Quadrupole Phonons in the Cadmium Isotopes

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A key question concerning the spherical-vibrator attributes of states in Cadmium isotopes is addressed by means of a boson Hamiltonian encompassing U(5) partial dynamical symmetry. The U(5) symmetry is preserved in a segment of the spectrum and is broken in particular non-yrast states, and the resulting mixing with the intruder states is small. The vibrational character is thus maintained in the majority of low-lying normal states, as observed in ^{110}Cd.

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The concept of a phonon is indispensable to understand collective behavior in quantum-mechanical many-body systems. In particular in condensed matter, the description of lattice excitations requires the introduction of elementary modes of vibration that are identified with phonons. Phonons also play a central role in nuclear physics, notably in the interpretation of the collective motion of nucleons in an atomic nucleus. A standard model of the nucleus is in terms of a quantum liquid drop that exhibits vibrations around an equilibrium shape, which, if deformed, can also rotate. In their seminal studies Bohr and Mottelson [1–3] argued that the collective low-energy properties of nuclei are dominated by quadrupole vibrations, whose nature depends on the equilibrium shape. Small oscillations about spherical equilibrium can be described in terms of a single type of quadrupole phonon while the oscillations about a quadrupole-deformed equilibrium require the introduction of two different phonons that generate so-called β and γ vibrations.

This Rapid Communication deals with vibrations of spherical nuclei. The first observation to be made is that, despite more than half a century of research, the phonon interpretation of low-energy nuclear structure remains controversial, as exemplified by the Cadmium isotopes. The latter since long have been considered as archetypal examples of nuclei that exhibit small-amplitude vibrations around a spherical shape, to the extent that they have become textbook material to illustrate nuclear phonon behavior [3–7]. Evidence for near-harmonic vibrational properties of Cd isotopes was reported for up to three [8] and even up to six [9] quadrupole phonons. Nevertheless, it was also realized early on [10] that not all low-energy levels of these isotopes can be considered as vibrational and that additional levels exist at low excitation energies. The latter, named coexisting or intruder states [11], were claimed to arise because of proton excitations across the $Z = 50$ shell closure, character that was later confirmed in two-proton transfer reactions [12]. Over the years intruder bands were identified in many even-mass Cd isotopes [13] and, in parallel, models were extended to include such states. Extensive $E2$ decay patterns were established in several Cd isotopes and reproduced theoretically, albeit laboriously, by allowing mixing between vibrational and intruder states, see e.g. Refs. [14, 15]. However, as more data on the Cd isotopes were collected, the interpretation in terms of vibration–intruder mixing became increasingly untenable: decay properties of ^{112}Cd could not be explained [16], those of ^{114}Cd were found to be "enigmatic" [17], of ^{116}Cd to be "puzzling" [18]. The crisis culminated in papers claiming the "breakdown of vibrational motion" not only in the Cd [19] but also in the neighboring Pd and Sn isotopes [20]. This paradoxical behavior, characterized in [11] as an unsolved problem, continues to attract considerable attention [21–27].

In this Rapid Communication, we suggest that the vibrational interpretation of the Cd isotopes can be resurrected not, as attempted previously, by mixing vibrational and intruder states but by mixing particular phonon states. From a formal point of view, the latter mechanism represents a departure from U(5), which is the dynamical symmetry (DS) of spherical nuclei in the collective model [3] and the interacting boson model (IBM) [4], and generalizes it to a U(5) partial dynamical symmetry (PDS) [28].

