

S&P 500 Index Direction Forecasting from 1976 to 2010: A Fuzzy System Approach

Cesar Duarte Souto-Maior Federal University of Santa Catarina. Brazil. cesaredm@yahoo.com.br

José Alonso Borba. Federal University of Santa Catarina. Brazil. jalonso@cse.ufsc.br

Newton C. A. da Costa Jr. Federal University of Santa Catarina. Brazil. newton@cse.ufsc.br

Abstract. Investors are not always completely rational and they do not always work only with numbers. Sometimes, they use linguistic concepts to make their decisions. Fuzzy logic is helpful in handling such situations. This article differs from other studies in this area by applying fuzzy logic for forecasting the direction/sign of stock market indexes. The article also presents a model for forecasting the direction/sign of stock market indexes using a fuzzy system, and this model is applied in the prediction of the S&P 500 Index. The rules of the model were established between 1970 and 2009, and the test period was from 1976 to 2010. Despite the fact that the estimated model produced a linguistic output, it was possible to delineate a statistically significant investment strategy, which outperformed a buy-and-hold one. In addition, the success rate calculated was also statistically significant. Another difference from other studies is that the proposed model does not return an exact output, but a probabilistic one of linguistic variables. The proposed model, with its probabilistic output, can be used as a support for investment decisions. In other words, the linguistic output does not force the investor to blindly follow the proposed strategy.

Keywords: Fuzzy logic; Stock market index; Forecasting; S&P 500.

1. INTRODUCTION

The rise of efficient market hypotheses brings popularity to stock market indexes trading. Beating the market is difficult, so a stock index investment seems to be a good alternative. For Chen, Leoung, & Daouk (2003), an index provides an effective way of reducing risk. According to Armano, Marchesi, & Murru (2005), this reduction occurs because the impact of bad news from a unique corporation is minimized. In addition, stock indexes provide a reference to investors about the profitability of a particular market.

Since the 1980s, the literature related to financial time series has produced important studies that have questioned the (weak) efficient market and the

random-walk hypotheses (e.g., see Lo & Mackinlay, 1988; Poterba & Summers, 1988; Fama & French, 1988). These authors claim that considerable evidence exists that stock returns are, to some extent, predictable. They show strong evidence of conditional heteroskedasticity in many financial time series, meaning that these time series returns are not independent and are identically distributed as established by the random-walk model. According to Yeh, Huang, & Lee (2011), the stock time series are inherently noisy and non-stationary. Chun, Kim, & Kim (2002) argue that financial markets can be predictable, but they exhibit nonlinear dynamics with a chaotic behavior.

These facts have provoked theoretical and practical interest in the nonlinear financial time series models based on techniques such as auto-regressive moving-average (ARMA) family, generalized auto-regressive conditional heteroskedasticity (GARCH) family, and, more recently, methods based on artificial intelligence such as artificial neural networks (ANN), genetic algorithms (GA), and fuzzy logic (FL).

Predictability of financial time series can be viewed in two different ways: forecasting by level (value) estimation models and forecasting by classification models. The first method relies on accurate prediction of the price level of stocks, indices, and other financial series instruments. (e.g., see Teixeira & Rodrigues, 1997; Chakraborty, 2006; Dutta *et al.*, 2006; Panda & Narasimhan, 2006; Sohn & Lim, 2007; Majhi *et al.*, 2009; Mostafa, 2009; Chen, Hsin & Wu, 2010, Shen *et al.*, 2011). The degree of accuracy and the acceptability of the forecast are measured by its deviation from the actual observations, thus minimizing forecasting errors. The second method is guided by forecasts on the direction/sign of the changes in price levels. This latter approach is defended by some authors (e.g., see Wu & Zhang, 1997; Aggarwal & Demaskey, 1997; Tsaih, Hsu & Lai, 1998; Leung, Daouk & Chen, 2000; Chen, Leung & Daouk, 2003; Kim, Min & Han, 2006; Faria *et al.*, 2009; Lu, 2010; Na & Sohn, 2011), who claim that trading driven by a certain forecast with a small forecast error may not be as profitable as trading based on an accurate prediction of the direction/sign of the movement.

