] Vissani, F. and Smirnov, A. Y. (1994). Neutrino masses and $b - \tau$ unification in the supersymmetric standard model. *Phys. Lett.*, B341:173–180.

[Weinberg, 1967] Weinberg, S. (1967). A Model of Leptons. *Phys. Rev. Lett.*, 19:1264–1266.

[Wolfenstein, 1978] Wolfenstein, L. (1978). Neutrino oscillations in matter. *Phys. Rev.*, D17:2369–2374.

[Wolfenstein, 1979] Wolfenstein, L. (1979). Neutrino Oscillations and Stellar Collapse. *Phys. Rev.*, D20:2634–2635.

[Yanagida, 1979] Yanagida, T. (1979). Horizontal gauge symmetry and masses of neutrinos. In Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories, Tsukuba, Japan, 13-14 Feb 1979.

[Yang and Mills, 1954] Yang, C.-N. and Mills, R. L. (1954). Conservation of isotopic spin and isotopic gauge invariance. *Phys. Rev.*, 96:191–195.

[Zlatev et al., 1999] Zlatev, I., Wang, L.-M., and Steinhardt, P. J. (1999). Quintessence, Cosmic Coincidence, and the Cosmological Constant. *Phys. Rev. Lett.*, 82:896–899.

[Zwicky, 1933] Zwicky, F. (1933). Spectral displacement of extra galactic nebulae. *Helv. Phys. Acta*, 6:110–127.

Glossary

The purpose of including a glossary traces back to the large number of both abbreviations and acronyms appearing throughout this work. Thus, a list covering those words whose impact in this work is sizeable may help towards a fluent reading.

- **BBN**: Big Bang Nucleosynthesis, namely the generation of light nuclei in the early Universe as result of thermodynamic reactions.
- **CMBR**: Cosmic Microwave Background Radiation, namely the photons left over by the Big Bang, *i. e.*, which have been crossing the Universe freely ever since.
- **CMSSM**: Constrained Minimal Supersymmetric Standard Model, namely a scenario where, for the sake of simplicity, universal values are considered for the (vast number of) MSSM parameters.
- **GUT**: Grand Unified Theories, *i. e.*, a theoretical framework to embed all of fundamental interactions into a single Lie group.
- **LSP**: Lightest Supersymmetric Particle, namely the least massive field among the supersymmetric modes.
- **mSUGRA**: minimal SUperGRAvity, *i.e.*, a particular framework in SUGRA theories (local SUSY) where universal conditions are called for.
- **MSSM**: Minimal Supersymmetric Standard Model, *i. e.*, a model which augments the SM content by the minimal ingredients to render the model supersymmetric.
- **QFT**: Quantum Field Theory, a broad theoretical framework built upon the principles of (standard) quantum mechanics and (special) relativity. Its domain not only includes particle physics, but also condensed matter and statistical physics alike.
- **SC**: Standard Cosmology, the cosmological analogue to the SM of particle physics. It is the most widely accepted model for the origin and evolution of the Universe as a whole.
- **SM**: Standard Model of elementary particle physics, thought of as a low-energy effective theory of a more fundamental theory operating beyond ~ 100 GeV.
- **SUSY**: SUperSYmmetry, namely a symmetry relating bosonic modes to fermionic ones (in QFT matter is described by fermionic degrees of freedom and interactions by bosonic

ones). Supersymmetry may also be regarded as a sort of unification between matter and interactions.

- **VEV**: Vacuum Expectation Value, *i.e.*, the average value that a given operator is assigned to when computed in the vacuum.

- **WMAP**: Wilkinson Microwave Anisotropy Probe, a very sophisticated satellite intended to reveal cosmological physics to an extraordinary precision. Its recent scrutiny is revolutionising the field.