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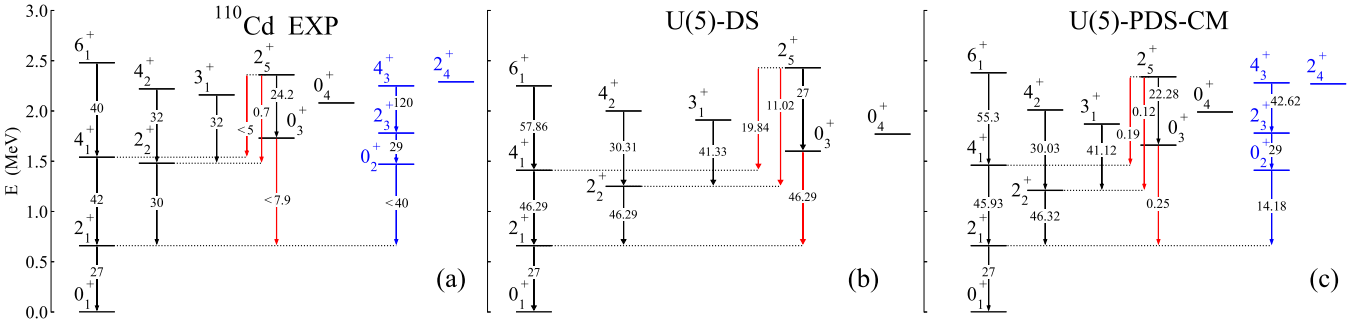

FIG. 1. (a) Experimental spectrum and representative $E2$ rates [21, 36] (in W.u.) of normal and intruder levels (0_2^+, 2_3^+, 4_3^+, 2_4^+) in ^{110}Cd. (b) Calculated U(5)-DS spectrum obtained from $\hat{H}_{\rm DS}$ (2) with parameters $t_1=715.75$, $t_2=-t_3=42.10$, $t_4=11.38$ keV and $N=7$. (c) Calculated U(5)-PDS-CM spectrum, obtained from $\hat{H}$ (11) with parameters $t_1=767.83$, $t_2=-t_3=73.62$, $t_4=18.47$, $r_0=2.15$, $e_0=-6.92$, $\kappa=-72.73$, $\Delta=9978.86$, $\alpha=-42.78$ keV and $N=7\,(9)$ in the normal (intruder) sector. For a complete listing of $B(E2)$ values and choice of $E2$ parameters, see Tables I-II.

A Hamiltonian with DS is written as a linear combination of Casimir operators of nested algebras, leading to complete solvability of its spectrum with exact quantum numbers for all eigenstates [4, 29]. This property, although very appealing, is rarely, if ever, satisfied in an existing quantum-mechanical system. However, more realistic Hamiltonians can be constructed, which satisfy the stringent DS conditions only partially. This leads to three different types of PDS: (i) some eigenstates retain all quantum numbers [30, 31], (ii) all eigenstates retain some quantum numbers [32, 33], and (iii) some eigenstates retain some quantum numbers [34].

In the following we apply a PDS of type (i) to explain the spectroscopic properties of ^{110}Cd. The starting point is the U(5) limit of the IBM, corresponding to the following chain of nested algebras [4, 35],

$$\mathrm{U}(6)\supset\mathrm{U}(5)\supset\mathrm{SO}(5)\supset\mathrm{SO}(3)\ . \qquad (1)$$

The basis states $|[N],n_d,\tau,n_\Delta,L\rangle$ have quantum numbers which are the labels of irreducible representations of the algebras in the chain. Here N is the total number of monopole (s) and quadrupole (d) bosons, n_d and τ are the d-boson number and seniority, respectively, and L is the angular momentum. The multiplicity label n_Δ counts the maximum number of d-boson triplets coupled to $L=0$. The U(5)-DS Hamiltonian has the form

$$\hat{H}_{\rm DS}=t_1\,\hat{n}_d+t_2\,\hat{n}_d^2+t_3\,\hat{C}_{\rm SO(5)}+t_4\,\hat{C}_{\rm SO(3)}\ , \qquad (2)$$

where $\hat{C}_{\rm G}$ denotes a Casimir operator of G, and $\hat{n}_d=\sum_m d_m^\dagger d_m=\hat{C}_{\rm U(5)}$. $\hat{H}_{\rm DS}$ is completely solvable for *any* choice of parameters t_i, with eigenstates $|[N],n_d,\tau,n_\Delta,L\rangle$ and eigen-energies