At the same time, several researches in the area of business have used fuzzy logic to handle imprecise information and improved results. Some of them are Shehab & Abdalla (2002) and Murcia, Borba, & Souto-Maior (2005) in a costing system; Sahin & Dogan (2003) in supplies and customers relationship; Jiang & Hsu (2003) in manufacturing and business cycle evaluations; Lin, Hwang, &

Becker (2003) in accounting frauds detection; Beynon, Peel, & Tang (2004) in price composition of auditing services; Wang & Hsu (2008) in tourism demand; Keles, Kolcak, & Keles (2008) in domestic debt; Wang, Chang, & Tzeng (2011) and Wu (2010, 2011) in automobile sales; and Cheng, Tsai, & Sudjono (2010) in project cash flow control.

In the area of finance, works related to stock selection (such as Tanaka & Guo, 1999; Inuiguchi & Tanino, 2000; León, Liern & Vercher, 2002; Wang & Zhu, 2002; Serguieva & Hunter, 2004; Tiryaki & Ahlatcioglu, 2009), option pricing (Thavaneswaran, Appadoo & Paseka, 2009), and insolvency prediction (Tseng & Lin, 2005; Vigier & Terceño, 2008; Chen, Huang & Lin, 2009) have also used fuzzy logic.

Works that use fuzzy logic to select and forecast specific stocks do not take advantage of the benefits from stock market index trading. According to the CAPM model, great performance could be generated by choosing stocks with high beta values and with bigger risks as well. Some of them are Simutis (2000) with an expert system; Dourra & Siv (2002) with technical indicators; and Atsalakis & Valavanis (2009) and Hadavandi, Shavandi, & Ghandari (2010) with neural networks.

In stock index prediction, some works deal with level forecasting using fuzzy logic (Huarng, 2001a, 2001b; Yu, 2005a, 2005b; Huarng & Yu, 2005, 2006; Chen, Cheng & Teoh, 2007, 2008; Chu *et al.*, 2009) and hybrid models with rough sets (Teoh *et al.*, 2008, 2009), Markov chain concept (Wang, Cheng & Hsu, 2010), or neural networks (Cheng, Wei & Chen, 2009; Yu & Huarng, 2008, 2010; Boyacioglu & Avci, 2010), but none of them deals with forecasts on the direction/sign of the changes in price levels. In addition, according to Leung, Daouk, & Chen (2000), forecasting strategies based on levels seem to be less profitable than forecasting strategies based on the change of direction/sign of a stock index.

This article differs from other studies in this area by applying fuzzy logic for forecasting the direction of the change in stock market indices level. Another difference is that the proposed model does not return an exact output, but a probabilistic one of linguistic variables. This linguistic output can be used with other economic and non-economic information, including intuition, to help in making investment decisions. Based on the output model, an investment strategy could be created and compared with a passive one.

The article is organized as follows. In Section 2, fuzzy logic and its concepts are briefly described. Section 3 presents the methodology used. Empirical results are presented in Section 4. Section 5 is a conclusion of the work.

2. FUZZY LOGIC

According to binary logic, which was initially formulated by the Greek philosopher Aristotle (384–322 A.C.), a proposition is either true or false. This type of logic assumes that the states of nature are well-defined events. However, in most contexts such as the accounting and business areas, the states of nature are vague, and transitions between ‘what is’ and ‘what is not’ are not very well defined.

Zadeh (1965) published the first paper about fuzzy logic called Fuzzy Sets. The model was developed to convert subjective values into objective ones. A fuzzy set does not have precise and limited boundaries; the difference between belonging and not belonging does not exist, only a degree of pertinence exists. According to Zebda (1998), the fuzzy sets theory is not a decision theory but rather a calculus (a modeling language) wherein vague humanistic events can be treated in a systematic manner.

The main objective of fuzzy logic is to provide concepts that perform approximate reasoning. Fuzzy logic assumes a degree of pertinence in the 0 to 1 range, which permits the fuzzy set element to be partially true or partially false.

Bojadziev & Bojadziev (1997, p. 9) define a fuzzy set as

$$A' = \{(x, \mu_A(x)) / x \in A, \mu_A(x) \in [0,1]\},$$

where $\mu_A(x)$ is called function of pertinence and specifies the grade in which each element x in A belongs to the fuzzy set A' .

According to von Altrock (1997), the theory of fuzzy sets is a generalization that covers the classical sets where $\mu_A(x) = 0$ or $\mu_A(x) = 1$. In other words, the classical sets are special cases of the fuzzy sets. Table 1 and Figure 1 show the differences between classical sets and fuzzy sets.