$$E_{\rm DS}=t_1\,n_d+t_2\,n_d^2+t_3\,\tau(\tau+3)+t_4\,L(L+1)\ . \qquad (3)$$

A typical U(5)-DS spectrum exhibits n_d-multiplets of a spherical vibrator, with a two-phonon ($n_d=2$) triplet of states ($L=4,2,0$) at an energy $E(n_d=2)\approx 2E(n_d=1)$ above the ground state ($n_d=L=0$) and first-excited state ($n_d=1,L=2$), and a three-phonon ($n_d=3$) quintuplet of states ($L=6,4,3,0,2$) at $E(n_d=3)\approx 3E(n_d=1)$. A quadrupole operator proportional to

$$\hat{Q}=d^\dagger s+s^\dagger\tilde{d}\ , \qquad (4)$$

enforces strong ($n_d+1\to n_d$) $E2$ transitions with particular ratios, *e.g.*, $\frac{B(E2;\,n_d=2,L=0,2,4\to n_d=1,L=2)}{B(E2;\,n_d=1,L=2\to n_d=0,L=0)}=2\frac{(N-1)}{N}$.

The empirical spectrum of ^{110}Cd, shown in Fig. 1(a), consists of both normal and intruder levels, the latter based on 2p-4h proton excitations across the $Z=50$ closed shell. Experimentally known $E2$ rates are listed in Tables I and II. A comparison of the calculated spectrum [Fig. (1b)] and $B(E2)$ values [Table I], obtained from $\hat{H}_{\rm DS}$ (2), demonstrates that most normal states have good spherical-vibrator properties, and conform well with the properties of U(5)-DS. However, the measured rates for $E2$ decays from the non-yrast states, 0_3^+ ($n_d=2$) and [0_4^+, 2_5^+ ($n_d=3$)], reveal marked deviations from this behavior. In particular, $B(E2;\,0_3^+\to 2_1^+)<7.9$, $B(E2;\,2_5^+\to 4_1^+)<5$, $B(E2;\,2_5^+\to 2_2^+)<0.7^{+0.5}_{-0.6}$ W.u., are extremely small compared to the U(5)-DS values: 46.29, 19.84, 11.02 W.u., respectively. Absolute $B(E2)$ values for transitions from the 0_4^+ state are not known, but its branching ratio to 2_2^+ is small.

Attempts to explain the above deviations in terms of mixing between the normal spherical [U(5)-like] states and intruder deformed [SO(6)-like] states have been shown to be unsatisfactory [19, 21]. The reasons are twofold. (i) An adequate description of the two-phonon 0_3^+ state requires strong (maximal $\sim 50\%$) normal-intruder mixing which, in turn, results in serious disagreements with the observed decay pattern of three-phonon yrast states. (ii) The discrepancy in the decays of the non-yrast two- and three-phonon states persists throughout the range $A=$ 110-126, including the heavier ^{A}Cd isotopes [22, 23], even though the energy of intruder states rises away from neutron mid-shell, and the mixing is reduced. These observations have led to the conclusion that the normal-intruder strong-mixing scenario needs to be rejected, and have raised serious questions on the valid-

TABLE I. Absolute (relative in square brackets) $B(E2)$ values in W.u. for $E2$ transitions from normal levels in ^{110}Cd. The experimental (EXP) values are taken from [21, 36]. The U(5)-DS values are obtained for an $E2$ operator $e_B\,\hat{Q}$, Eq. (4), with $e_B = 1.964$ (W.u.)$^{1/2}$. The U(5)-PDS-CM values are obtained using $\hat{T}(E2)$, Eq. (13), with $e_B^{(N)} = 1.956$ and $e_B^{(N+2)} = 1.195$ (W.u.)$^{1/2}$. In both calculations the boson effective charges were fixed by the empirical $2_1^+ \to 0_1^+$ rate. Intruder states $0_{2;i}^+$ $2_{3;i}^+$, $4_{3;i}^+$, $2_{4;i}^+$, are marked by a subscript i.