Classical sets – Binary logic	Fuzzy sets – Fuzzy logic
Precise boundaries	Imprecise boundaries
Rough transition between belonging and not belonging	Smooth transition between belonging and not belonging
Represents well-defined concepts	Represents imprecise and vague concepts

Table 1. Classical sets versus fuzzy sets.

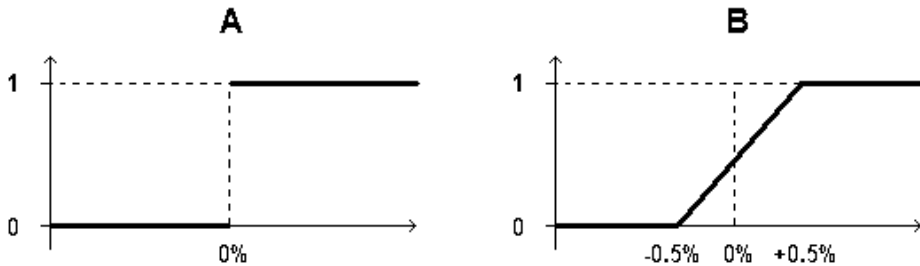


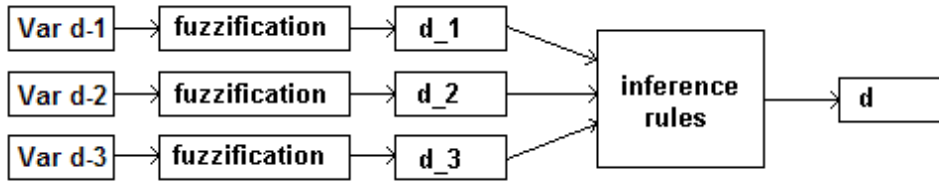
Figure 1. Classical sets versus fuzzy sets

Figure 1, part A, shows an example of classical logic. If the index variation is positive, the variation is considered “up” and if it is negative, the variation is considered “down”. There is a rough transition between “up” and “down”. Infinitesimal negative values are classified as “down”, and infinitesimal positive values are classified as “up”. However, the market could classify both values as the same information, and a system based on classical logic could not present a good performance. Figure 1, part B, shows how fuzzy logic can be used to delineate a smooth transition between “up” and “down” variations.

3. METHODOLOGY

3.1 The model

Figure 2 illustrates the proposed model. It is divided into three main parts: the fuzzification of input variables, inference rules, and output variables. Here, fuzzification (defuzzification) implies the conversion of a numeric (linguistic) value into a linguistic (numeric) one.



Note:

Var d-1, Var d-2, and Var d-3 are the percentage variations (in numeric values) of the index on days d, d-1, d-2, and d-3, respectively. d_1, d_2, and d_3 are the percentage variations (in linguistic values) of the index on days d, d-1, d-2, and d-3, respectively. d is the linguistic output of the model.

Figure 2. General view of a fuzzy model for stock market index forecasting

The conceptual model is divided into four main parts: (1) choice of variables for the forecasting model; (2) fuzzification; (3) inference rules; and (4) output variables. Each of these parts is described next.

3.1.1 Choice of variables of the forecasting model

In the work by Kamitsuji & Shibata (2003), the percentage variations of the index on the three days before the day to be forecasted were chosen as input variables. Kamitsuji & Shibata (2003) also tested their model for four and five daily variations, but these additional data produced worse results. These authors argue that these additional data cause an over-fitting in neural network training; in other words, the system becomes very specific and loses generalization to the subsequent periods.

Percentage variations of the index on the three days before the day to be forecasted were chosen as input variables. This choice is based on O'Connor, Remus and Griggs (1997). They show that individuals present different tendencies and behavior for up and down series. In this method, an investor could make decisions based on recent information. As an example, if a stock price increases during many consecutive days, it could provoke a selling behavior that would drive the stock price down, despite a positive macroeconomic scenario. On the other hand, if the stock price decreases during many consecutive days, it could provoke a buying behavior and a tendency to increase the stock price, despite a negative macroeconomic scenario.

In order to calculate the percentage variation on day d, the following formula was used:

$$Var_d = \frac{(V_d - V_{d-1})}{V_{d-1}}$$

where Var_d is the percentage variation (in numeric values) of the daily variation or return of the index on day d ; V_d and V_{d-1} are the closing index values on days d and $d-1$, respectively.

The same logic will be applied to calculate the percentage variation of daily variation of the index on days $d-1$, $d-2$, and $d-3$.