L_i	L_f	EXP	U(5)-DS	U(5)-PDS-CM
2_1^+	0_1^+	27.0 (8)	27.00	27.00
4_1^+	2_1^+	42 (9)	46.29	45.93
2_2^+	2_1^+	30 (5); 19 (4)[a]	46.29	46.32
	0_1^+	1.35 (20); 0.68 (14)[a]	0.00	0.00
0_3^+	2_2^+	< 1680[a]	0.00	55.95
	2_1^+	< 7.9[a]	46.29	0.25
6_1^+	4_1^+	40 (30); 62 (18)[a]	57.86	55.30
	4_2^+	< 5[a]	0.00	0.00
	$4_{3;i}^+$	14 (10); 36 (11)[a]		2.39
4_2^+	4_1^+	12^{+4}_{-6}; [a]$10.7^{+4.9}_{-4.8}$	27.55	27.45
	2_2^+	32^{+10}_{-14}; 22 (10)[a]	30.31	30.03
	2_1^+	$0.20^{+0.06}_{-0.09}$; 0.14 (6)[a]	0.00	0.00
	$2_{3;i}^+$	< 0.5[a]		0.005
3_1^+	4_1^+	$5.9^{+1.8}_{-4.6}$; [a]$2.4^{+0.9}_{-0.8}$	16.53	16.48
	2_2^+	32^{+8}_{-24}; 22.7 (69)[a]	41.33	41.12
	2_1^+	$1.1^{+0.3}_{-0.8}$; 0.85 (25)[a]	0.00	0.00
	$2_{3;i}^+$	< 5[a]		0.012
0_4^+	2_2^+	[< 0.65[a]]	57.86	1.24
	2_1^+	[0.010[a]]	0.00	31.76
	$2_{3;i}^+$	[100[a]]		16.32
2_5^+	0_3^+	24.2 (22)[a]	27.00	22.28
	4_1^+	<5[a]	19.84	0.19
	2_2^+	[a]$0.7^{+0.5}_{-0.6}$	11.02	0.12
	2_1^+	$2.8^{+0.6}_{-1.0}$	0.00	0.00
	$2_{3;i}^+$	< 5[a]		0.002
	$0_{2;i}^+$	< 1.9[a]		0.20

[a] From Ref. [21]

ity of the multi-phonon interpretation [19, 21]. In what follows, we consider a possible explanation for the "Cd problem", based on U(5)-PDS. The latter corresponds to a situation in which the U(5) dynamical symmetry [U(5)-DS] is obeyed by only a subset of states and is broken in other states. Similar PDS-based approaches have been implemented in nuclear spectroscopy, in conjunction with the SU(3)-DS [31, 37–39] and SO(6)-DS [33, 34, 40] chains of the IBM. Here we focus on U(5)-PDS associated with the chain (1).

The lowest spherical-vibrator levels comprise three classes of states,

$$\text{Class A}: \quad n_d = \tau = 0,1,2,3 \quad (n_\Delta = 0)\ , \tag{5a}$$

$$\text{Class B}: \quad n_d = \tau + 2 = 2,3 \quad (n_\Delta = 0)\ , \tag{5b}$$

$$\text{Class C}: \quad n_d = \tau = 3 \quad (n_\Delta = 1)\ . \tag{5c}$$

TABLE II. $B(E2)$ values (in W.u.) for $E2$ transitions from intruder levels in ^{110}Cd. Notation and relevant information on the observables shown, are as in Table I.

L_i	L_f	EXP	U(5)-PDS-CM
$0_{2;i}^+$	2_1^+	< 40[a]	14.18
$2_{3;i}^+$	$0_{2;i}^+$	29 (5)[a]	29.00
	0_1^+	$0.31^{+0.08}_{-0.12}$	0.08
	2_1^+	$0.7^{+0.3}_{-0.4}$	0.00
	2_2^+	< 8[a]	0.96
$2_{4;i}^+$	2_1^+	$0.019^{+0.020}_{-0.019}$	0.10
$4_{3;i}^+$	2_1^+	$0.22^{+0.09}_{-0.19}$	0.49
	2_2^+	$2.2^{+1.4}_{-2.2}$	0.00
	$2_{3;i}^+$	120^{+50}_{-110}	42.62
	4_1^+	$2.6^{+1.6}_{-2.6}$	0.00

[a] From Ref. [21]

In the U(5)-DS calculation of Fig 1(b), applicable to normal states only, the "problematic" states [0_3^+ ($n_d = 2$) and 2_5^+ ($n_d = 3$)] belong to class B, and 0_4^+ ($n_d = 3$) belongs to class C. The remaining "good" spherical-vibrator states [0_1^+ ($n_d = 0$); 2_1^+ ($n_d = 1$); $4_1^+, 2_2^+$ ($n_d = 2$); $6_1^+, 4_2^+, 3_1^+$ ($n_d = 3$)] belong to class A. As mentioned, the spherical-vibrator interpretation is valid for most normal states in Fig. 1(a), but not all. We are thus confronted with a situation in which some states in the spectrum (assigned to class A) obey the predictions of U(5)-DS, while other states (assigned to classes B and C) do not. These empirical findings signal the presence of a partial dynamical symmetry, U(5)-PDS.

The construction of Hamiltonians with U(5)-PDS follows the general algorithm [30, 40], by identifying operators which annihilate particular sets of U(5) basis states. In the present case, this leads to the following interaction:

$$\hat{V}_0 = r_0\, G_0^\dagger G_0 + e_0\, \left(G_0^\dagger K_0 + K_0^\dagger G_0 \right)\ , \tag{6}$$

where $G_0^\dagger = [(d^\dagger d^\dagger)^{(2)} d^\dagger]^{(0)}$, $K_0^\dagger = s^\dagger (d^\dagger d^\dagger)^{(0)}$ and standard notation of angular momentum coupling is used. $\hat{V}_0$ of Eq. (6) is in normal-ordered form and satisfies

$$\hat{V}_0 |[N], n_d = \tau, \tau, n_\Delta = 0, L\rangle = 0\ , \tag{7}$$

with $L = \tau, \tau+1, \ldots, 2\tau-2, 2\tau$, for any choice of parameters r_0 and e_0. Equation (7) follows from the fact that the indicated states have $n_d = \tau$ and $n_\Delta = 0$, hence do not contain a pair or a triplet of d-bosons coupled to $L = 0$ and, as such, are annihilated by K_0 [4] and G_0 [41].

The states of Eq. (7), which include those of class A, form a subset of U(5) basis states, hence remain solvable eigenstates of the Hamiltonian

$$\hat{H}_{\rm PDS} = \hat{H}_{\rm DS} + \hat{V}_0\ , \tag{8}$$

with good U(5) symmetry and energies given in Eq. (3) with $n_d = \tau$. It should be noted that while $\hat{H}_{\rm DS}$ (2) is diagonal in the U(5)-DS chain (1), the r_0-term (e_0-term) in $\hat{V}_0$ connects states with $\Delta n_d = 0$ and $\Delta\tau = 0, \pm 2, \pm 4, \pm 6$

($\Delta n_d = \pm 1$ and $\Delta\tau = \pm 1, \pm 3$). Accordingly, the remaining eigenstates of $\hat{H}_{\rm PDS}$ (8), in particular those of classes B and C, are mixed with respect to U(5) and SO(5). The U(5)-DS is therefore preserved in a subset of eigenstates, for any choice of parameters in $\hat{H}_{\rm PDS}$, but is broken in others. By definition, $\hat{H}_{\rm PDS}$ exhibits U(5)-PDS. Cubic terms of the type present in $\hat{V}_0$, Eq. (6), were previously encountered in IBM studies of triaxiality [42, 43], signature splitting [39, 44], band anharmonicity [40, 45], and shape-coexistence [46, 47] in deformed nuclei. Such higher-order terms show up naturally in microscopic-inspired IBM Hamiltonians derived by a mapping from self-consistent mean-field calculations [48, 49].