3.1.2 Fuzzyfication

An investor does not always think mathematically with numbers. Sometimes an investor rationalizes with linguistic terms such as “up”, “down”, “expensive” or “cheap”. To use fuzzy logic, all numerical input variables should be converted to linguistic variables. In this work, the input linguistic variables adopted were “up” and “down”.

To accomplish this task, pertinence functions were developed. Figure 3 shows a pertinence function for the linguistic variable d_1 , related to the numeric variation of day $d-1$, the previous day.

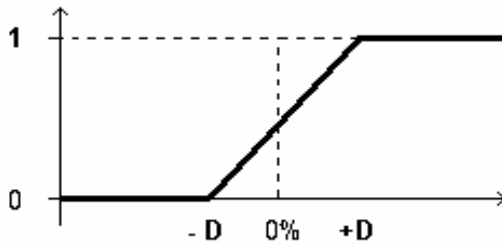


Figure 3. Graphic depiction of pertinence function “up” to d_1 linguistic variable

This pertinence function can be described in the following manner:

$$\mu_S = \begin{cases} 0 & \text{if } Var_{d-1} \leq -D \\ 1/2 + Var_{d-1}/2D & \text{if } -D < Var_{d-1} < +D \\ 1 & \text{if } Var_{d-1} \geq +D \end{cases}$$

where μ_S is the pertinence function “up”, and Var_{d-1} is the percentage variation (in numeric value) of day $d-1$.

In other words, beyond a certain value $+D$, the index variation is “up”, below D is not “up”, and between $-D$ and $+D$ is “partially up”. The pertinence function “down” is very similar.

The choice of D value is important. Beyond what would D value investors classify a percentage variation as “up”? Souto-Maior (2007) tests several values of D . Small and big values are not good choices. As a rule-of-thumb, D could be chosen as the fifth of daily percentage variation standard deviation. In the initial period from 1970 to 1974 (five years) the daily percentage variation standard deviation was around 1%. Therefore, in this work, we use 0.2% as the D value. Finally, we test other D values to verify the effectiveness of this rule-of-thumb.

3.1.3 Inference Rules

After the fuzzification of all input values, the step that follows involves the establishment of inference rules. These rules represent the way humans make decisions, inferring from linguistic premises. These rules are logical statements, and each rule can be assigned a value from zero to one, called Degree of Support (DoS), which depends on the characteristics of the sample. When a rule is assigned a DoS value equal to zero (one), the rule is considered insignificant (significant). The DoS also allows for values between zero and one for partially significant rules. Given next is an example of the rules used in our model.

If ($d_3 = \text{“up”}$ and $d_2 = \text{“up”}$ and $d_1 = \text{“up”}$),

Then $d = \text{“down”}$, with a DoS of 0.53.

To create the rules used in the model, the period from 1970 to 1974 (five years) was used as the estimation period for forecasting year 1975 and so on. Therefore, the period from 1970 to 2009 was used as the estimation period, and the period from 1976 to 2010 (35 years) was used as the forecasting or test period. In this work, 840 rules were created.

The training period comprises five years and provides sets with the input variables (d_1 , d_2 , and d_3) and the variation d . To create the rules, we identified the sets where each input variable (d_1 , d_2 , and d_3) is 100% down or 100% up. These sets were called characteristics sets.

In characteristics sets, each input variable can be down or up. So, corresponding to the sequence d_1 , d_2 , and d_3 , one can find eight different types of characteristics sets: (down, down, down), (down, down, up), (down, up,

down), (down, up, up), (up, down, down), (up, down, up), (up, up, down), and (up, up, up).

The variation d was analyzed for each type of sequence to find out patterns of behaviors and create the rules. For example, analyzing the sequence (up, up, up) and the corresponding variation d in the period from 1995 to 1999, there are 53% chances of down, 11% chances of flat, and 36% chances of up behaviors. With this information, inference rules were created to use in the year 2000.

The inference rules can reflect the behavior of traders in a specific market. If correct, a strategy based on these rules could help investors forecast the direction sign of the change.

3.1.4 Output Variables

The output of the model is denominated d , which represents the variation of the index on the day of the forecast. The outputs of the linguistic values adopted were down, flat, and up. In this work, defuzzification was not necessary. The linguistic values were used as output variables, corresponding to the probability of the index showing a down, flat, or up behavior.

The output variable d can be represented as a vector of dimension 3×1 , as shown next:

$$d = \begin{bmatrix} d1 \\ d2 \\ d3 \end{bmatrix},$$

where $d1$ is the linguistic component “down” of output d , $d2$ is the linguistic component “flat” of output d , and $d3$ is the linguistic component “up” of output d .

3.2 Metrics

In order to evaluate the results obtained from the proposed model, two different metrics were used: (1) the comparison of an investment strategy based on the fuzzy model with a passive strategy, and (2) a success ratio. These metrics are described next.

3.2.1 Investment Strategy

In general, it is a standard practice to test the created model through a real application via an investment strategy (e.g., see Brooks, Rew & Ritson, 2001; Kuo, Chen & Hwang, 2001; Haefke & Helmestein, 2002; Kim & Han, 2001; Leung, Daouk & Chen, 2000; Chen, Leung & Daouk, 2003; Leigh, Purvis & Ragusa, 2002; Leigh, Modani & Hightower, 2004; Leigh, Hightower & Mondani, 2005; Armano, Marchesi & Murru; 2005).

In this study, after the model estimation had been completed, a buying-and-selling strategy was applied to the S&P 500 index. This strategy can be viewed as a market timing strategy where one can switch between asset classes (from stock market index to risk-free cash and vice versa) in anticipation of the turning points in the market.

The two indices served as surrogates for an investment fund in risky assets. On the days on which the linguistic component “down” is greater than a variable called ε , the money is withdrawn from the fund, and applied again the next day. For example, if $\varepsilon = .5$, it means that the money will be withdrawn from the fund when the “down” possibility, indicated by the fuzzy model, is greater than 50%. Implicit in this strategy is the supposition that the fund is able to buy and sell stocks in the same proportion as the stocks in the market index.

Concisely, the algorithm used was

If $(d1_i > \varepsilon)$,

Then $\alpha_i = 0$.

Else $\alpha_i = 1$,

where $d1_i$ is the linguistic component “down” for day i , $\alpha_i = 1$ means that the money should be applied in the fund on day i , and $\alpha_i = 0$ means that the money should be withdrawn from the fund on day i .

The value of variable ε is important. For one year, $\varepsilon = .5$ could be the value of ε that maximizes the return. However, that value of ε could not be the most profitable to other years. The investor wants to know what value of ε will be adopted before trading. In other words, the ε should be defined ex-ante. In this study, the value of ε that maximizes the return in a year should be used the next year. For example, the value $\varepsilon = .52$ maximizes the return in the year 1975, so this value of ε should be used in the year 1976. The year 1975 was the first year with inference rules (estimated from the 1970–1974 period) and has no value of ε to be

adopted. So, this year is not used to test the investment strategy. The value of ϵ that maximizes the return in the year 1975 will be used in the strategy by the year 1976 and so on.

The fuzzy strategy should be compared with a passive one in order to check its usefulness. The following formula was used to calculate the buy-and-hold (passive) strategy returns:

$$\prod_{i=1}^n (1 + Var_{di}/100)$$

where Var_d is the percentage variation (in numeric value) of the daily variation or return of the index in day di . Actually, the value depicted in the equation just cited is the cumulative return from the index during the test period (with n trading days).

The return with the fuzzy-based strategy was calculated by the following formula:

$$\prod_{i=1}^n [\alpha_i \times (1 + Var_{di}/100) + (1 - \alpha_i)]$$

For the simulation, the premise of a “frictionless market” was adopted. In other words, all transaction costs were considered nonexistent. This is not a problem, because when our investment strategy is compared with the returns of the stock market index, the costs related to the quarterly rebalancing of these indices were not taken into account. In addition, the return that could be achieved by investing in the open market on days when the money is withdrawn from the fund was not taken into account.

3.2.2 Success Ratio and Chi-Square test

Besides testing a forecasting model through an investment strategy, it is also possible to test the “success ratio” of the model. Several authors used this approach (e.g., see Brooks, Rew & Ritson, 2001; Kuo, Chen & Hwang, 2001; Pérez-Rodrigues, Torra & Andrada-Félix, 2005; Chun & Park, 2004; Leung, Daouk & Chen, 2000; Chen, Leung & Daouk, 2003; Kim, 2004; Kim, Min & Han, 2006; Armano, Marchesi & Murru, 2005; Panda & Narasimhan, 2006).

The success ratio was used to analyze the model’s efficiency. This ratio compares the number of correct predictions with the total predictions. The statistical test used to check this efficiency was the chi-square test. It verified

whether the set of days with “down” predictions presented a distribution of “down” and “up” outputs and was statistically different from the set of days with no “down” predictions.