The effect of intruder levels can be studied in the framework of the interacting boson model with configuration mixing (IBM-CM) [50, 51]. The latter involves the space of normal states described by a system of N bosons representing valence nucleon pairs, and the space of intruder states described by a system of $N+2$ bosons, accounting for particle-hole shell-model excitations. This procedure has been used extensively in describing coexistence phenomena in nuclei [52–55]. In the present study of ^{110}Cd, the Hamiltonian in the normal sector is taken to be $\hat{H}_{\rm PDS}$ of Eq. (8), acting in a space of $N = 7$ bosons. The Hamiltonian in the intruder sector is taken to be of SO(6)-type [52],

$$\hat{H}_{\rm intrud} = \kappa \hat{Q} \cdot \hat{Q} + \Delta \ , \qquad (9)$$

acting in a space of $N = 9$ bosons, with $\hat{Q}$ given in Eq. (4). A mixing term between the $[N]$ and $[N+2]$ boson spaces is defined as [52–55],

$$\hat{V}_{\rm mix} = \alpha \left[(s^\dagger)^2 + (d^\dagger d^\dagger)^{(0)} \right] + {\rm H.c.} \ , \qquad (10)$$

where H.c. means Hermitian conjugate. The combined Hamiltonian for the two configurations has the form

$$\hat{H} = \hat{H}^{(N)}_{\rm PDS} + \hat{H}^{(N+2)}_{\rm intrud} + \hat{V}^{(N,N+2)}_{\rm mix} \ . \qquad (11)$$

Here $\hat{\mathcal{O}}^{(N)} = \hat{P}^\dagger_N \hat{\mathcal{O}} \hat{P}_N$ and $\hat{\mathcal{O}}^{(N,N')} = \hat{P}^\dagger_N \hat{\mathcal{O}} \hat{P}_{N'}$ for an operator $\hat{\mathcal{O}}$, with $\hat{P}_N$ a projection operator onto the $[N]$ boson space. In general, an eigenstate of $\hat{H}$,

$$|\Psi\rangle = a|\Psi^{(N)}_n\rangle + b|\Psi^{(N+2)}_i\rangle \ , \qquad (12)$$

involves a mixture of normal (n) and intruder (i) components with N and $N+2$ bosons, respectively. Similarly, the $E2$ operator is defined as

$$\hat{T}(E2) = e^{(N)}_B \, \hat{Q}^{(N)} + e^{(N+2)}_B \, \hat{Q}^{(N+2)} \ , \qquad (13)$$

with boson effective charges, $e^{(N)}_B$ and $e^{(N+2)}_B$.