3.3 Choice of D

At first, the strategy was developed with $D = 0.2\%$. New rules were generated for a series of values of D (0.0% ; 0.1% ; 0.3% ; 0.4%). Based on these rules, we can calculate the cumulative returns associated with each D value and verify whether 0.2% is a good choice.

In addition, we can verify whether fuzzy logic use brings better results compared with classical logic. The performance could have originated from rules and not by fuzzy logic use. When $D = 0\%$, we are using classical logic. The fuzzy logic strategy (with $D = 0.2\%$) should be compared with a classical logic strategy ($D = 0\%$) in order to check its usefulness.

4. RESULTS

The strategy was simulated in the period from 1976 to 2010. Table 2 presents these results using formulas from section 3.2.1.

The buy-and-hold strategy shows an average annual return of 7.82% for the test period. Table 2 shows that the fuzzy-based strategy presents an average annual return of 13.04% in the same period. In the case of absolute values, the fuzzy model outperforms the passive strategy by 5.22% a year, with a one-tailed t statistics of 2.23 , significant at the 5% level.

During the test period, the fuzzy-based strategy outperforms the passive strategy in 19 years, underperforms in 7 years, and has the same return in 9 years. The strategy is particularly profitable in the years of crises, when S&P 500 exhibits a negative return.

Figure 4 shows the difference between passive and fuzzy-based strategy returns during the period from 1976 and 2010.

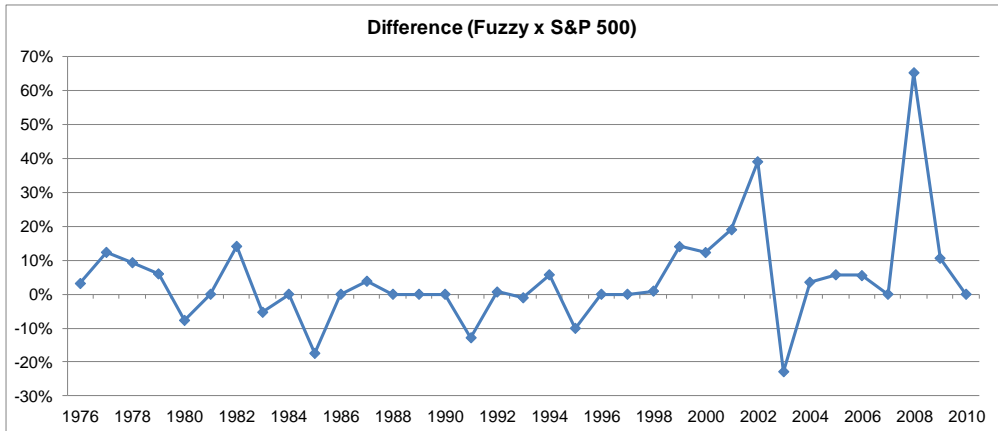
Year	(a) Fuzzy	(b) S&P 500	Differential return (a-b)
1976	22.31%	19.15%	3.16%
1977	0.82%	-11.50%	12.32%
1978	10.34%	1.06%	9.28%
1979	18.30%	12.31%	5.99%
1980	18.08%	25.77%	-7.70%
1981	-9.73%	-9.73%	0.00%
1982	28.85%	14.76%	14.09%
1983	11.97%	17.27%	-5.31%
1984	1.41%	1.40%	0.01%
1985	8.94%	26.33%	-17.40%
1986	14.62%	14.62%	0.00%
1987	5.86%	2.03%	3.83%
1988	12.40%	12.40%	0.00%
1989	27.25%	27.25%	0.00%
1990	-6.56%	-6.56%	0.00%
1991	13.49%	26.31%	-12.81%
1992	5.12%	4.46%	0.66%
1993	6.01%	7.06%	-1.05%
1994	4.19%	-1.54%	5.72%
1995	24.08%	34.11%	-10.03%
1996	20.26%	20.26%	0.00%
1997	31.01%	31.01%	0.00%
1998	27.59%	26.67%	0.92%
1999	33.54%	19.53%	14.02%
2000	2.14%	-10.14%	12.28%
2001	5.93%	-13.04%	18.98%
2002	15.63%	-23.37%	38.99%
2003	3.58%	26.38%	-22.80%
2004	12.52%	8.99%	3.53%
2005	8.72%	3.00%	5.72%
2006	19.09%	13.62%	5.47%
2007	3.53%	3.53%	0.00%
2008	26.70%	-38.49%	65.18%
2009	34.05%	23.45%	10.59%
2010	12.78%	12.78%	0.00%
Mean	13.04%	7.82%	5.22% (*)

Notes:

(*) means statistically significant at 5%, one-tailed t-test;

S&P 500 is the proxy for the buy-and-hold (passive) strategy.