The Hamiltonian of Eq. (11) has nine parameters. However, most of them only improve the fit to energies, but do not affect the structure of the states nor the calculated $E2$ rates, which are the challenge in the Cd problem. The 6 parameters (t_i, r_0, e_0) of $\hat{H}_{\rm PDS}$ (8) do not affect the U(5) purity of class-A states and, for small α, the deviations from U(5)-DS in a few non-yrast states, is governed solely by the r_0 and e_0 terms. The spectrum and $B(E2)$ values obtained with $\hat{H}$ (11) and $\hat{T}(E2)$ (13), are shown in Fig. 1(c) and Tables I-II. As seen, the IBM-PDS-CM calculation provides a good description of the empirical data in ^{110}Cd. The normal states of class A retain good U(5) symmetry and quantum numbers, to a good approximation. Their $|\Psi^{(N)}_n\rangle$ part involves a single component with n_d value as in Eq. (5a). The mixing with the intruder states is weak [small b^2 in Eq. (12)] and increases with L. Specifically, $b^2 = 0.9,\ 2.2,\ 3.6,\ 5.9,\ 4.6,\ 6.1\,\%$ for the 0^+_1, 2^+_1, 2^+_2, 4^+_1, 3^+_1, 4^+_2 states, respectively. The 6^+_1 state experiences a somewhat larger mixing ($b^2 = 17.3\%$), consistent with its enhanced decay to the intruder $4^+_{3;i}$ state. The high degree of purity is reflected in the calculated $B(E2)$ values for transitions between class A states which, as seen in Table I, are very similar to those of U(5)-DS. In contrast, the structure of the non-yrast states assigned originally to classes B and C, whose decay properties show marked deviations from the U(5)-DS limit, changes dramatically. Specifically, the 0^+_3 and 0^+_4 states, which in the U(5)-DS classification are members of the two-phonon triplet and three-phonon quintuplet, interchange their character, and the U(5) decomposition of their $|\Psi^{(N)}_n\rangle$ parts peaks at $n_d = 3$ and $n_d = 2$, respectively. Similarly, the 2^+_5 and 2^+_6 states, which in the U(5)-DS classification are members of the three-phonon quintuplet and four-phonon octet, interchange their character, and the U(5) decomposition of their $|\Psi^{(N)}_n\rangle$ parts peaks at $n_d = 4$ and $n_d = 3$, respectively. The mixing with the intruder states is weak ($b^2 = 5.1\%,\ 2.9\%,\ 4.4\%$) for the ($0^+_3$, 2^+_5, 2^+_6) states, and somewhat larger ($b^2 = 18\%$) for the 0^+_4 state. The resulting calculated values: $B(E2; 0^+_3 \to 2^+_1) = 0.25$, $B(E2; 2^+_5 \to 4^+_1) = 0.19$ and $B(E2; 2^+_5 \to 2^+_2) = 0.12$ W.u., are consistent with the measured upper limits: 7.9, 5 and $0.7^{+0.5}_{-0.6}$ W.u., respectively. The calculated decay $0^+_4 \to 2^+_2$ is weaker than $0^+_4 \to 2^+_1$, however, the observed dominant branching to the intruder $2^+_{3;i}$ state is not reproduced. This may indicate a different structure for the 0^+_4 state (*e.g.*, a 4p-6h proton excitation as speculated in [21]). The dominant $E2$ decays of the 2^+_6 state (not shown in Fig. 1) are predicted to be $B(E2; 2^+_6 \to 0^+_4) = 24.28$, $B(E2; 2^+_6 \to 4^+_1) = 15.73$, $B(E2; 2^+_6 \to 2^+_2) = 9.27$ and $B(E2; 2^+_6 \to 4^+_{3;i}) = 3.56$ W.u.

A few monopole transition rates are experimentally known in ^{110}Cd [56], expressed in terms of the quantity $\rho(E0) = \frac{\langle f|\hat{T}(E0)|i\rangle}{eR^2}$, with $R = 1.2A^{1/3}$ fm. The corresponding $E0$ operator in the IBM-CM, can be transcribed [57] in the form $\hat{T}(E0) = (e_n N' + e_p Z)\eta\,(\hat{n}^{(N)}_d + \hat{n}^{(N+2)}_d)$, where N' (Z) are neutron (proton) numbers and $e_p = 2e_n = e$. The measured [56] and calculated (in curly brackets) strengths are given by $\rho^2(E0; 0^+_{2;i} \to 0^+_1) \cdot 10^3 < 31\,\{0.75\}$, $\rho^2(E0; 2^+_{3;i} \to 2^+_1) \cdot 10^3 = 9(8)\,\{10\}$, $\rho^2(E0; 4^+_{3;i} \to 4^+_1) \cdot 10^3 = 106^{+98}_{-91}\,\{36\}$. The calculated

strengths, obtained with $\eta = 0.063$, reproduce the measured values, within the quoted error bars. The calculation predicts $\rho^2(E0)\cdot 10^3 \sim 10$ for $0^+_4 \to 0^+_{2;i}$ and $2^+_{4;i} \to 2^+_2$ $E0$ transitions, which have not been measured so far.

In summary, we have addressed a key question concerning the phonon structure of states in Cd isotopes. Our results suggest that the vibrational interpretation of ^{110}Cd can be salvaged by introducing a boson Hamiltonian that mixes particular phonon states while keeping the mixing with coexisting intruder levels weak. The proposed scheme relies on a partial dynamical U(5) symmetry, in which most low-lying normal states in ^{110}Cd maintain their spherical-vibrational character and only a few specific non-yrast states exhibit a departure from U(5), in line with the empirical data. Work currently in progress appears to indicate that the same PDS-based approach can be implemented also in other neutron-rich Cd isotopes, at least as an appropriate starting point for further refinements.

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