Table 2. Returns for the fuzzy and passive strategies, from 1976 to 2010.



Notes:

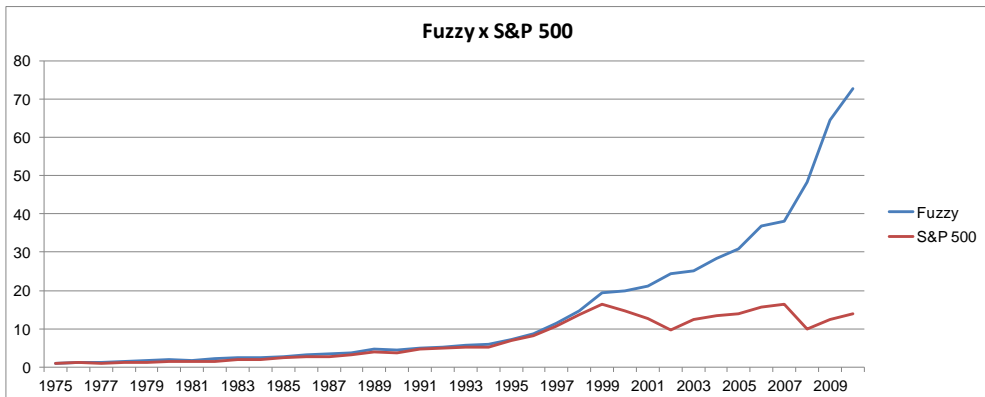
Horizontal axis represents the years of the test period;

Vertical axis represents the difference between fuzzy strategy and S&P 500 in each year.

Figure 4. Difference between passive and fuzzy-based strategy from 1976 and 2010.

Figure 5 shows the passive (S&P 500) and fuzzy strategy returns of one monetary unit invested at the beginning of the test period (January 1, 1976).

At the end of the test period (December 31, 2010), the passive S&P 500 would return 13.94 monetary units, and the fuzzy-based strategy would return 72.87 monetary units.



Notes:

Horizontal axis represents the years of the test period;

Vertical axis represents the value of one monetary unit invested at the beginning of the test period.

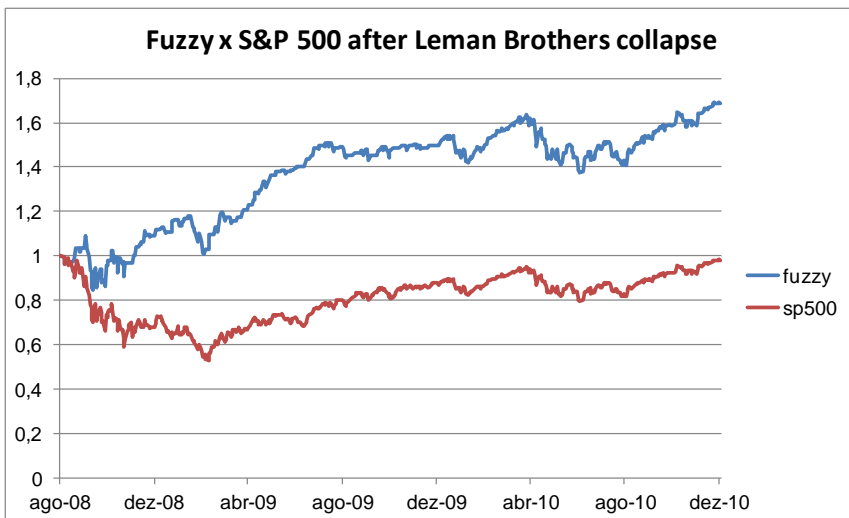
Figure 5. Cumulative returns of the passive (S&P 500) and fuzzy-based strategy.

In addition, the success rate on the days of money withdrawal was calculated. A success ratio of 50.81% was achieved. The money would be withdrawn in 2647 days, and the index would fall on 1345 of these days. The chi-square ($\chi^2 = 20.12$) was statistically significant at the 1% level.

4.1 Strategy after Lemman Brothers Failure

Figure 6 presents a graphic depiction with the cumulative returns of one monetary unit invested using a fuzzy and passive strategy on August 28, 2008, a few days before the Lemman Brothers collapse. At the end of the test period (December 31, 2010), the passive S&P 500 would return 0.98 monetary units, and the fuzzy-based strategy would return 1.69 monetary units.

The fuzzy strategy outperforms the passive one, with a one-tailed t statistics of 1.77, significant at the 5% level.



Notes:

Horizontal axis represents the days of the test period;

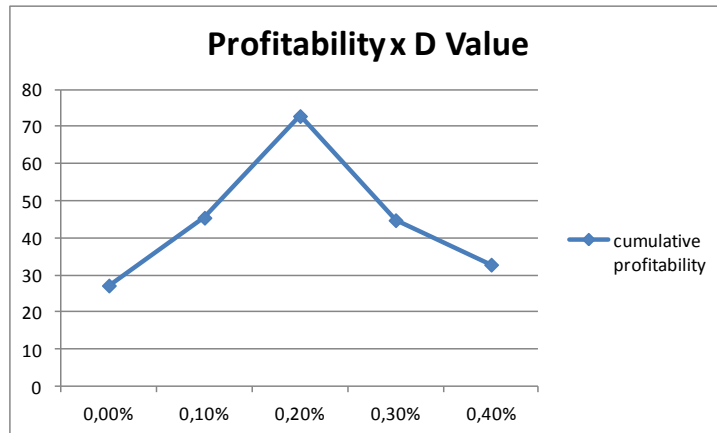
Vertical axis represents the value of one monetary unit invested at the beginning of the test period.

Figure 6. Cumulative returns of the fuzzy-based strategy and S&P 500 after Lemman Brothers collapse

4.2 Choice of D

At first, the strategy was developed with $D = 0.2\%$. New rules were generated for a series of values of D (0.0%; 0.1%; 0.3%; 0.4%).

Figure 7 presents a graphic depiction with the cumulative returns (of one monetary unit invested at the beginning of the test period) for each D value.



Notes:

Horizontal axis represents the D values;

Vertical axis represents the final value of one monetary unit invested at the Beginning of the test period.

Figure 7. Fuzzy strategy cumulative returns for each D value.

As observed in Figure 7, if the D value is too small, then the profitability of the strategy is similar to that obtained with classical logic. On the other hand, if the D value are too large, then the profitability falls.

The profitability of fuzzy logic (using $D = 0.2\%$) is superior to classical logic ($D = 0\%$), with a one-tailed t statistics of 1.88, significant at the 5% level.

5. CONCLUSION

This article presents a new approach, based on fuzzy logic, for forecasting the direction of movements of the S&P 500 index. The period from 1970 to 2009 was used to establish the rules of the model, and the period from 1976 to 2010 was used to test the model.

The proposed model returned a non-exact answer, with a probabilistic output. Despite this imprecision, it was possible to estimate a statistically significant (5%)

buying-and-selling strategy that outperformed a buy-and-hold (passive) one for the test period. The fuzzy-based strategy outperformed the passive strategy. Further, the success rate calculated was statistically significant (1%).

Why does the fuzzy strategy outperform the passive one? The strategy seeks to avoid financial loss on days where the probability of the stock index fall is high. Therefore, the strategy is particularly profitable in the years of crises, when the S&P 500 exhibits a negative return. For example, after the Lemman Brothers collapse.

However, the fuzzy logic model outperforms the application of a similar classical logic model. Fuzzy logic outperformed classical logic at the 5% significance level.

The proposed model, with its probabilistic output, can be used as a support to investment decisions, as investors could have other information, secrets, or otherwise or even intuitions about political influences or economic tendencies. For example, if the “down” possibility (indicated by the fuzzy model) is greater than ε the money should be withdrawn from the fund, however the investor can use complementary information and decides do not withdrawn. In other words, the linguistic output does not force the investor follow blindly the strategy.

There are many new research possibilities that could be derived from this work. One of them is the choice of new input variables and linguistic terms. For example, the procedure could be applied to weekly and monthly forecasts that could bring good results and reduce transaction costs. Another possibility would be the inclusion in the model of some well-known stock market anomalies (e.g. day-of-the-week or month-of-the-year effects), as well as rules from technical analysis. In addition, other artificial intelligence techniques, such as ANN, could be associated with the models based on fuzzy logic.

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